



Oxford Cambridge and RSA

# A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

**Friday 15 June 2018 – Afternoon**

**Time allowed: 2 hours**



**You must have:**

- Printed Answer Booklet

**You may use:**

- a scientific or graphical calculator

## INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by  $g\text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

## INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [ ].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **8** pages.

## Formulae

### A Level Mathematics A (H240)

#### Arithmetic series

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

#### Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

#### Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

#### Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$$\text{Quotient rule } y = \frac{u}{v}, \quad \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

#### Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

#### Integration

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

$$\text{Integration by parts } \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

#### Small angle approximations

$\sin \theta \approx \theta$ ,  $\cos \theta \approx 1 - \frac{1}{2}\theta^2$ ,  $\tan \theta \approx \theta$  where  $\theta$  is measured in radians

**Trigonometric identities**

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi\right)$$

**Numerical methods**

Trapezium rule:  $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$ , where  $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving  $f(x) = 0$ :  $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

**Probability**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B) \quad \text{or} \quad P(A|B) = \frac{P(A \cap B)}{P(B)}$$

**Standard deviation**

$$\sqrt{\frac{\sum(x-\bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

**The binomial distribution**

If  $X \sim B(n, p)$  then  $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$ , Mean of  $X$  is  $np$ , Variance of  $X$  is  $np(1-p)$

**Hypothesis test for the mean of a normal distribution**

If  $X \sim N(\mu, \sigma^2)$  then  $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$  and  $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

**Percentage points of the normal distribution**

If  $Z$  has a normal distribution with mean 0 and variance 1 then, for each value of  $p$ , the table gives the value of  $z$  such that  $P(Z \leq z) = p$ .

$p$	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
$z$	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

**Kinematics**

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

**Section A: Pure Mathematics**Answer **all** the questions.

- 1 A circle with centre  $C$  has equation  $x^2 + y^2 + 8x - 2y - 7 = 0$ .

Find

- (i) the coordinates of  $C$ , [2]  
(ii) the radius of the circle. [1]

- 2 Solve the equation  $|2x - 1| = |x + 3|$ . [3]

- 3 **In this question you must show detailed reasoning.**

A gardener is planning the design for a rectangular flower bed. The requirements are:

- the length of the flower bed is to be 3 m longer than the width,
- the length of the flower bed must be at least 14.5 m,
- the area of the flower bed must be less than  $180\text{m}^2$ .

The width of the flower bed is  $x$  m.

By writing down and solving appropriate inequalities in  $x$ , determine the set of possible values for the width of the flower bed. [6]

- 4 **In this question you must show detailed reasoning.**

The functions  $f$  and  $g$  are defined for all real values of  $x$  by

$$f(x) = x^3 \quad \text{and} \quad g(x) = x^2 + 2.$$

- (i) Write down expressions for
- (a)  $fg(x)$ , [1]  
(b)  $gf(x)$ . [1]
- (ii) Hence find the values of  $x$  for which  $fg(x) - gf(x) = 24$ . [6]

- 5 (i) Use the trapezium rule, with two strips of equal width, to show that

$$\int_0^4 \frac{1}{2 + \sqrt{x}} dx \approx \frac{11}{4} - \sqrt{2}. \quad [5]$$

- (ii) Use the substitution  $x = u^2$  to find the exact value of

$$\int_0^4 \frac{1}{2 + \sqrt{x}} dx. \quad [6]$$

- (iii) Using your answers to parts (i) and (ii), show that

$$\ln 2 \approx k + \frac{\sqrt{2}}{4},$$

where  $k$  is a rational number to be determined. [2]

- 6 It is given that the angle  $\theta$  satisfies the equation  $\sin\left(2\theta + \frac{1}{4}\pi\right) = 3 \cos\left(2\theta + \frac{1}{4}\pi\right)$ .

(i) Show that  $\tan 2\theta = \frac{1}{2}$ . [3]

- (ii) Hence find, in surd form, the exact value of  $\tan \theta$ , given that  $\theta$  is an obtuse angle. [5]

- 7 The gradient of the curve  $y = f(x)$  is given by the differential equation

$$(2x - 1)^3 \frac{dy}{dx} + 4y^2 = 0$$

and the curve passes through the point  $(1, 1)$ . By solving this differential equation show that

$$f(x) = \frac{ax^2 - ax + 1}{bx^2 - bx + 1},$$

where  $a$  and  $b$  are integers to be determined. [9]

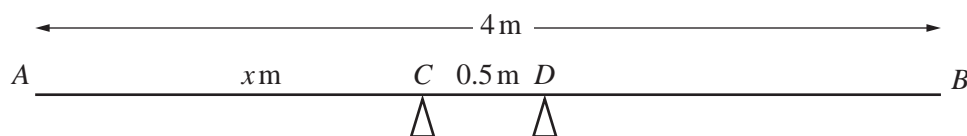
## Section B: Mechanics

Answer **all** the questions.

- 8 In this question  $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$  and  $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$  denote unit vectors which are horizontal and vertically upwards respectively.

A particle of mass 5 kg, initially at rest at the point with position vector  $\begin{pmatrix} 2 \\ 45 \end{pmatrix}$  m, is acted on by gravity and also by two forces  $\begin{pmatrix} 15 \\ -8 \end{pmatrix}$  N and  $\begin{pmatrix} -7 \\ -2 \end{pmatrix}$  N.

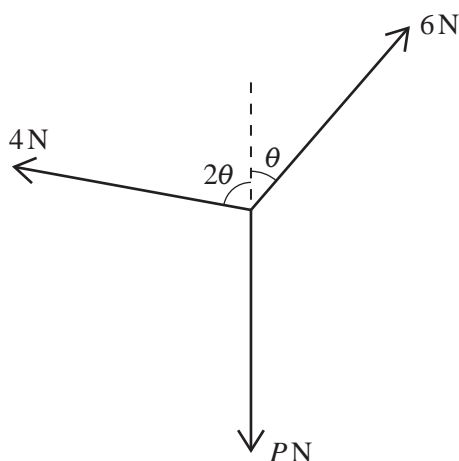
- (i) Find the acceleration vector of the particle. [3]
- (ii) Find the position vector of the particle after 10 seconds. [3]
- 9 A uniform plank  $AB$  has weight 100 N and length 4 m. The plank rests horizontally in equilibrium on two smooth supports  $C$  and  $D$ , where  $AC = x$  m and  $CD = 0.5$  m (see diagram).



The magnitude of the reaction of the support on the plank at  $C$  is 75 N. Modelling the plank as a rigid rod, find

- (i) the magnitude of the reaction of the support on the plank at  $D$ , [1]
- (ii) the value of  $x$ . [3]
- A stone block, which is modelled as a particle, is now placed at the end of the plank at  $B$  and the plank is on the point of tilting about  $D$ .
- (iii) Find the weight of the stone block. [3]
- (iv) Explain the limitation of modelling
- (a) the stone block as a particle, [1]
- (b) the plank as a rigid rod. [1]

- 10 Three forces, of magnitudes 4 N, 6 N and  $P$  N, act at a point in the directions shown in the diagram.



The forces are in equilibrium.

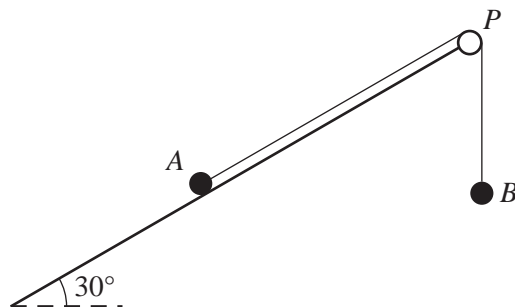
- (i) Show that  $\theta = 41.4^\circ$ , correct to 3 significant figures. [4]
- (ii) Hence find the value of  $P$ . [2]

The force of magnitude 4 N is now removed and the force of magnitude 6 N is replaced by a force of magnitude 3 N acting in the same direction.

- (iii) Find
- (a) the magnitude of the resultant of the two remaining forces, [3]
- (b) the direction of the resultant of the two remaining forces. [2]

- 11 The velocity  $v \text{ m s}^{-1}$  of a car at time  $t$  s, during the first 20 s of its journey, is given by  $v = kt + 0.03t^2$ , where  $k$  is a constant. When  $t = 20$  the acceleration of the car is  $1.3 \text{ m s}^{-2}$ . For  $t > 20$  the car continues its journey with constant acceleration  $1.3 \text{ m s}^{-2}$  until its speed reaches  $25 \text{ m s}^{-1}$ .
- (i) Find the value of  $k$ . [3]
- (ii) Find the total distance the car has travelled when its speed reaches  $25 \text{ m s}^{-1}$ . [7]

- 12 One end of a light inextensible string is attached to a particle  $A$  of mass  $m$  kg. The other end of the string is attached to a second particle  $B$  of mass  $\lambda m$  kg, where  $\lambda$  is a constant. Particle  $A$  is in contact with a rough plane inclined at  $30^\circ$  to the horizontal. The string is taut and passes over a small smooth pulley  $P$  at the top of the plane. The part of the string from  $A$  to  $P$  is parallel to a line of greatest slope of the plane. The particle  $B$  hangs freely below  $P$  (see diagram).



The coefficient of friction between  $A$  and the plane is  $\mu$ .

- (i) It is given that  $A$  is on the point of moving down the plane.
- (a) Find the exact value of  $\mu$  when  $\lambda = \frac{1}{4}$ . [7]
- (b) Show that the value of  $\lambda$  must be less than  $\frac{1}{2}$ . [2]
- (ii) Given instead that  $\lambda = 2$  and that the acceleration of  $A$  is  $\frac{1}{4}g \text{ m s}^{-2}$ , find the exact value of  $\mu$ . [5]

**END OF QUESTION PAPER**

# OCR

Oxford Cambridge and RSA

## Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website ([www.ocr.org.uk](http://www.ocr.org.uk)) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.