



Oxford Cambridge and RSA

Wednesday 5 June 2019 – Morning**A Level Mathematics A****H240/01 Pure Mathematics****Time allowed: 2 hours****You must have:**

- Printed Answer Booklet

You may use:

- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **8** pages.

Formulae

A Level Mathematics A (H240)

Arithmetic series

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N})$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$$\text{Quotient rule } y = \frac{u}{v}, \quad \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

$$\text{Integration by parts } \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Small angle approximations

$$\sin \theta \approx \theta, \quad \cos \theta \approx 1 - \frac{1}{2}\theta^2, \quad \tan \theta \approx \theta \quad \text{where } \theta \text{ is measured in radians}$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi\right)$$

Numerical methods

Trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B) \quad \text{or} \quad P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Standard deviation

$$\sqrt{\frac{\sum(x - \bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np , variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ and $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p , the table gives the value of z such that $P(Z \leq z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

Answer **all** the questions.

1 In this question you must show detailed reasoning.

Solve the inequality $10x^2 + x - 2 > 0$.

[4]

Factorising $10x^2 + x - 2 = (5x - 2)(2x + 1)$

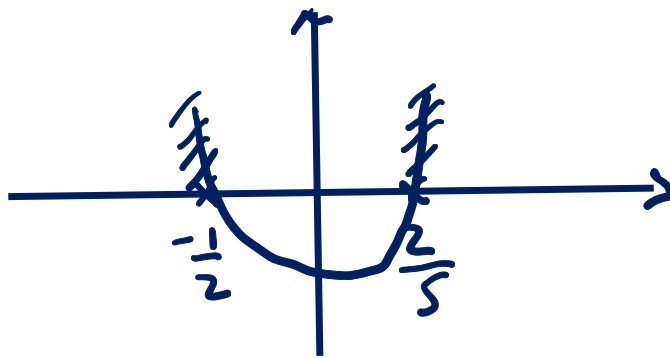
$$(5x - 2)(2x + 1) > 0$$

The roots of the equation are;

$$(5x - 2)(2x + 1) = 0$$

$$x = \frac{2}{5} \quad x = -\frac{1}{2}$$

Solve
Sketch
Range



→ From the diagram above the values of x that satisfy the inequality are;

$$x < -\frac{1}{2} \quad \text{and} \quad x > \frac{2}{5}$$

2 The point A is such that the magnitude of $\vec{OA}$ is 8 and the direction of $\vec{OA}$ is 240° .

(a) (i) Show the point A on the axes provided in the Printed Answer Booklet. [1]

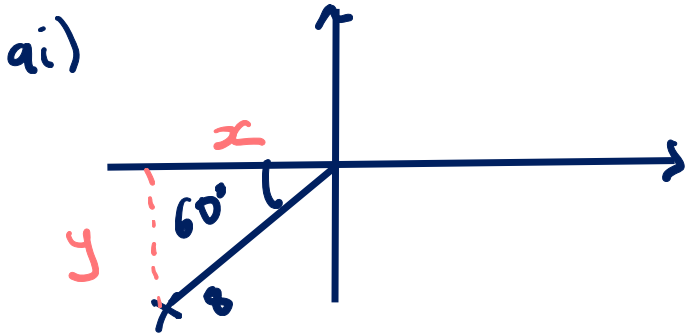
(ii) Find the position vector of point A .
Give your answer in terms of $\mathbf{i}$ and $\mathbf{j}$. [3]

The point B has position vector $6\mathbf{i}$.

(b) Find the exact area of triangle AOB . [2]

The point C is such that $OABC$ is a parallelogram.

(c) Find the position vector of C .
Give your answer in terms of $\mathbf{i}$ and $\mathbf{j}$. [2]



ii) Finding x and y components

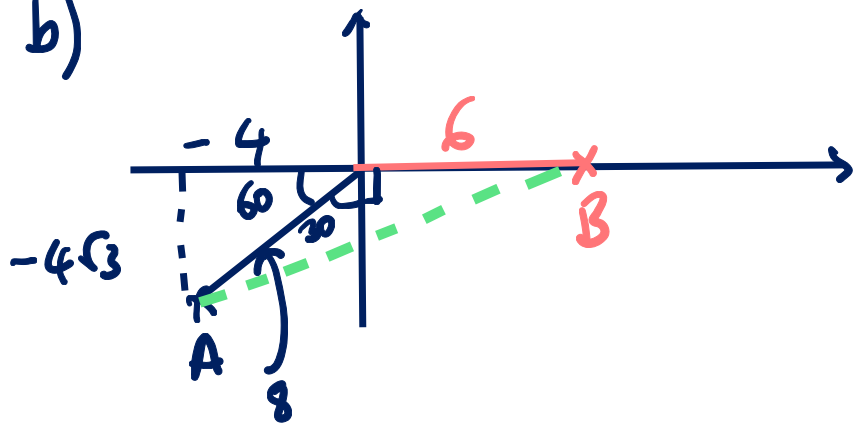
$$x = 8 \cos 60 = 4$$

$$y = 8 \sin 60 = 8 \times \frac{\sqrt{3}}{2} = 4\sqrt{3}$$

* Since they are in the 3rd quadrant both $\mathbf{i}$ and $\mathbf{j}$ components are negative *

$$\Rightarrow -4\mathbf{i} - 4\sqrt{3}\mathbf{j}$$

b)



$$* \text{Area} = \frac{1}{2} ab \sin \theta *$$

$$\Rightarrow \frac{1}{2} \times 8 \times 6 \times \sin(90 + 30)$$

$$= 12\sqrt{3} \text{ units}^2$$

$$(c) \Rightarrow 6i - (-4i - 4\sqrt{3}j) \quad (\text{as } \vec{AO} = \vec{CB})$$

$$= 10i + 4\sqrt{3}j.$$

3 The function f is defined by $f(x) = (x-3)^2 - 17$ for $x \geq k$, where k is a constant.

(a) Given that $f^{-1}(x)$ exists, state the least possible value of k . [1]

(b) Evaluate $ff(5)$. [2]

(c) Solve the equation $f(x) = x$. [3]

(d) Explain why your solution to part (c) is also the solution to the equation $f(x) = f^{-1}(x)$. [1]

$$a) \quad x-3=0 \quad x=3 \quad \therefore k=3$$

$$b) \quad f(5) = (5-3)^2 - 17 \Rightarrow (2)^2 - 17 \Rightarrow 4 - 17 = -13$$

$\Rightarrow -13$ is not in the domain
so $f(-13)$ i.e. $ff(5)$ cannot be defined.

$$\begin{aligned} (c) \quad (x-3)^2 - 17 &= x \\ x^2 - 6x + 9 - 17 &= x \\ x^2 - 6x - 8 &= x \\ x^2 - 7x - 8 &= 0 \\ \text{factorising this;} \end{aligned}$$

$$(x-8)(x+1) = 0 \quad \therefore x = 8, -1$$

this is not a valid soln. as $x \geq 3$

$$\therefore x = 8 \text{ only.}$$

d) $f(x)$ and $f^{-1}(x)$ are reflections on the line $y=x$, so the point of intersection must be on the line $y=x$.

- 4 Sam starts a job with an annual salary of £16 000. It is promised that the salary will go up by the same amount every year. In the second year Sam is paid £17 200.
- (a) Find Sam's salary in the tenth year. [2]
- (b) Find the number of complete years needed for Sam's **total** salary to first exceed £500 000. [4]
- (c) Comment on how realistic this model may be in the long term. [1]

a) This is an arithmetic progression with;

$$a = 16,000$$

$$d = 1200 \quad (17,200 - 16,000)$$

$$u_n = a + d(n-1) \quad \therefore u_{10} = 16,000 + 1200(10-1) = \underline{26,000}$$

b) $S_n > 500,000$

$$S_n = \frac{n}{2} [2a + d(n-1)]$$

$$\frac{n}{2} [2(16,000) + 1200(n-1)] > 500,000$$

$$\frac{n}{2} [32,000 + 1200n - 1200] > 500,000$$

$$n[30,800 + 1200n] > 1,000,000$$

$$1200n^2 + 30,800n - 1,000,000 > 0$$

↑ equating this to zero gives; $n = 18.8, (\text{or } -44.4)$
 $\therefore \underline{n = 19}$

c) Unrealistic - as Sam is unlikely to stay in the same role that long.

5 A curve has equation $x^3 - 3x^2y + y^2 + 1 = 0$.

(a) Show that $\frac{dy}{dx} = \frac{6xy - 3x^2}{2y - 3x^2}$. [4]

(b) Find the equation of the normal to the curve at the point (1, 2). [4]

a) Using implicit differentiation;

$$\frac{dy}{dx} = 3x^2 - 3x^2 \cdot \frac{dy}{dx} - 6xy + 2y \cdot \frac{dy}{dx} = 0.$$

Bringing like terms together:

$$2y \cdot \frac{dy}{dx} - 3x^2 \cdot \frac{dy}{dx} = 6xy - 3x^2$$

$$\frac{dy}{dx} (2y - 3x^2) = 6xy - 3x^2$$

$$\frac{dy}{dx} = \frac{6xy - 3x^2}{2y - 3x^2} \quad \text{as required.}$$

$$b) \left. \frac{dy}{dx} \right|_{\substack{x=1 \\ y=2}} = \frac{6(1)(2) - 3(1)^2}{2(2) - 3(1)^2} = \frac{9}{1} = 9. \quad \text{gradient of tangent}$$

$$\therefore \text{gradient of normal} = -\frac{1}{9}.$$

$$y - y_0 = m(x - x_0)$$

$$y - 2 = -\frac{1}{9}(x - 1)$$

$$y = -\frac{1}{9}x + \frac{1}{9} + 2 \Rightarrow y = -\frac{1}{9}x + \frac{19}{9}$$

$$x + 9y = 19$$

6 Let $f(x) = 2x^3 + 3x$. Use differentiation from first principles to show that $f'(x) = 6x^2 + 3$. [6]

$$(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$$

$$f(x+h) - f(x) =$$

$$[2(x+h)^3 + 3(x+h)] - [2x^3 + 3x]$$

↑ From above

$$[2(x^3 + 3x^2h + 3xh^2 + h^3) + 3x + 3h] - 2x^3 - 3x$$

Expanding

$$~~2x^3~~ + 6x^2h + 6xh^2 + 2h^3 + ~~3x~~ + 3h - ~~2x^3~~ - ~~3x~~$$

$$\Rightarrow 6x^2h + 6xh^2 + 2h^3 + 3h$$

$$\frac{f(x+h) - f(x)}{h} = \frac{6x^2h + 6xh^2 + 2h^3 + 3h}{h}$$

$$\Rightarrow \frac{\cancel{h}(6x^2 + 6xh + 2h + 3)}{\cancel{h}} = 6x^2 + 6xh + 2h + 3$$

BUT $f'(x)$ is when $h \rightarrow 0$

$$\therefore f'(x) = 6x^2 + 6x(0) + 2(0) + 3$$

$$= 6x^2 + 3 \text{ as required.}$$

7 In this question you must show detailed reasoning.

A sequence $u_1, u_2, u_3 \dots$ is defined by $u_n = 25 \times 0.6^n$.

Use an algebraic method to find the smallest value of N such that $\sum_{n=1}^{\infty} u_n - \sum_{n=1}^N u_n < 10^{-4}$. [8]

This is a geometric progression with;

$$a = 25$$

$$r = 0.6$$

Sum to infinity

$$\frac{a}{1-r} = \frac{25}{1-0.6} = \frac{125}{2}$$

Sum from $1 \rightarrow N$

$$\frac{a(1-r^N)}{1-r} = \frac{25(1-0.6^N)}{1-0.6}$$

$$= \frac{125}{2} (1-0.6^N)$$

$$\frac{125}{2} - \frac{125}{2} (1-0.6^N) < 10^{-4}$$

$$\cancel{\frac{125}{2}} - \cancel{\frac{125}{2}} + \frac{125}{2} \cdot 0.6^N < 10^{-4}$$

$$\Rightarrow 0.6^N < 10^{-4} \times \frac{2}{125} \Rightarrow 0.6^N < \frac{1}{625000}$$

$$N > \frac{\log(1/625000)}{\log(0.6)} \Rightarrow N > 25.125 \dots$$

hence $N = 26$

- 8 A cylindrical tank is initially full of water. There is a small hole at the base of the tank out of which the water leaks.

The height of water in the tank is x m at time t seconds. The rate of change of the height of water may be modelled by the assumption that it is proportional to the square root of the height of water.

When $t = 100$, $x = 0.64$ and, at this instant, the height is decreasing at a rate of 0.0032 ms^{-1} .

(a) Show that $\frac{dx}{dt} = -0.004\sqrt{x}$. [2]

(b) Find an expression for x in terms of t . [4]

(c) Hence determine at what time, according to this model, the tank will be empty. [2]

a) $\frac{dx}{dt} \propto \sqrt{x}$

$\therefore \frac{dx}{dt} = k\sqrt{x}$

$\uparrow$
 0.64

$\uparrow$
 -0.0032

$\uparrow$ Because height is decreasing

$-0.0032 = k\sqrt{0.64}$

$k = \frac{-0.0032}{\sqrt{0.64}}$

$= -0.004$

$\therefore \frac{dx}{dt} = -0.004\sqrt{x}$
as required.

b) $\int \frac{dx}{\sqrt{x}} = \int -0.004 dt$

$\int x^{-1/2} dx = \int -0.004 dt$

$\frac{x^{1/2}}{1/2} = -0.004t + c$

$2x^{1/2} = -0.004t + c$

$x^{1/2} = -0.002t + c$

When $x = 0.64$ $t = 100$

$(0.64)^{1/2} = -0.002(100) + c$
 $c = 1$

$x^{1/2} = -0.002t + 1$

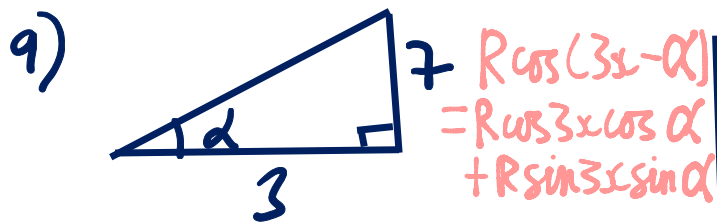
$x = (1 - 0.002t)^2$

(c) $x = 0$

$1 = 0.002t$

$t = 500 \text{ s}$

- 9 (a) Express $3 \cos 3x + 7 \sin 3x$ in the form $R \cos(3x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. [3]
- (b) Give full details of a sequence of three transformations needed to transform the curve $y = \cos x$ to the curve $y = 3 \cos 3x + 7 \sin 3x$. [4]
- (c) Determine the **greatest** value of $3 \cos 3x + 7 \sin 3x$ as x varies and give the smallest positive value of x for which it occurs. [2]
- (d) Determine the **least** value of $3 \cos 3x + 7 \sin 3x$ as x varies and give the smallest positive value of x for which it occurs. [2]



$$R^2 = 7^2 + 3^2$$

$$R^2 = 58$$

$$R = \sqrt{58}$$

$$\tan \alpha = \frac{7}{3} \quad \alpha = 1.17$$

$3 \leftarrow R \cos \alpha$

$$\therefore \sqrt{58} \cos(3x - 1.17)$$

b) → Stretch in the y direction by s.f $\sqrt{58}$
 → Translation in the x -direction by 1.17
 → Stretch in the x direction by s.f $\frac{1}{3}$.

c) The greatest value $\cos$ can take is 1
 $\therefore$ the greatest value
 $= \sqrt{58}$.

This occurs when

$$3x - 1.17 = 0$$

$$x = 0.39$$

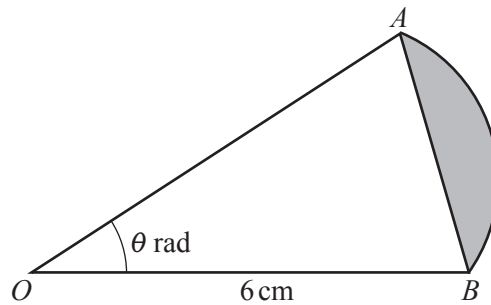
d) The least value $\cos$ can take is -1
 $\therefore$ the least value
 $= -\sqrt{58}$

This occurs when

$$3x - 1.17 = \pi$$

$$x = \frac{\pi + 1.17}{3} = 1.44$$

10



The diagram shows a sector AOB of a circle with centre O and radius 6 cm.

The angle AOB is θ radians.

The area of the segment bounded by the chord AB and the arc AB is 7.2 cm^2 .

(a) Show that $\theta = 0.4 + \sin \theta$. [3]

(b) Let $F(\theta) = 0.4 + \sin \theta$.

By considering the value of $F'(\theta)$ where $\theta = 1.2$, explain why using an iterative method based on the equation in part (a) will converge to the root, assuming that 1.2 is sufficiently close to the root. [2]

(c) Use the iterative formula $\theta_{n+1} = 0.4 + \sin \theta_n$ with a starting value of 1.2 to find the value of θ correct to 4 significant figures. You should show the result of each iteration. [3]

(d) Use a change of sign method to show that the value of θ found in part (c) is correct to 4 significant figures. [3]

a) Area of segment = Area of sector - Area of triangle.

$\Rightarrow$ Area of sector

$$\frac{1}{2} r^2 \theta = \frac{1}{2} \times 6^2 \times \theta = 18\theta$$

Area of triangle

$$\frac{1}{2} ab \sin \theta = \frac{1}{2} \times 6 \times 6 \times \sin \theta = 18 \sin \theta$$

$$18\theta - 18 \sin \theta = 7.2 \Rightarrow \frac{18(\theta - \sin \theta)}{18} = \frac{7.2}{18}$$

$$\theta - \sin \theta = 0.4 \Rightarrow \theta = 0.4 + \sin \theta \text{ as required.}$$

$$b) F'(1.2) = \cos(1.2)$$

$|F'(1.2)| < 1 \therefore$ iteration will converge.

$$c) \theta_{n+1} = 0.4 + \sin \theta_n$$

$$\begin{array}{l} n=1 \\ \theta_2 = 0.4 + \sin \theta_1 = 0.4 + \sin(1.2) = 1.3320 \dots \end{array}$$

$$\begin{array}{l} n=2 \\ \theta_3 = 0.4 + \sin \theta_2 = 0.4 + \sin(1.3320 \dots) = 1.3716 \dots \end{array}$$

Repeating this process gives

$$\Rightarrow 1.3802, 1.3819, 1.3822, 1.3823$$

$$\Rightarrow \theta = 1.382 \text{ (4sf)}$$

$$d) \begin{array}{l} \text{Upper bound} = 1.3825 \\ \text{Lower bound} = 1.3815 \end{array}$$

$$f(1.3825) = -0.000637$$

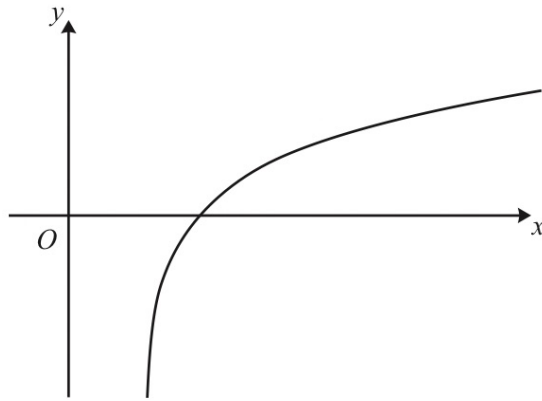
$$f(1.3815) = 0.000175$$

$\rightarrow$ The change in sign in the interval

$1.3815 < \theta < 1.3825$, it shows us $\theta = 1.382$

to 4.s.f.

11



The diagram shows part of the curve $y = \ln(x-4)$.

- (a) Use integration by parts to show that $\int \ln(x-4) dx = (x-4)\ln|x-4| - x + c$. [5]
- (b) State the equation of the vertical asymptote to the curve $y = \ln(x-4)$. [1]
- (c) Find the total area enclosed by the curve $y = \ln(x-4)$, the x -axis and the lines $x = 4.5$ and $x = 7$. Give your answer in the form $a\ln 3 + b\ln 2 + c$ where a , b and c are constants to be found. [4]

$$a) \int 1 \cdot \ln(x-4) dx$$

$$u = \ln(x-4)$$

$$u' = \frac{1}{x-4}$$

$$v' = 1$$

$$v = x$$

Integration by parts formula.

$$uv - \int v u' dx$$

$$x \ln|x-4| - \int \frac{x}{x-4} dx.$$

↑
need to turn this into
a proper fraction

$$\frac{x}{x-4} \equiv A + \frac{B}{x-4}$$

$$\frac{A(x-4) + B}{x-4} \equiv \frac{x}{x-4}$$

$$A(x-4) + B = x$$

$$\text{let } x = 4$$

$$4 = B.$$

$$\text{let } x = 0$$

$$-4A + B = 0$$

$$\text{But } B = 4$$

$$-4A + 4 = 0$$

$$A = 1$$

$$\therefore \Rightarrow 1 + \frac{4}{x-4}$$

$$x \ln|x-4| - \int \left(1 + \frac{4}{x-4}\right) dx$$

$$\Rightarrow x \ln|x-4| - x + 4 \ln|x-4| + C$$

Bringing like terms together.

$$(x-4) \ln|x-4| - x + C \quad \text{as required.}$$

b) $x=4$

c) $\left| \int_5^7 \ln(x-4) dx \right| = \int_{4.5}^5 \ln(x-4) dx$
As below x-axis

$$\left[(x-4) \ln|x-4| - x \right]_5^7 - \left[(x-4) \ln|x-4| - x \right]_{4.5}^5$$

$$(3\ln 3 - 7) - (\underbrace{1\ln 1}_{=0} - 5) - (\underbrace{1\ln 1}_{=0} - 5) + \left(-\frac{1}{2}\ln \frac{1}{2} - \frac{9}{2}\right)$$

$$= 3\ln 3 - \frac{1}{2}\ln \frac{1}{2} - \frac{3}{2}$$

12 A curve has equation $y = a^{3x^2}$, where a is a constant greater than 1.

(a) Show that $\frac{dy}{dx} = 6xa^{3x^2} \ln a$. [3]

(b) The tangent at the point $(1, a^3)$ passes through the point $(\frac{1}{2}, 0)$.

Find the value of a , giving your answer in an exact form. [4]

(c) By considering $\frac{d^2y}{dx^2}$ show that the curve is convex for all values of x . [5]

$$a) \text{ let } 3x^2 = v \Rightarrow \frac{dv}{dx} = 6x$$

$$y = a^v \Rightarrow \frac{dy}{dv} = a^v \ln a$$

Using chain rule

$$\frac{dy}{dx} = \frac{dy}{dv} \times \frac{dv}{dx}$$

$$= a^{\overset{3x^2}{v}} \ln a \times 6x$$

$$= a^{3x^2} \ln a \times 6x$$

$$= 6x a^{3x^2} \ln a \text{ as required.}$$

$$b) \frac{dy}{dx} \Big|_{x=1} = 6(1) a^{3(1)^2} \ln a$$

$$= 6a^3 \ln a.$$

$$y - y_0 = m(x - x_0)$$

$$y - a^3 = 6a^3 \ln a (x - 1)$$

$$y = 6a^3 \ln a x - 6a^3 \ln a + a^3.$$

↑ passes through $(\frac{1}{2}, 0)$

$$0 = 6a^3 \ln a \left(\frac{1}{2}\right) - 6a^3 \ln a + a^3$$

$$0 = 3a^3 \ln a - 6a^3 \ln a + a^3.$$

$$0 = a^3 - 3a^3 \ln a.$$

$$0 = a^3 (1 - 3 \ln a)$$

$$a^3 = 0 \quad a = 0$$

OR.

$$1 - 3 \ln a = 0 \Rightarrow \ln a = \frac{1}{3} \quad a = e^{1/3}$$

$$\begin{aligned}
 \text{c) } \frac{dy}{dx} &= 6x a^{3x^2} \ln a & \text{but } a &= e^{1/3} \\
 &= 6x e^{x^2} \ln e^{1/3} & &= 6x e^{x^2} \times \frac{1}{3} \\
 & & &= 2x e^{x^2}
 \end{aligned}$$

$$\frac{d^2y}{dx^2} = ?$$

using product rule ;

$$\begin{aligned}
 &2x \cdot 2x e^{x^2} + 2e^{x^2} \\
 &= 4x^2 e^{x^2} + 2e^{x^2}
 \end{aligned}$$

$$\begin{aligned}
 \rightarrow e^{x^2} &> 0 \quad \text{for all values of } x, \text{ so} \\
 2e^{x^2} &> 0.
 \end{aligned}$$

$$\begin{aligned}
 \rightarrow x^2 &\geq 0 \quad \text{for all values of } x, \text{ so} \\
 4x^2 e^{x^2} &\geq 0
 \end{aligned}$$

$$\therefore 4x^2 e^{x^2} + 2e^{x^2} \geq 0$$

$$\therefore \frac{d^2y}{dx^2} > 0 \quad \text{for all } x. \therefore \text{the curve is always convex}$$