Oxford Cambridge and RSA

Wednesday 07 October 2020 – Afternoon

AS Level Mathematics B (MEI)

H630/01 Pure Mathematics and Mechanics

Time allowed: 1 hour 30 minutes

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **70**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

Formulae AS Level Mathematics B (MEI) (H630)

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1} b + {}^n\mathrm{C}_2 a^{n-2} b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r} b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Sample variance

$$s^2 = \frac{1}{n-1} S_{xx} \text{ where } S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = r) = {}^n\mathrm{C}_r p^r q^{n-r}$ where $q = 1 - p$

Mean of X is np

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

1 Celia states that $n^2 + 2n + 10$ is always odd when n is a prime number.

Prove that Celia's statement is false. **[2]**

Let
$n = 2$

$(2)^2 + 2(2) + 10$

$= 18$ which is not an odd number

$\therefore$ Celia is wrong.

2 Fig. 2 shows a quadrilateral ABCD. The lengths AB and BC are 5 cm and 6 cm respectively. The angles ABC, ACD and DAC are 60°, 60° and 75° respectively.

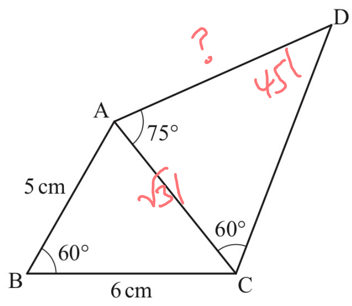


Fig. 2

Calculate the exact value of the length AD. **[4]**

① Finding AC using cosine rule

$a^2 = b^2 + c^2 - (2bc\cos A)$

$AC^2 = 5^2 + 6^2 - (2 \times 5 \times 6 \cos 60)$

$AC^2 = 31$

$AC = \sqrt{31}$

② $A\hat{D}C = 180 - (75+60)$

$180 - 135 = 45$

② Using sine Rule Find AD

$\frac{\sin A}{A} = \frac{\sin B}{B}$

$$\frac{\sin 45}{\sqrt{31}} = \frac{\sin 60}{x}$$

$$x = \frac{\sin 60 \times \sqrt{31}}{\sin 45}$$

$$= \frac{\sqrt{186}}{2}$$

3 Fig. 3 shows a triangle PQR. The vector $\overrightarrow{PQ}$ is $\mathbf{i}+7\mathbf{j}$ and the vector $\overrightarrow{QR}$ is $4\mathbf{i}-12\mathbf{j}$.

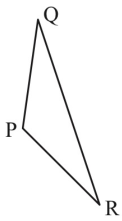


Fig. 3

(a) Show that the triangle PQR is isosceles. **[3]**

The point P has position vector $-3\mathbf{i}-\mathbf{j}$. The point S is added so that PQRS is a parallelogram.

(b) Find the position vector of S. **[2]**

a) $\overrightarrow{PQ} = i + 7j$

$\overrightarrow{QR} = 4i - 12j$

$\therefore \overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR}$

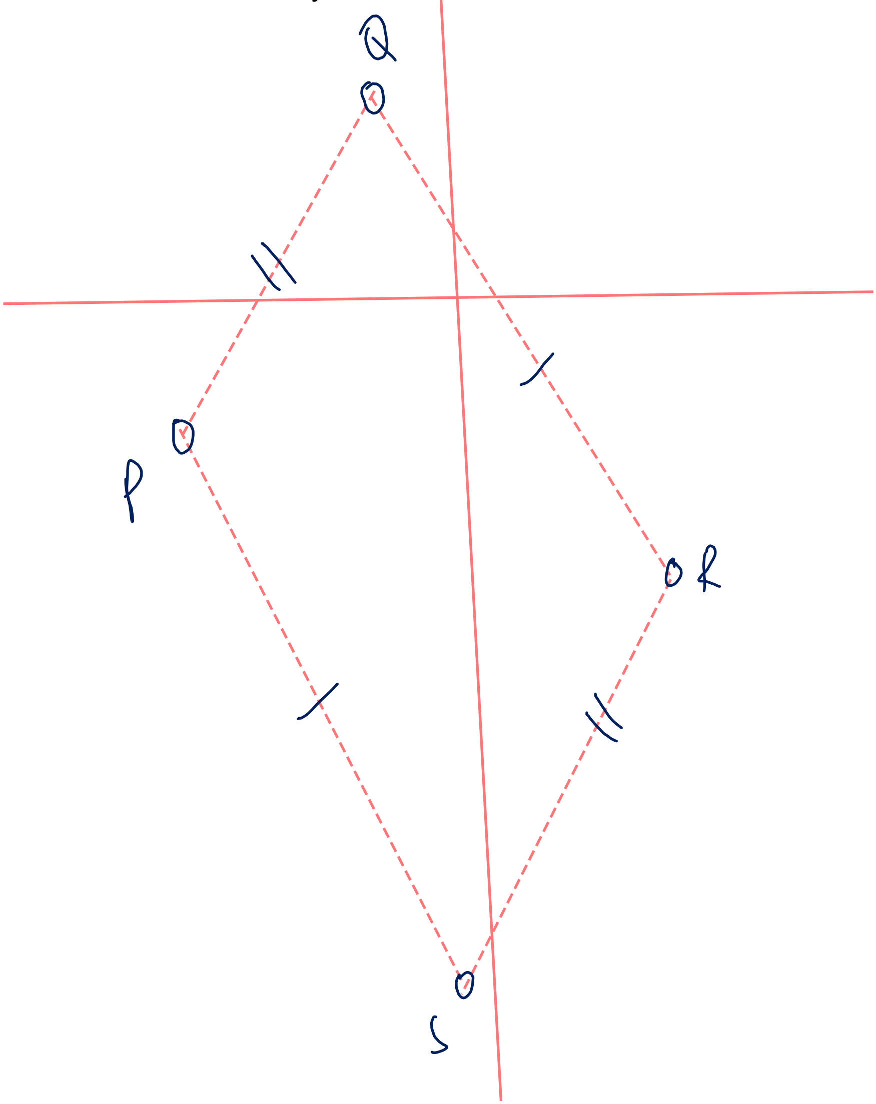
$= i + 7j + 4i - 12j$

$= 5i - 5c$

$|\overrightarrow{PQ}| = \sqrt{(1)^2 + (7)^2} = \sqrt{50}$

$|\overrightarrow{PR}| = \sqrt{(5)^2 + (-5)^2} = \sqrt{50}$

$\therefore$ since $|\overrightarrow{PQ}| = |\overrightarrow{PR}|$ the Δ is isosceles b)



If PQRS is a parallelogram, $\overrightarrow{QR} = \overrightarrow{PS}$

$\overrightarrow{OS} = \overrightarrow{OP} + \overrightarrow{PS}$

$$-3i - j + \overrightarrow{QR} = \overrightarrow{OS}$$

$$-3i - j + 4i - 12j$$

$$\boxed{i - 13j} = \overrightarrow{OS}$$

4 In this question, the x and y directions are horizontal and vertically upwards respectively.

A particle of mass 1.5 kg is in equilibrium under the action of its weight and forces $\mathbf{F}_1 = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$N and $\mathbf{F}_2$.

(a) Find the force $\mathbf{F}_2$. **[3]**

The force $\mathbf{F}_2$ is changed to $\begin{pmatrix} 2 \\ 20 \end{pmatrix}$N.

(b) Find the acceleration of the particle. **[2]**

a) Weight only acts downwards ∴ -ve

$$\therefore F_1 + F_2 + \text{Weight} = 0$$

$$\begin{pmatrix} 4 \\ -2 \end{pmatrix} + \begin{pmatrix} 0 \\ -1.5g \end{pmatrix} + F_2 = 0$$

$$\begin{pmatrix} 4 \\ -2-1.5g \end{pmatrix} + F_2 = 0$$

$$F_2 = \begin{pmatrix} -4 \\ 2+1.5g \end{pmatrix} = \begin{pmatrix} -4 \\ 16.7 \end{pmatrix} N$$

b) $F = ma$

$$\begin{pmatrix} 4 \\ -2 \end{pmatrix} + \begin{pmatrix} 0 \\ -1.5g \end{pmatrix} + \begin{pmatrix} 2 \\ 20 \end{pmatrix} = 1.5 \times a$$

$$\Rightarrow \begin{pmatrix} 6 \\ -16.7 + 20 \end{pmatrix} = 1.5 \times a$$

$$\frac{1}{1.5}\begin{pmatrix} 6 \\ 3.3 \end{pmatrix} = a$$

$$a = \begin{pmatrix} 4 \\ 2.2 \end{pmatrix} \text{ ms}^{-2}$$

5 Fig. 5.1 shows part of the curve $y = x^{\frac{1}{2}}$. P is the point (1, 1) and Q is the point on the curve with x-coordinate $1 + h$.

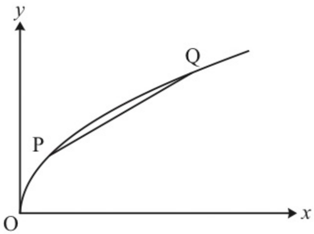


Fig. 5.1

Table 5.2 shows, for different values of h, the coordinates of P, the coordinates of Q, the change in y from P to Q and the gradient of the chord PQ.

x for P	y for P	h	x for Q	y for Q	change in y	gradient PQ
1	1	1	2	1.414214	0.414214	0.414214
1	1	0.1	1.1	1.048 809	0.048 809	0.488 088
1	1	0.01	1.01	1.004 988	0.004 988	0.498 756
1	1	0.001	1.001	1.000 500	0.000 500	0.499 875

Table 5.2

(a) Fill in the missing values for the case $h = 1$ in the copy of Table 5.2 in the Printed Answer Booklet. Give your answers correct to 6 decimal places where necessary. **[1]**

(b) Explain how the sequence of values in the last column of Table 5.2 relates to the gradient of the curve $y = x^{\frac{1}{2}}$ at the point P. **[1]**

(c) Use calculus to find the gradient of the curve at the point P. **[2]**

b) The sequence of gradients as h tends to zero is the gradient of the tangent of the curve. The sequence of gradient tends to 0.5

c) $y = x^{1/2}$

$\frac{dy}{dx} = ?$

$\frac{dy}{dx} = nx^{n-1} \Rightarrow \frac{1}{2}x^{-1/2}$

when $x = 1$ $\frac{dy}{dx} = \frac{1}{2}(1)^{-1/2} = \frac{1}{2}$

6 **In this question you must show detailed reasoning.**

A particle moves in a straight line. Its velocity $v\,\mathrm{m\,s^{-1}}$ after t s is given by $v = t^3 - 5t^2$.

(a) Find the times at which the particle is stationary. **[2]**

(b) Find the total distance travelled by the particle in the first 6 seconds. **[3]**

a) Particle is stationary when velocity = 0

$V = 0$

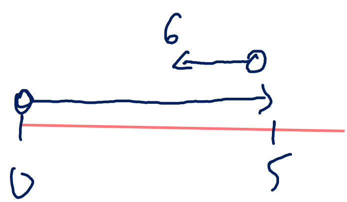
$t^3 - 5t^2 = 0$

$t^2(t-5) = 0$

$t^2 = 0 \qquad \boxed{t = 0}$

$t - 5 = 0 \qquad \boxed{t = 5}$

b) distance $= \int v \, dt$



(moves in +ve direction then -ve direction)

$$\int_0^5 (t^3 - 5t^2)\, dt$$

$$\left[\frac{t^4}{4} - \frac{5t^3}{3}\right]_0^5 \Rightarrow \left[\frac{5^4}{4} - \frac{5(5)^3}{3}\right] - 0$$

$$= 193/12$$

$$\int_5^6 (t^3 - 5t^2)\,dt$$

$$\left[\frac{t^4}{4} - \frac{5t^3}{3}\right]_5^6$$

$$\left[\frac{6^4}{4} - \frac{5(6)^3}{3}\right] - \left[\frac{(5)^4}{4} - \frac{5(5)^3}{3}\right]$$

$$-36 - \frac{193}{12} = -\frac{652}{12}$$

$\Rightarrow$ But distance is scalar $\therefore = +\frac{652}{12}$

$$\frac{652}{12} + \frac{193}{12} = \frac{409}{6}\,m$$

$$= 68.2\,m$$

7 In this question you must show detailed reasoning.

A curve has equation $y = 4x^3 - 6x^2 - 9x + 4$.

(a) Sketch the gradient function for this curve, clearly indicating the points where the gradient is zero. **[4]**

(b) Find the set of values of x for which the gradient function is decreasing. Give your answer using set notation. **[2]**

a) $\frac{dy}{dx} = 3(4)x^2 - 6(2)x - 9$

$= 12x^2 - 12x - 9$ y int $= -9$

Factorising

$3(4x^2 - 4x - 3)$

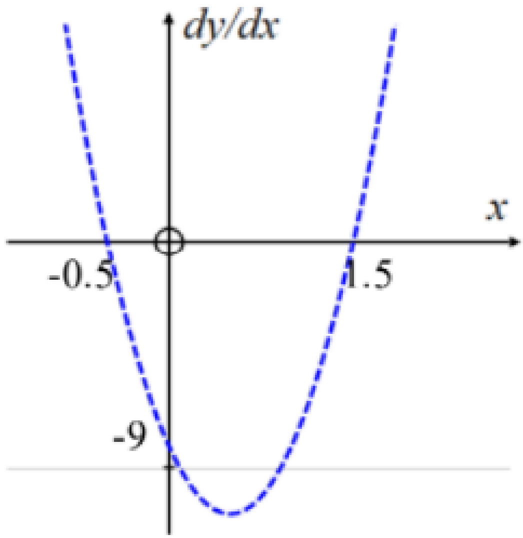
$3(2x+1)(2x-3)$

$\therefore$ X intercepts $=$ $x = -1/2$

$x = 3/2$

$\rightarrow$ Since co-efficient of $x^2 > 0$

the shape of the curve $=$ U



b) $12x^2 - 12x - 9$

$3(4x^2 - 4x - 3)$

completing the square

$3\left[4\left(x^2 - x - 3/4\right)\right]$

$3\left(4\left[\left(x - 1/2\right)^2 - \left(1/2\right)^2 - 3/4\right]\right)$

$12\left(x - 1/2\right)^2 - 12.$

$\therefore$ Min point $=$ $(1/2, -12)$

$\therefore$ From diagram $\frac{dy}{dx}$ is decreasing for

$x < \frac{1}{2}$

$\therefore$ in set notation

$\{x : x < \frac{1}{2}\}$

8 The point A has coordinates $(-1, -2)$ and the point B has coordinates $(7, 4)$. The perpendicular bisector of AB intersects the line $y+2x=k$ at P.

Determine the coordinates of P in terms of k. **[7]**

Perpendicular means $m_1 \times m_2 = -1$
Bisector means the line passes through the midpoint of the 2 given points.

Gradient AB:

$(-1, -2)$ $(7, 4)$
x_1 y_1 x_2 y_2

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - -2}{7 - -1} = \frac{6}{8} = 3/4$$

$$\frac{3}{4} \times m_2 = -1$$

$$m_2 = -4/3$$

Midpoint of AB

$$x = \frac{x_1 + x_2}{2} \qquad y = \frac{y_1 + y_2}{2}$$

$\frac{-1+7}{2} = 3 \qquad \frac{-2+4}{2} = 1$

$\therefore \ MP = (3, 1)$

$\therefore \ y = mx + c$ with $m = -4/3$
and co-ordinates $(3, 1)$

$y = -\frac{4}{3}x + c \quad (3, 1)$

$1 = -\frac{4}{3}(3) + c$

$1 = -4 + c \qquad c = 5.$

$y = -\frac{4}{3}x + 5.$

$\underbrace{y = -\frac{4}{3}x + 5}$ substitute intersects with $y + 2x = k$ in that

$-\frac{4}{3}x + 5 + 2x = k$

$$\frac{2}{3}x + 5 = k$$

$$\frac{2}{3}x = k - 5$$

$$x = \frac{3}{2}(k-5) \quad = \frac{3}{2}k - \frac{15}{2}$$

$$y = -\frac{4}{3}x + 5$$

$$y = -\frac{\cancel{4}^{2}}{\cancel{3}}\left[\frac{\cancel{3}}{2}(k-5)\right] + 5$$

$$y = -2(k-5) + 5$$

$$y = -2k + 10 + 5$$

$$y = -2k + 15$$

$\therefore$ Co-ordinates of Point of intersection $= \left(\frac{3}{2}k - \frac{15}{2}, -2k + 15\right)$

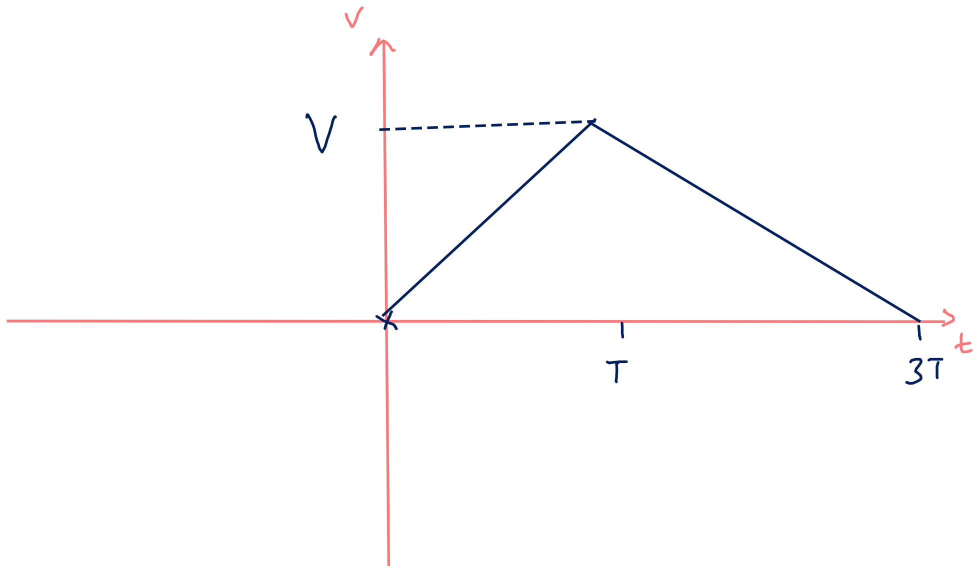
9 A car travelling in a straight line accelerates uniformly from rest to $V \text{ m s}^{-1}$ in T s. It then slows down uniformly, coming to rest after a further $2T$ s.

(a) Sketch a velocity-time graph for the car. **[2]**

The acceleration in the first stage of the motion is 2.5 m s^{-2} and the total distance travelled is 240 m.

(b) Calculate the values of V and T. $T+2T=3T$ **[4]**

a)



b) 1 Acceleration = gradient of the graph

$(0,0) \quad (T,V)$

$$\frac{V-0}{T-0} = 2.5 \qquad \frac{V}{T} = 2.5 \qquad V = 2.5T \quad \text{-- } 1$$

2 Distance travelled = Area under the graph

$$\text{Area} = \frac{1}{2} bh.$$

$$= \frac{1}{2} \times 3T \times V$$

$$\frac{3TV}{2} = 240$$

$$TV = \frac{2 \times 240}{3} = 160 \qquad \therefore TV = 160 \text{ --- } (2)$$

Substitute 1 into 2

$$T(2.5T) = 160$$

$$2.5T^2 = 160$$

$$T^2 = \frac{160}{2.5} = 64 \qquad T = \pm\sqrt{64} = \pm 8$$

since time is scalar

$$T = +8$$

$$V = 2.5(8) = 20 ms^{-1}$$

$$T = 8s$$

$$V = 20 ms^{-1}$$

10 An astronaut on the surface of the moon drops a ball from a point 2 m above the surface.

(a) Without any calculations, explain why a standard model using $g = 9.8\,\text{m}\,\text{s}^{-2}$ will not be appropriate to model the fall of the ball. **[1]**

The ball takes 1.6 s to hit the surface.

(b) Find the acceleration of the ball which best models its motion. Give your answer correct to 2 significant figures. **[2]**

(c) Use this value to predict the maximum height of the ball above the point of projection when thrown vertically upwards with an initial velocity of $15\,\text{m}\,\text{s}^{-1}$. **[2]**

a) Because acceleration due to gravity is only $9.8\,ms^{-2}$ on earth, and the value changes depending where you are in the universe.

b) $s = 2$

$u = 0$

v

$a = ?$

$t = 1.6$

$S = ut + \frac{1}{2}at^2$

$\downarrow = +ve$

$2 = 0 + \frac{1}{2}(a)(1.6)^2$

$2 = 1.28a \qquad a = \frac{2}{1.28} = \frac{25}{16}\,ms^{-2}$

$a = 1.56\,ms^{-2}$ (3sf)

c) $s = ?$

$\uparrow = +ve$

$u = 15$

$v = 0$

$a = -\frac{25}{16}$

t

$v^2 = u^2 + 2as$

$0 = 15^2 + 2(-25/16)s$

$\frac{25}{8}s = 225$

$s = 72m.$

11 In this question you must show detailed reasoning.

(a) A student is asked to solve the inequality $x^{\frac{1}{2}} < 4$.

The student argues that $x^{\frac{1}{2}} < 4 \Leftrightarrow x < 16$, so that the solution is $\{x : x < 16\}$.

Comment on the validity of the student's argument. **[1]**

(b) Solve the inequality $\left(\frac{1}{2}\right)^x < 4$. **[3]**

(c) Show that the equation $2\log_2(x+8) - \log_2(x+6) = 3$ has only one root. **[5]**

a) - The argument is incorrect.

- The statement $x < 16$ includes negative values for x for which $x^{1/2} < 4$ doesn't exist $\therefore$ the stament doesn't imply $x^{1/2} < 4$

b) $\left(\frac{1}{2}\right)^x < 4$

Applying logs to both sides

$x \log(1/2) < \log 4$

$x > \dfrac{\log 4}{\log(1/2)}$

the sign flips if what you are dividing is less than 0.

$x > -2$

c) $2\log_2(x+8) - \log_2(x+6) = 3.$

Laws of logs

Product Rule	$\log_a(xy) = \log_a x + \log_a y$
Quotient Rule	$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$
Power Rule	$\log_a x^p = p\log_a x$

$\therefore \log_2(x+8)^2 - \log_2(x+6) = 3$

$\log_2\left[\frac{(x+8)^2}{(x+6)}\right] = 3$

$2^3 = \frac{(x+8)^2}{(x+6)}$

$8 = \frac{(x+8)^2}{(x+6)}$

$$8(x+6) = (x+8)^2$$

$$8x+48 = x^2+16x+64$$

$$x^2+16x+64-8x-48=0$$

$$x^2+8x+16=0$$

$$\frac{-b \pm \sqrt{(b)^2-4(a)(c)}}{2 \times a}$$

$$\frac{-8 \pm \sqrt{(8)^2-4(16)}}{2 \times 1} = \frac{-8 \pm \sqrt{64-64}}{2}$$

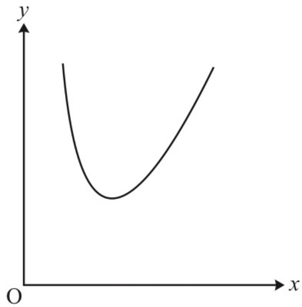
discriminant $= 0$

$$= \frac{-8}{2} = -4$$

$\therefore$ only 1 root as required

12 In this question you must show detailed reasoning.

Fig. 12 shows part of the graph of $y = x^2 + \frac{1}{x^2}$.



$\frac{1}{x^2} = x^{-2}$

Fig. 12

The tangent to the curve $y = x^2 + \frac{1}{x^2}$ at the point $(2, \frac{17}{4})$ meets the x-axis at A and meets the y-axis at B. O is the origin.

(a) Find the exact area of the triangle OAB. **[6]**

(b) Use calculus to prove that the complete curve has two minimum points and no maximum point. **[6]**

a) $\frac{dy}{dx} = 2x + (-2)x^{-3}$

$= 2x - 2x^{-3}$

@ $x = 2$

$\frac{dy}{dx} = 2(2) - 2(2)^{-3}$

$= \frac{15}{4}$

$y = mx + c$

with $m = \frac{15}{4}$

$c = (2, 17/4)$

$y = \frac{15}{4}x + c$

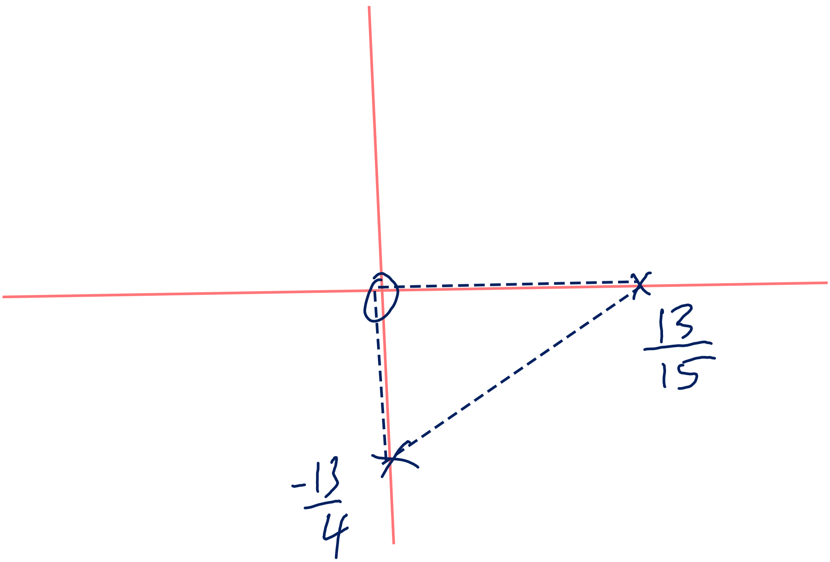
$\frac{17}{4} = \frac{15}{4}(2) + c$

$c = \frac{17}{4} - \frac{15}{2} = -\frac{13}{2}$

$\therefore y = \frac{15}{4}x - \frac{13}{4}$ $\quad$ $y_{int} = (0, -13/4)$

Xint $y=0$

$\frac{15}{4}x = \frac{13}{4}$ $\quad$ $x = \frac{13}{15}$ $\quad$ $\therefore \left(\frac{13}{15}, 0\right)$



$$\frac{1}{2} \times \frac{13}{15} \times \frac{13}{4} = \frac{169}{120}$$

b) $\frac{dy}{dx} = 0$ @ stationary point.

$$2x - 2x^{-3} = 0$$

$$2x = \frac{2}{x^3} \qquad x^4 = 1$$

$$x = \pm 1$$

$$\frac{d^2y}{dx^2} = 2 - 2(-3)x^{-4}$$

$$2 + 6x^{-4}$$

when $x = +1$

$2+6(1)^{-4} = 8$ $\quad 8 > 0 \therefore$ this is a min. point

When $x = -1$

$2+6(-1)^{-4} = 8$

Again this is a min point

Two stationary points are minimum $\therefore$ there is no max point (As required)

OCR
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