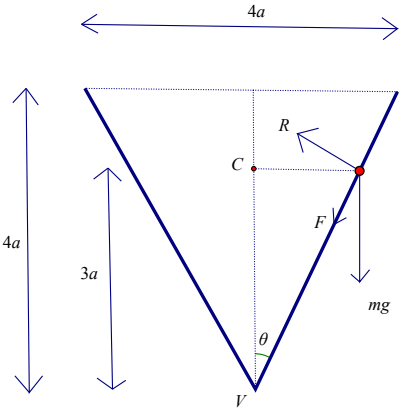


Paper 4F: Further Mechanics 2 Mark Scheme

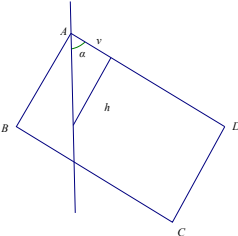
Question	Scheme	Marks	AOs
1(a)	Total mass = $\int_0^{15} 10\left(1 - \frac{x}{25}\right) dx$	M1	2.1
	$= \left[10x - \frac{x^2}{5}\right]_0^{15}$	A1	1.1b
	$= 150 - \frac{225}{5} = 105 \text{ (kg) } *$	A1*	1.1b
		(3)	
(b)	Taking moments about the base: $\int_0^{15} 10x\left(1 - \frac{x}{25}\right) dx$	M1	3.4
	$= \left[5x^2 - \frac{2}{15}x^3\right]_0^{15} (= 675)$	A1	1.1b
	$\Rightarrow 105d = 675$	M1	3.4
	$d = 6.43 \text{ (m) } \quad 6\frac{3}{7} \text{ (m)}$	A1	1.1b
		(4)	
(7 marks)			
Notes:			
(a)			
M1: Use integration (usual rules)			
A1: Correct integration			
A1*: Use limits and show sufficient working to justify given answer			
(b)			
M1: Use the model to find the moment about the base (usual rules for integration)			
A1: Correct integration			
M1: Use the model to complete the moments equation Require 105 and their 675 used correctly			
A1: 6.43 or better			

Question	Scheme	Marks	AOs
2			
	Complete overall strategy	M1	3.1b
	Resolve vertically	M1	3.3
	$mg + F \cos \theta = R \sin \theta$	A1	1.1b
	Horizontal equation of motion	M1	3.3
	$mr\omega^2 = R \cos \theta + F \sin \theta$	A1	1.1b
	Use of limiting friction since maximum ω	M1	3.3
	Substitute for trig ratios: $\frac{3a\omega^2}{2g} = \frac{9}{2}$	M1	1.1b
	Maximum $\omega = \sqrt{\frac{3g}{a}}$	A1	1.1b
(8 marks)			
<p>Notes:</p> <p>M1: Overall strategy to form equation in ω only e.g. consider vertical and horizontal motion and limiting friction</p> <p>M1: Needs all 3 terms. Condone sign errors and sin/cos confusion</p> <p>A1: Correct unsimplified equation</p> <p>M1: Needs all 3 terms. Condone sign errors and sin/cos confusion</p> <p>A1: Correct unsimplified equation</p> <p>M1: Seen or implied</p> <p>M1: Substitute to achieve equation in a, ω and g only</p> <p>A1: Or equivalent exact form</p>			

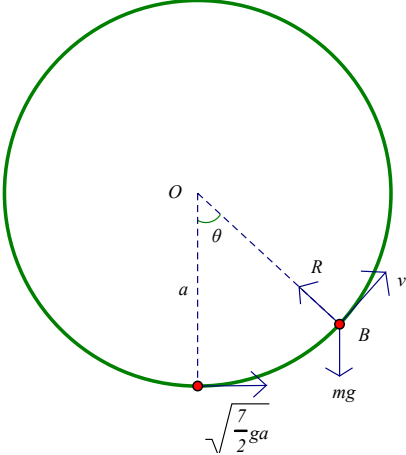
Question	Scheme	Marks	AOs												
3(a)	<table border="1"> <thead> <tr> <th></th> <th>mass</th> <th>c of m from O</th> </tr> </thead> <tbody> <tr> <td>cylinder</td> <td>$4\pi a^2 h$</td> <td>$\frac{h}{2}$</td> </tr> <tr> <td>hemisphere</td> <td>$\frac{2}{3}\pi a^3$</td> <td>$\frac{3}{8}a$</td> </tr> <tr> <td>V</td> <td>$4\pi a^2 h - \frac{2}{3}\pi a^3$</td> <td>$d$</td> </tr> </tbody> </table>		mass	c of m from O	cylinder	$4\pi a^2 h$	$\frac{h}{2}$	hemisphere	$\frac{2}{3}\pi a^3$	$\frac{3}{8}a$	V	$4\pi a^2 h - \frac{2}{3}\pi a^3$	d		
		mass	c of m from O												
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	V	$4\pi a^2 h - \frac{2}{3}\pi a^3$	d												
	Mass ratios		B1	1.2											
Correct distances		B1	1.2												
Moments about a diameter through O		M1	2.1												
$4\pi a^2 h \times \frac{h}{2} - \frac{2}{3}\pi a^3 \times \frac{3}{8}a = 2\pi a^2 \left(2h - \frac{1}{3}a\right) \times d$		A1	1.1b												
$d = \frac{h^2 - \frac{a^2}{8}}{2h - \frac{a}{3}} = \frac{3(8h^2 - a^2)}{8(6h - a)} *$		A1*	2.2a												
		(5)													
(b)															
	$h = 5a \Rightarrow d = 2.573...a$		B1	1.1b											
	About to topple so c of m above tipping point		M1	2.2a											
	$\Rightarrow \tan \phi = \frac{2a}{5a - 2.573a}$		A1ft	1.1b											
	$\phi = 39.5^\circ$ or 0.689 rads		A1	1.1b											
		(4)													
(9 marks)															

Question 3 notes:**(a)****B1:** Correct mass ratios**B1:** Correct distances**M1:** All three terms & dimensionally correct. Could use a parallel axis but final answer must be for the distance from O **A1:** Correct unsimplified equation**A1*:** Deduce the given answer. Their working must make it clear how they reached their answer**(b)****B1:** Distance of com from base**M1:** Condone tan the wrong way up**A1ft:** Correct unsimplified expression for trig ratio for ϕ following their d **A1:** 39.5° or 0.689 rads

Question	Scheme	Marks	AOs
4(a)	Equation of motion: $1800 - 2v^2 = 500a$ (when seen)	B1	2.1
	Select form for a : $= 500 \frac{dv}{dt}$	M1	2.5
	$\int \frac{2}{500} dt = \int \frac{1}{900 - v^2} dv = \frac{1}{60} \int \frac{1}{30 + v} + \frac{1}{30 - v} dv$	M1	2.1
	$\frac{t}{250} = \frac{1}{60} \ln(30 + v) - \frac{1}{60} \ln(30 - v) (+C)$	A1	1.1b
	$T = \frac{25}{6} \ln\left(\frac{30+10}{30-10}\right) = \frac{25}{6} \ln 2$ *	M1 A1*	2.1 2.2a
		(6)	
(b)	Equation of motion: $500v \frac{dv}{dx} = 1800 - 2v^2$	M1	2.5
	$\int \frac{500v}{1800 - 2v^2} dv = \int 1 dx$	M1	2.1
	$-125 \ln(1800 - 2v^2) = x (+C)$	A1	1.1b
	Use boundary conditions: $x = -125 \ln 1600 + 125 \ln 1800$	M1	2.1
	$x = 125 \ln \frac{9}{8} (\text{m})$ *	A1*	2.2a
		(5)	
(11 marks)			
Notes:			
(a)			
B1: All three terms & dimensionally correct			
M1: Use of correct form for acceleration to give equation in v, t only			
M1: Separate variables and integrate			
A1: Condone missing C			
M1: Use boundary conditions correctly			
A1*: Show sufficient working to justify given answer and a 'statement' that the required result has been achieved			
(b)			
M1: Correct form of acceleration in the equation of motion to give equation in v, x only			
M1: Separate variables and integrate			
A1: Condone missing C			
M1: Extract and use boundary conditions			
A1*: Show sufficient working to justify given answer and a 'statement' that the required result has been achieved			

Question	Scheme	Marks	AOs												
5(a)	<table border="1"> <thead> <tr> <th></th> <th>Mass</th> <th>From AD</th> </tr> </thead> <tbody> <tr> <td>Rectangle</td> <td>$8a^2$</td> <td>a</td> </tr> <tr> <td>Semicircle</td> <td>$\frac{1}{2}\pi a^2$</td> <td>$\frac{4a}{3\pi}$</td> </tr> <tr> <td>Sign</td> <td>$a^2\left(8 - \frac{\pi}{2}\right)$</td> <td>$h$</td> </tr> </tbody> </table>		Mass	From AD	Rectangle	$8a^2$	a	Semicircle	$\frac{1}{2}\pi a^2$	$\frac{4a}{3\pi}$	Sign	$a^2\left(8 - \frac{\pi}{2}\right)$	h		
		Mass	From AD												
	Rectangle	$8a^2$	a												
	Semicircle	$\frac{1}{2}\pi a^2$	$\frac{4a}{3\pi}$												
	Sign	$a^2\left(8 - \frac{\pi}{2}\right)$	h												
	Mass ratios		B1	1.2											
Moments about AD		M1	2.1												
$a^2\left(8 - \frac{\pi}{2}\right)h = 8a^2 \times a - \frac{1}{2}\pi a^2 \times \frac{4a}{3\pi} \left(= 8a^3 - \frac{2}{3}a^3 = \frac{22}{3}a^3\right)$		A1	1.1b												
$\Rightarrow h = \frac{22}{3}a \div \left(8 - \frac{\pi}{2}\right) = \frac{44a}{3(16 - \pi)}$ *		A1*	2.2a												
		(4)													
(b)	Moments about A $2aT = \frac{44a}{3(16 - \pi)}W$	M1	3.1b												
	$T = \frac{hW}{2a} = \frac{22W}{3(16 - \pi)}$	A1	1.1b												
		(2)													
(c)															
	Take moments about AB to find distance of com from AB	M1	3.1b												
	$8a^2 \times 2a - \frac{1}{2}\pi a^2 \times d = \left(8 - \frac{1}{2}\pi\right)a^2 \times v$	A1	1.1b												
	$v = \frac{32a - \pi d}{16 - \pi}$	A1	1.1b												
	Correct trig for the given angle	M1	3.1b												
	$\tan \alpha = \frac{11}{18} = \frac{h}{v} = \frac{44a}{3(32a - \pi d)}$	A1ft	1.1b												
	$(24a = 32a - \pi d, \quad 8a = \pi d) \quad d = \frac{8a}{\pi}$	A1	1.1b												
		(6)													
(12 marks)															

Question 5 notes:**(a)****B1:** Correct mass ratios**M1:** Need all three terms, must be dimensionally correct**A1:** Correct unsimplified equation**A1*:** Show sufficient working to justify the given answer and a 'statement' that the required result has been achieved**(b)****M1:** Could also take moments about B **or** about the c.o.m. and use**A1:** cso**(c)****M1:** All terms and dimensionally correct**A1:** Correct unsimplified equation**A1:** Or equivalent**M1:** Condone tan the wrong way up**A1:** Equation in a and d; follow through on their v**A1:** cao

Question	Scheme	Marks	AOs
6(a)			
	Conservation of energy	M1	2.1
	$\frac{1}{2}mv^2 + mga(1 - \cos \theta) = \frac{1}{2}m\left(\frac{7}{2}ga\right)$	A1	1.1b
	$v^2 = ga\left(\frac{3}{2} + 2\cos \theta\right)^*$	A1*	2.2a
		(3)	
(b)	Resolve parallel to OB and use $\frac{mv^2}{a}$	M1	3.1b
	$R - mg \cos \theta = \frac{mv^2}{a}$	A1	1.1b
	Use $R = 0$ $g \cos \theta = -\frac{v^2}{a}$	M1	3.1b
	Solve for $\theta \Rightarrow g \cos \theta = -g\left(\frac{3}{2} + 2\cos \theta\right)$	M1	1.1b
	$\theta = 120^\circ$	A1	1.1b
		(5)	
(c)	Any appropriate comment e.g. the hoop is unlikely to be smooth	B1	3.5b
		(1)	

Question	Scheme	Marks	AOs
6(d)	At rest $\Rightarrow v = 0$	M1	3.1b
	$\Rightarrow \cos \theta = -\frac{3}{4}$	A1	1.1b
	Acceleration is tangential	M1	3.1b
	Magnitude $ g \cos(\theta - 90) = 6.48 \text{ m s}^{-2}$ or $\frac{\sqrt{7}}{4} g$	A1	1.1b
	At $\left(\cos^{-1}\left(-\frac{3}{4}\right) - 90 = \right) 48.6^\circ$ to the downward vertical	A1	1.1b
		(5)	
(14 marks)			
Question 6 notes:			
(a)			
M1: All terms required. Must be dimensionally correct			
A1: Correct unsimplified equation			
A1*: Show sufficient working to justify the given answer and a 'statement' that the required result has been achieved			
(b)			
M1: Resolve parallel to OB			
A1: Correct equation			
M1: Use $R=0$ seen or implied			
M1: Solve for θ			
A1: Accept $\frac{2\pi}{3}$			
(c)			
B1: Any appropriate comment e.g. - hoop may not be smooth; - air resistance could affect the motion			
(d)			
M1: $v = 0$ seen or implied			
A1: Correct equation in θ			
M1: Correct direction for acceleration			
A1: Accept 6.48, 6.5 or exact in g			
A1: Accept 0.848 (radians)			

Question	Scheme	Marks	AOs
7(a)			
	$T_A = \frac{20e}{2}, T_B = \frac{50(2-e)}{2} e$	M1	3.1a
	In equilibrium $T_A = T_B$, $10e = 25(2-e)$	M1	3.1a
	$(35e = 50), e = \frac{10}{7}$	A1	1.1b
	Equation of motion for P when distance x from equilibrium position towards B :	M1	3.1a
	$3.5\ddot{x} = T_B - T_A = \frac{50(2-e-x)}{2} - \frac{20(e+x)}{2}$	A1 A1	1.1b 1.1b
	$= \frac{50\left(\frac{4}{7}-x\right)}{2} - \frac{20\left(\frac{10}{7}+x\right)}{2}$		
	$\Rightarrow 3.5\ddot{x} = -35x, \ddot{x} = -10x$ <p>and hence SHM about the equilibrium position</p>	A1	3.2a
		(7)	
(b)	Amplitude $= 2 - \frac{10}{7} = \frac{4}{7}$	B1 ft	2.2a
	Use of max speed $= a \omega$	M1	1.1b
	$= \frac{4}{7} \sqrt{10} = 1.81 \text{ (m s}^{-1}\text{)}$	A1 ft	1.1b
		(3)	

Question	Scheme	Marks	AOs
7(c)	Nearer to A than to B : $x < -\frac{3}{7}$	B1	3.1a
	Solve for $\sqrt{10}t$: $\cos \sqrt{10}t = -\frac{3}{4}$, $\sqrt{10}t = 2.418\dots\dots\dots$	M1	3.1a
	Length of time: $\frac{2}{\sqrt{10}}(\pi - 2.418\dots)$	M1	1.1b
	0.457 (seconds)	A1	1.1b
	Alternative: $\frac{3.864 - 2.419}{\sqrt{10}} = 0.457$		
	Alternative: $x = \frac{4}{7} \sin \sqrt{10}t = \frac{3}{7} \Rightarrow \sqrt{10}t = 0.8481$ or $\sqrt{10}t = 2.29353$ $t_1 = 0.2682$, $t_2 = 0.72527$ \Rightarrow time = 0.457 (seconds)		
		(4)	
(14 marks)			
Notes:			
(a)			
M1: Use of $T = \frac{\lambda x}{a}$			
M1: Dependent on the preceding M1. Equate their tensions			
A1: cao			
M1: Condone sign error			
A1: Correct unsimplified equation in e and x A1A1 Equation with one error A1A0			
A1: Full working to justify conclusion that it is SHM about the equilibrium position			
(b)			
B1ft: Seen or implied. Follow their e			
M1: Correct method for max. speed			
A1ft: 1.81 or better. Follow their a, ω			
(c)			
B1: Seen or implied			
M1: Use of $x = a \cos \omega t$			
M1: Correct strategy for the required interval			
A1: 0.457 or better			

