Strand E.
Waves

Unit 2.
The Nature of Waves

Text

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E.2.1. Interference and the Principle of Superposition

When waves meet, they pass through each other. Once they have passed through each other, they continue in their direction of travel unchanged. At the point of meeting however, whilst the two waves are occupying the same space or the same region of the medium (a string for example), the two waves combine. This combination is known as *superposition*, and it can be said that the two waves *superimpose* for an instant as they travel through each other. This leads us to the Principle of Superposition.

> *The Principle of Superposition states that when two waves meet, the total displacement at a point is equal to the sum of the individual displacements at that point*

In order to fully understand the Principle of Superposition, consider *Figure 2.1.1*(a). Here we see two transverse wave pulses travelling along a piece of rope (each pulse is a single crest, which could be made by flicking the end of the rope up and down). The two wave crests are initially travelling in opposite directions toward each other and have the same amplitude. As they meet the two pulses superimpose, and the displacements of the individual wave crests add together vectorially to form a '*super crest*'. The waves pass through each other and then continue in their respective direction unaffected.

Figure 2.1.1 (b) shows an identical situation except this time, it is two wave troughs travelling toward each other. Again, as the two troughs meet they superimpose to form a '*super trough*' that has amplitude equal to the sum of the two individual troughs.

Figure 2.1.1(c) shows a wave crest moving from left to right and a wave trough moving right to left. As the wave crest and trough meet, the amplitude of the resulting superimposed pulse is the vector sum of the two individual pulses. Since the wave crest represents positive displacement (lets assume an amplitude of +1) and the wave trough negative displacement (-1), the vector sum is 1+(-1), which is zero, and the two cancel completely. This is represented on the diagram by the straight line. As the wave crest and trough pass through

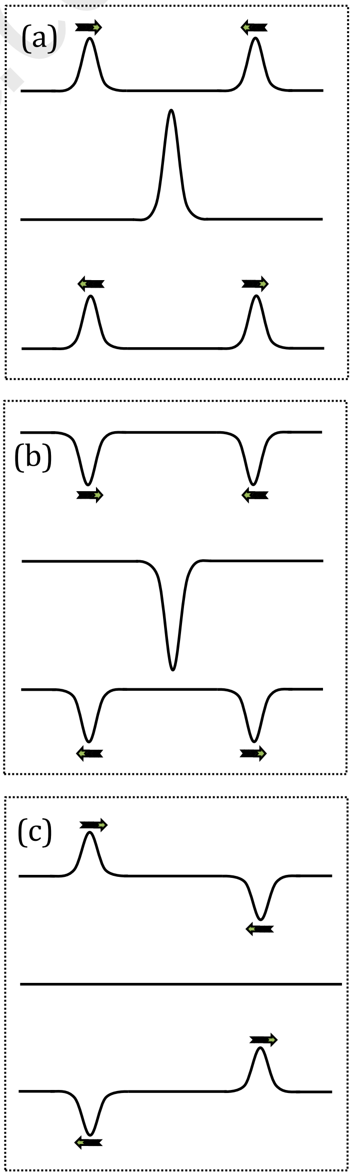


Figure 2.1.1

each other, they reappear and continue travelling in their respective directions unaffected.

Superposition of waves occur all around us, all of the time, and can be readily observed in light, surface water waves, and sound. For example, superposition is the underlying physical principle that makes colours visible in the reflected light from the bottom of a CD. When wave peaks or wave troughs meet, a super peak or super trough is created. The superposition of waves is known as *interference*. When waves are reinforced, creating a single peak or trough with an amplitude equal to the sum of the individual amplitudes, this type of interference is known as *constructive interference*.

Constructive interference occurs when two or more waves meet and add together constructively to form a new wave with larger amplitude

When a wave peak and a wave trough meet, the two either cancel out completely or partially, resulting in a wave of zero, or smaller amplitude. This situation is known as *destructive interference.*

Destructive interference occurs when two or more waves meet and add together destructively (subtract) to form a new wave with smaller, or zero amplitude.

For destructive interference to occur and completely cancel the two propagating waves, the amplitudes of the two waves when superimposed must be *exactly* equal in magnitude and opposite in sign (think back to *Figure 2.1.1* (c) where the amplitudes of the two pulses were +1 and -1). If they are not, we get the situation shown by *Figure 2.1.2*. Here let's assume the wave crest has an amplitude of +2 and the wave trough an amplitude of -1. When superimposed, +2 + (-1) = 1. The amplitude of the resultant pulse is reduced to 1. For complete cancellation to occur, the wave peak would require an amplitude of +1, or the wave trough an amplitude of -2.

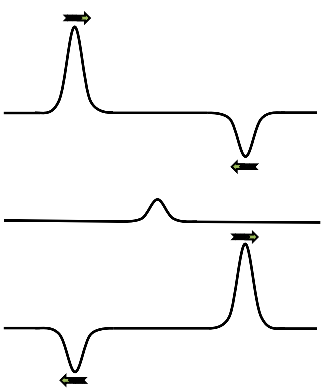


Figure 2.1.2

So far we have restricted our argument to single wave crests and troughs travelling in opposite directions to each other, but the Principle of Superposition holds for continuous waves, which may be travelling in either the opposite, or the same direction. Consider *Figure 2.1.3* (a). Here we see a single source emitting continuous circular wave fronts at an instant in time. This could be thought of as a dipper moving up and down in a water tank creating a continuous source of ripples. Each bright band is a wave peak, and each dark band is a wave trough.

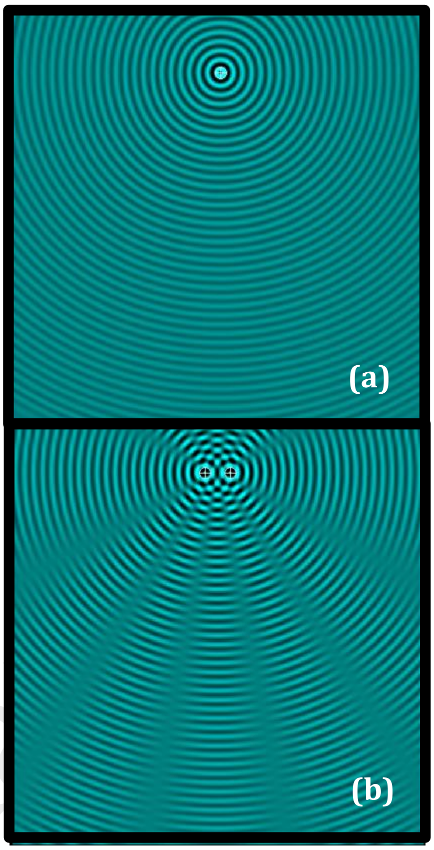


Figure 2.1.3

In *Figure 2.1.3* (b) a second source is added, Identical to the first. Two sources of the same frequency, wavelength, and equal phase (or constant phase difference) are said to be ***coherent sources***. Because the two sources are coherent, wave troughs and peaks overlap at fixed positions in space as the waves travel. If the two sources were not coherent the positions of troughs meeting peaks would constantly change. At the positions where troughs meet troughs and peaks meet peaks, constructive interference occurs. At the positions where peaks meet troughs, destructive interference occurs, giving lines of cancellation as shown by the interference pattern of *Figure2.1.3* (b). Note that changing the phase difference, or the frequency between the two sources, would alter the positions at which peaks and troughs overlap. Increasing the amplitude of one source with respect to the other would result in incomplete cancellation.

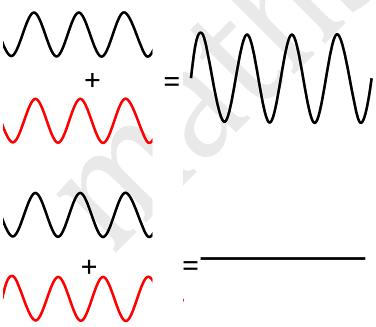


Figure 2.1.4

This is in fact a phase argument. For two coherent continuous waves, when the phase difference between the two is zero (or multiples of 360°), maximum constructive interference occurs as shown by *Figure 2.1.4.* Destructive interference occurs when the two waves are in anti-phase, or odd integer multiples of 180°. Note that from Unit 1, the two waves would be a whole number of wavelengths out of phase for constructive interference and a half number of wavelengths out of phase for destructive interference.

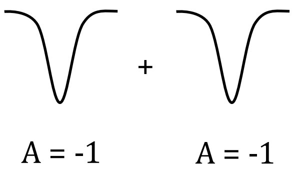
Maximum constructive interference occurs when two waves are in phase with each other (phase difference is an integer multiple of 360° (whole wavelength)).

Maximum destructive interference occurs when two waves are in anti-phase (phase difference is an odd integer multiple of 180° (half wavelength))

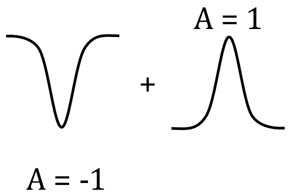
Exercise E.2.1

1. Sketch the resulting waveform stating amplitude A when the following are superimposed:

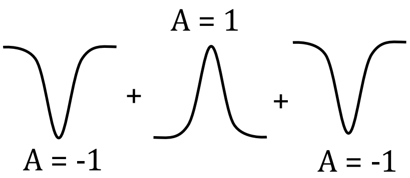
(a)



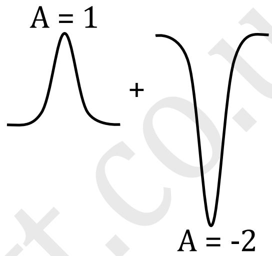
(b)



(c)



(d)



2. Two waves superimpose. If the phase difference between the two waves is 540°, is the resulting interference destructive or constructive?

3. When the two waves in question two cease to interact with each other they

(a) trade energy

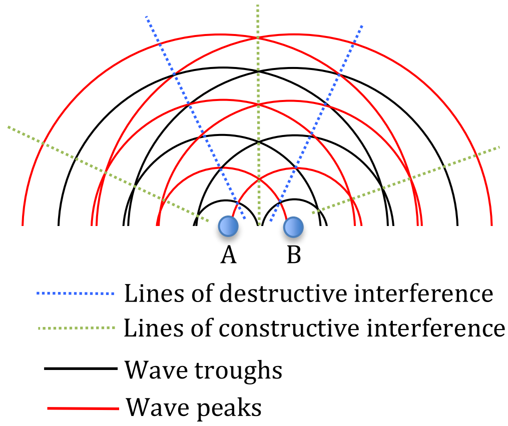
(b) return to their original size

(c) double in size

(d) one is consumed by the other

4. When two waves interact, what is the resultant displacement? What are the phase conditions for (a) destructive and (b) constructive interference?

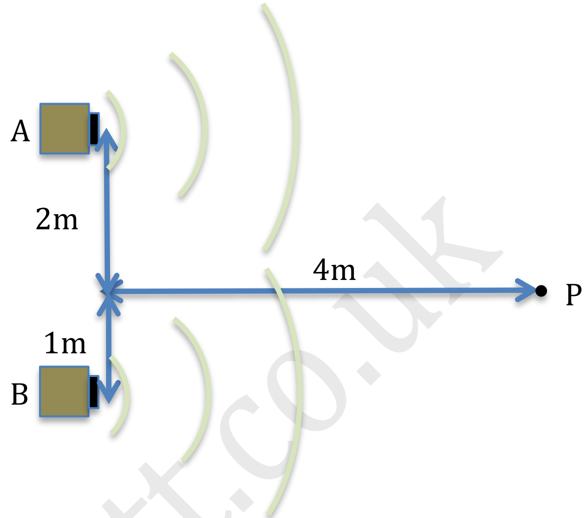
5. This question requires a compass and two coloured pencils but is well worth the effort. Consider the schematic of the super position pattern from two coherent sources A and B emitting troughs (black lines) and peaks (red lines). Look at the crossing points. Where red meets black, troughs are meeting peaks and we can connect these points with a line of destructive interference. Similarly red meets red or black meets black provides a line of constructive interference. Once you are happy that this is indeed the case, sketch



your own super position pattern with (a) the same wavelength (distance between peaks and troughs) but with the sources further apart. What happens to the lines of interference? (b) Now increase the wavelength of source B by small amounts. What happens to the interference pattern?

Challenge Question

6. Two loudspeakers A and B are driven by the same amplifier and produce an identical sine wave. What is the lowest frequency for which constructive interference occurs at point P? Assume the speed of sound in air is 350m/s. *Hint: You may find it useful to review the previous unit, and possibly review Pythagoras!*



E.2.2. Stationary Waves

In the previous sections, waves were defined as oscillations or disturbances that transfer energy from one region of space to another, like a surface sea wave transferring the energy from a storm out at sea to the beach on which it crashes. These waves are *progressive* waves, since they 'progress' through space, or through a medium. It is possible however, through the Principle of Superposition, for two progressive waves to combine and form a standing wave, a *stationary wave* that does not travel, nor transfer energy.

Progressive waves are travelling waves, transferring energy from one region to another.

Standing waves are stationary, they do not travel through space and thus do not transfer energy from one region to another.

Such waves are the source of musical notes, since they are waves of constant frequency. Consider the example of a plucked guitar string shown in *Figure 2.2.1*, which may be considered as a string under tension, fixed at both ends.

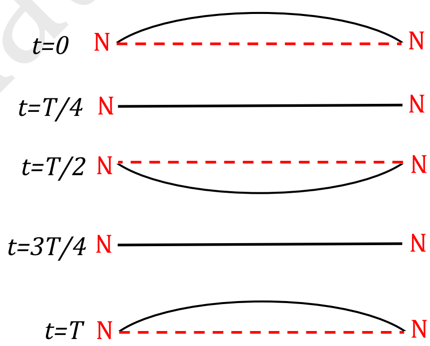


Figure 2.2.1.

When the string is plucked at the centre and released, it vibrates between the fixed ends. The string oscillates from maximum positive amplitude at $t = 0$, through equilibrium one quarter of a cycle later, to negative maximum amplitude at half a cycle, before returning to maximum positive amplitude via equilibrium at $t = T$ where T is the time period of oscillation. This is the fundamental stationary wave of the string (the lowest frequency standing wave that the string can support). Notice at the two fixed ends, zero displacement occurs. Regions of zero displacement are ***nodes (N)***. The point at which maximum amplitude occurs is the ***antinode***.

*The lowest frequency standing wave that may be supported is known as the **fundamental** mode of oscillation (and is also known as the first harmonic). It has two regions of zero displacement and one region of maximum displacement.*

*A **node** (N) is a point of **zero displacement**.*

*An **antinode** is a point of **maximum displacement**.*

In the example of a guitar string, the act of plucking the string sends a progressive wave up the string where it meets the fixed end and reflects. Now there are two waves travelling in opposite directions on the string, and since they are from the same source, they are coherent. These two waves interact. When they are in phase (*Figure 2.2.2 (a)*) they reinforce. One quarter of a cycle later the right travelling wave has moved on by ¼ wavelength and so has the left travelling wave. Hence the difference between them is now ½ wavelength and cancellation occurs (*Figure 2.2.2 (b)*). Another ¼ of a cycle later and we get reinforcement again, but with the amplitude in the opposite direction (*Figure 2.2.2 (c)*). If you struggle visualizing the formation of the standing wave, try sketching two waves on separate pieces of tracing paper and then sliding them over the top of each other.

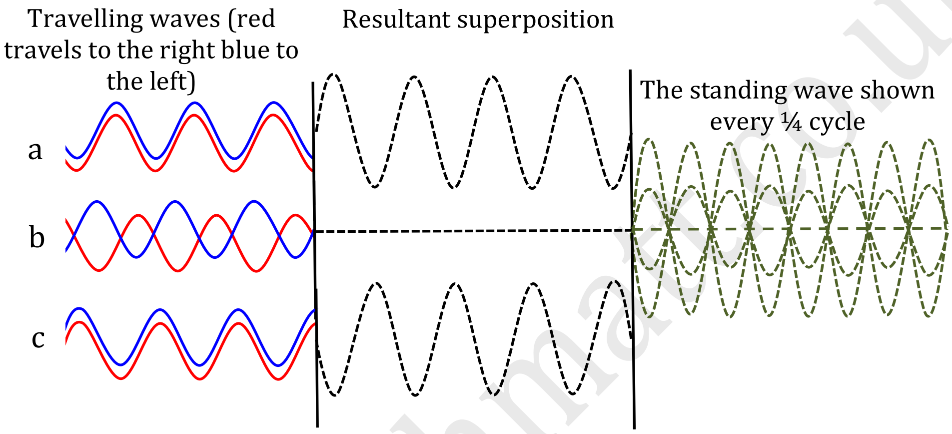
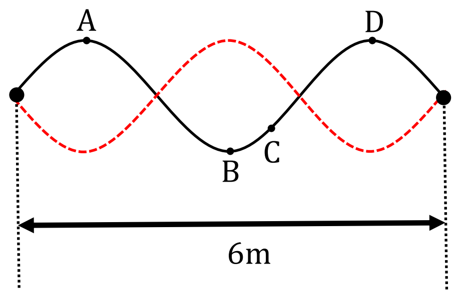


Figure 2.2.2

The net result is a standing wave which oscillates in the vertical direction, about fixed nodes of zero displacement, with the nodes separated by ½ of the wavelength (180° of phase), with all points on the wave (except nodes) vibrating at the same frequency.

Worked Example:

The stationary wave pattern shown is set up on a string of length 6m. Calculate the wavelength, and the phase difference between a particle at A, and a particle vibrating at B, C and D.



Answer

Each node is separated by ½ wavelength. There are therefore 3 × ½ = 1.5 wavelengths over 6m. The wavelength of the standing wave is thus 6/1.5 = 4m. Again, since each node is separated by ½ wavelength or 180° of phase, Point B oscillates 180° out of phase with A. Point C oscillates 180°+90° = 270° out of phase, and point D oscillates in phase (360°) with A.

The wavelength and therefore frequency of a standing wave on a string or any other system is determined by the dimensions of the system (the length L in the case of the string), since it is this dimension that determines the distance between the two nodes at the fixed ends. Since for the lowest frequency (*fundemental*) standing wave mode m_1 with two nodes at the fixed ends, there is a ½ λ seperation between the two nodes as shown by *Figure 2.2.3*. Therefore

$$\frac{1}{2}\lambda_0 = L \Longrightarrow \lambda_0 = 2L$$

and

$$f_0 = \frac{v}{2L}$$

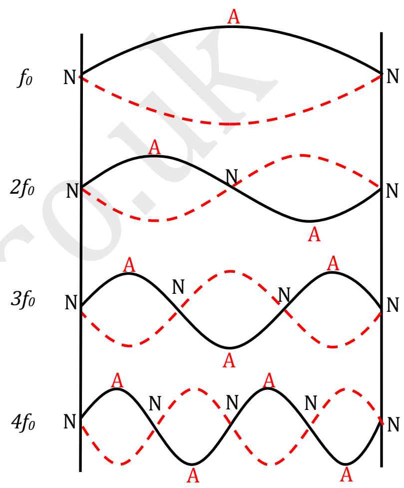


Figure 2.2.3

where λ_0 is the wavelength of the fundamental mode. Since the fundamental mode is also the first harmonic, the next allowed mode is known as the 2nd harmonic m_2, with 3 nodes and two antinodes. Since there are 3 nodes, 2 × ½ λ must fit across the distance L;

$$\lambda = L \qquad f = \frac{v}{L} = 2f_0$$

As shown by *Figure 2.2.3*, each successive mode (m_3, m_4...... m_n) has one more node and therefore must fit in an extra ½ wavelength, and in general we can write:

The frequencies of allowed standing wave modes increase as

f_0, $2f_0$, $3f_0$, $4f_0$.........

Where f_0 is the fundamental frequency.

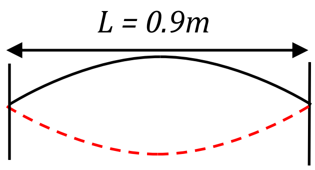
The wavelength is given by

λ=2L/m

Where L is the length of the system and m is the mode number (for the fundamental mode m = 1, for the 2nd harmonic m = 2 and so on).

Worked Example

A wire, stretched between two fixed points has a length L = 0.9m. The wire vibrates in its fundamental mode at a frequency of 350Hz. calculate (a) the wavelength of the progressive waves on the wire, (b) the speed of the progressive waves on the wire and (c) the frequency of the 5th harmonic.



Answer

(a) The fundamental mode m_1 has a wavelength $\lambda = 2L$. Therefore

$$\lambda = 2L = 2 \times 0.9m = 1.8m$$

(b) The velocity v of the waves on the wire is given by $v = f\lambda$ where f is the frequency and λ is the wavelength;

$$v = f\lambda = 350Hz \times 1.8m = 630ms^{-1}$$

(c) The frequency of the modes supported increase as mf_0 where m is the mode number. Thus for the fundamental $m=1$, and

$m=1$ (fundamental mode) $f_0 = 350\text{Hz}$

$m=2$ (2nd harmonic) $f = mf_0 = 2\times350\text{Hz} = 700\text{Hz}$

$m=3$ (3rd harmonic) $f = mf_0 = 3\times350\text{Hz} = 1050\text{Hz}$

$m=4$ (4th harmonic) $f = mf_0 = 4\times350\text{Hz} = 1400\text{Hz}$

and lastly,

$m=5$ (5th harmonic) $f = mf_0 = 5\times350\text{Hz} = 1750\text{Hz}$

Exercise E.2.2.

1. State the difference between progressive and stationary waves, and then briefly describe how two progressive waves on a string fixed at both ends could form a stationary wave.

2. Two progressive waves combine to form a standing wave in a tube that is blocked at both ends. The result is a transverse standing wave. The tube length is 2.6m.

(a) Sketch the 4th harmonic, clearly showing all nodes and antinodes, state the distance between adjacent nodes, and the order number of the mode.

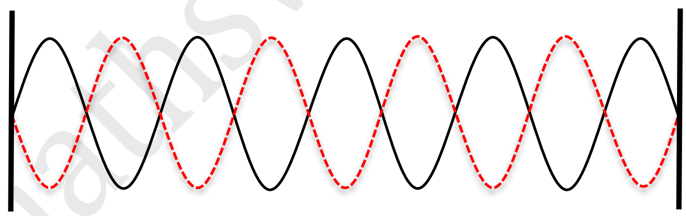
(b) At what frequency does the 4th harmonic oscillate given that the velocity of the waves in the tube is 364m/s?

3. For the standing wave in question 2, sketch the fundamental and 2nd harmonic. At what frequency do these two modes oscillate?

4. Two science teachers decide two demonstrate interference and superposition patterns to their students. They decide to each throw a stone into the school pond to act as two sources of transverse surface water waves, and see the lines of constructive and destructive interference in the interacting ripples. Describe why it is highly unlikely that their demonstration will work.

Challenge Question

5. Consider the following standing wave on a string, formed by two progressive waves propagating at a velocity v = 6m/s.



(a) State the order number m.

(b) Calculate the length of the string if this m^{th} order mode has a wavelength of 0.1m.

(c) Calculate the frequency of the fundamental mode and use this to state the frequency of oscillation of all modes up to and including the mode shown in the diagram.

E.2.3 Reflection, Diffraction and Refraction.

As previously discussed, a plane wave is one for which the displacement x of the wave is in the same plane as the wave velocity or propagation. *Figure 2.3.1* (a) shows a transverse plane wave since its displacement is in the same plane as wave velocity – the plane of the page. It is useful here to outline an alternative way of representing a plane wave, a '*top down*' or *plan* view of the wave as shown by *Figure 2.3.1* (b). Here we are representing the same wave, with each straight line being the crest of the wave. Each straight line also represents a line of constant phase (since each point on the line is a peak), and both the wave velocity and direction of propagation is always perpendicular to these lines. In addition, since each line is a peak, the distance between two adjacent lines represents one whole wavelength. It may be useful to think of this representation as a corrugated sheet. When you look down on the sheet you see parallel lines but when looking at the side of the sheet you see the individual peaks and troughs as a transverse wave. Although representing waves as in *Figure 2.3.1* (b) gives no information on amplitude since the displacement is into and out of the page, this representation is extremely useful in optics and discussions involving diffraction and interference, since point sources can be represented in exactly the same way, only the lines of constant phase form concentric circles.

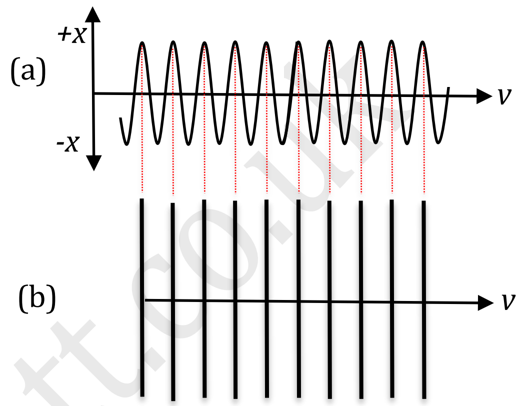


Figure 2.3.1

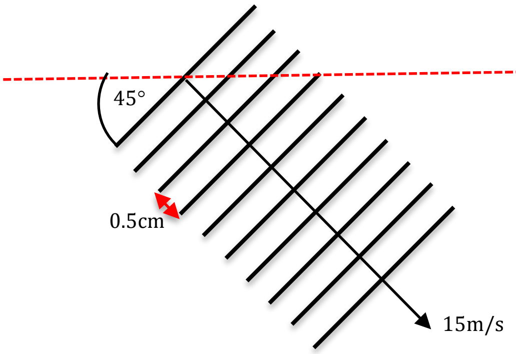
Worked Example

A plane wave of wavelength λ = 0.5cm propagates at 15m/s in a direction 45° below the horizontal from left to right. Represent this wave diagrammatically. In addition sketch a wave of the same wavelength from a point source.

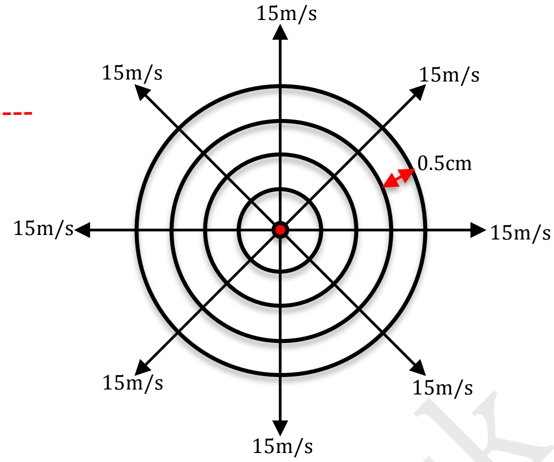
Answer

For the plane wave, the wavelength is represented by the distance between any two adjacent lines of constant phase (red arrow) and the wave velocity is perpendicular to the lines. The horizontal is represented by the red dotted line.

For the point source, the source is represented by the central red dot. Since the waves emanate from a point, the wave fronts (lines of constant phase) form concentric circles, with any two adjacent lines being separated by one wavelength. The wave velocity is perpendicular to the wave fronts and is therefore radial.



plane wave



point source

Reflection

When a wave is incident on a surface that it cannot cross, it is reflected. This could be a light wave reflecting off a mirror or equally a sea wave reflecting off a harbor wall. The incoming wave is called the *incident wave* to distinguish it from the *reflected wave*. When normally incident (wave crests parallel to the surface) the wave is reflected back on its self, propagating in the opposite direction. When incident at a non-zero angle of incidence θ_i, it is reflected at an angle θ_r. Just like a snooker ball rebounding from the cushion, upon reflection the incident angle is equal to the reflected angle. This is summarised by the law of reflection, which is true for all plane waves.

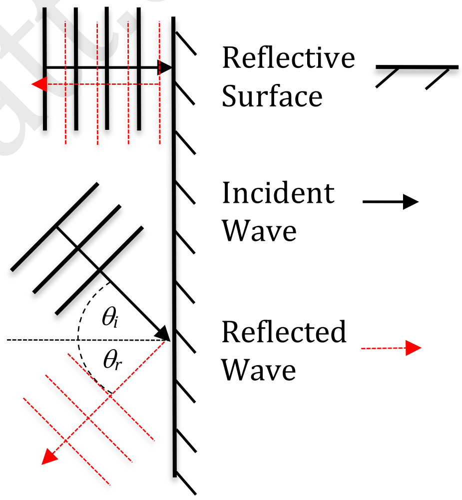


Figure 2.3.2

The Law of Reflection states that the angle of reflection θ_r is equal to the angle of incidence θ_i

$$\theta_r = \theta_i$$

where θ_r and θ_i are measured from the normal to the reflecting surface

Refraction

The speed of waves is not fixed (not even for light). The speed of the wave depends on the medium the wave is travelling in. For example, at 20°C sound waves travel at 343m/s in air and 1484m/s in water. Some waves travel faster in a dense medium and slower in a less dense medium (sound does this). Some waves travel slower in a denser medium (light does this). When a wave approaches a boundary between two media with different densities that it can cross, it is transmitted, from one medium to the other, and its speed must therefore change. This causes the wave direction to change as shown by *Figure 2.2.3*. This change in direction of the wave (or bending of the wave) is known as refraction, and can be most easily understood through an analogy. Imagine driving a land rover along a country lane for which the road surface is good. Along side the lane is a very muddy verge. Your two drivers side wheels go into the verge on the right of the lane as you are driving along, and these two wheels slow down, because the mud is sticky. The wheels on the left continue at the same speed because they are on the road. The Land rover therefore starts to turn to the right because the wheels on the left are going faster than the wheels on the right. This is what is happening to the wave. As the wave front crosses the boundary it changes speed. The part of the wave front that is still in the first medium must however continue at the same speed as shown in *Figure 2.2.3.* The wave front is a line of constant phase and must stay connected, but now half of the wave front is travelling slower than the other half. The result is that the wave front bends. Note from the diagram that in order to accommodate the change in speed and the bending, the wavelength must also change. The frequency of the wave however DOES NOT change. Also, if the wave is parallel to the boundary between the two media refraction can not occur since no two parts of the wave front is travelling at a different speed. Refraction in the case of light will be covered in more detail in strand F, but is a common occurrence. For example, if you look at a pencil side on in a glass of water, the part of the pencil that is out of the water looks disjointed with the part in the water due to the refraction of light.

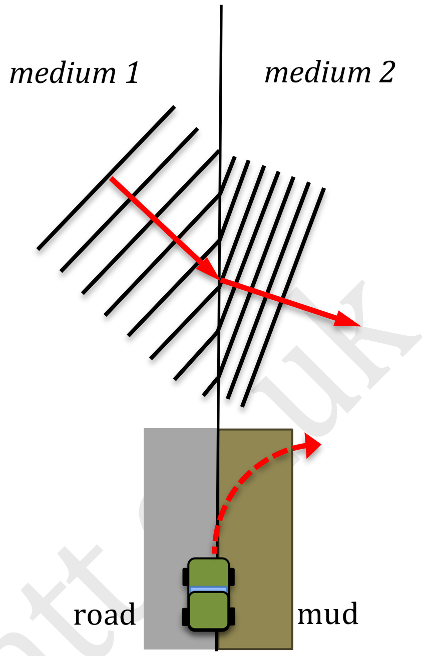


Figure 2.3.3

Diffraction

So far we have learnt that reflection is the 'rebounding' of waves at a barrier they can not cross, and refraction is the bending of waves when they pass through the boundary between two different media. *Diffraction* describes waves passing through gaps or around objects. All types of waves diffract. When waves meet a gap in a barrier they pass through the opening. As the wave front passes through it curves, and spreads out. The curving and spreading out is diffraction, and the amount of diffraction that occurs depends on the size of the gap / obstacle compared to the size of the wavelength.

For gaps which are large compared to the wavelength, only a small amount of diffraction occurs (*Figure2.3.4 (a)*). For example, light has a much smaller wavelength than most everyday objects and gaps. Because the light does not spread out much (it doesn't diffract much) we get sharp shadows. For gaps that are small compared to the wavelength, not much of the wave gets through. Maximum diffraction occurs when the gap is roughly the same size as the wavelength (*Figure 2.3.4* (b)).

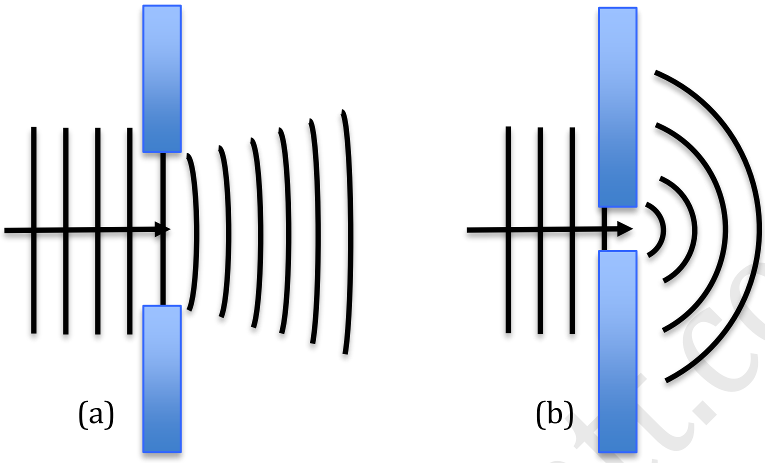


Figure 2.3.4

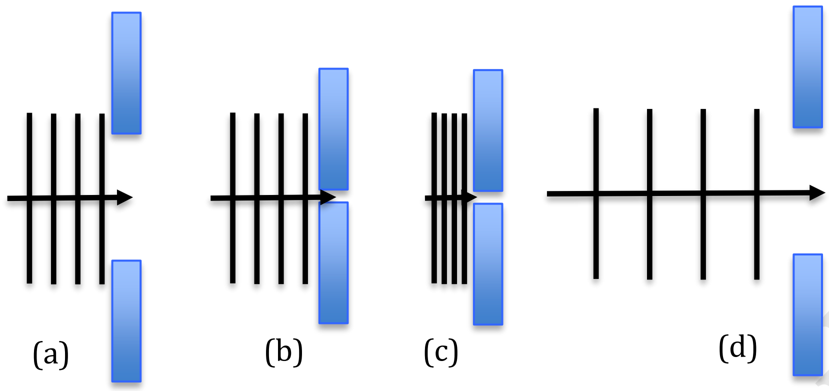
Note that increasing the wavelength of the wave to be on the order of the gap width shown in *Figure 2.3.4* (a) would be equivalent to reducing the size of the gap. Examples of diffraction include under water features or harbour mouths causing diffraction of sea waves and creating curved or crescent shaped beaches, or the diffraction of sound around doorways or other obstacles. Radio waves are chosen for television and radio signals because they are of a similar size to gaps between buildings and natural landscape features such as hills. This means that these signals diffract heavily, reducing poor reception areas due to shadow. Diffraction will be covered for electromagnetic waves in more detail in Strand F.

Exercise E.2.3.

1. A sea wave of frequency 0.1Hz travels at 2m/s. It collides with a harbour wall at an incident angle of 30° measured from the normal to the wall and reflects. The wave can be considered a plane wave. (a) State the frequency and angle of reflection of the reflected wave. (b) Calculate the wavelength of the reflected wave.

2. An electromagnetic plane wave travels at $c = 3\times10^8$ m/s in air at a frequency of 4×10^{14}Hz. (a) Calculate its wavelength. (b) The plane wave is incident on a glass block at 30° to the normal, and is transmitted, refracting in the process. If the speed of the electromagnetic wave in the glass is 8×10^7m/s, what is the frequency and wavelength of the refracted wave?

3. Consider the following schematic representations of a plane wave approaching a gap.

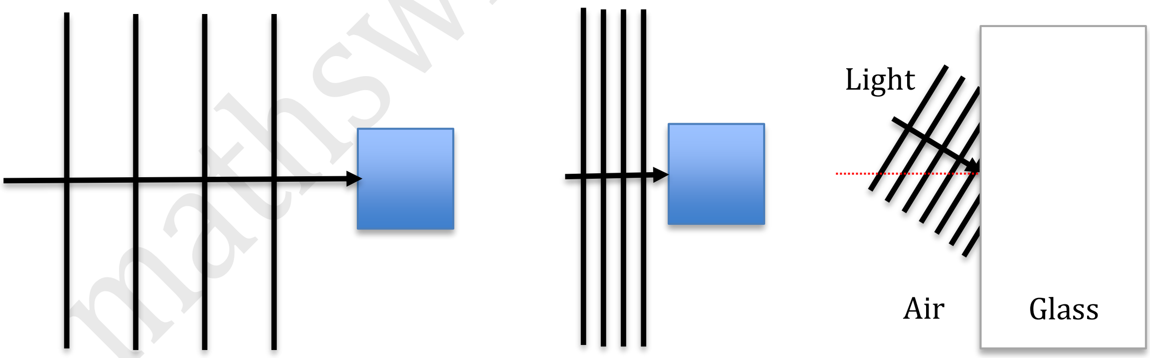


Giving your reasons, describe how much diffraction would be expected in each case.

4. A broadcasting tower transmits television, mobile phone and radio signals in a very hilly area. People who live in the area complain about reception. The most common complaint is that the residents get a poor TV signal, and no mobile phone signal, but the radio signal seems fine. Explain.

Challenge Question

5. Copy and complete the following diagrams clearly showing the expected diffraction pattern or refraction in each case. Assume the blue blocks to be solid objects.



(a) (b) (c)