Learning Objectives

At the end of this unit you should be able to

- Describe the main features of diffraction, refraction and reflection, and use the law of reflection.
- Describe how two progressive waves can combine and apply the principle of super position to two waves that interact.
- Describe using phase and coherence arguments how constructive and destructive interference can occur.
- Calculate frequencies and wavelengths of standing wave modes.
- Understand how the frequency of higher harmonics are related to the fundamental frequency of oscillation.
- Determine mode numbers of standing wave modes from the arrangement and number of nodes and antinodes.

Introduction.

The interaction of waves (superposition) is a very important physical property of waves that underpins many advances in modern physics and technology. Through superposition, two or more interacting waves combine, resulting in a wave that has a size (amplitude) that is the sum of the amplitudes of the individual waves that are interacting. This ability of waves to add together or cancel out in a stable way gives rise to interference, a phenomena responsible for many effects observed throughout nature, including diffraction (the bending of waves around obstacles or through gaps) and other effects such as the colours seen on the surface of a soap bubble or on the bottom (grooved surface) of a CD, and in addition, the formation of standing waves.

These effects were readily observable to early scientists since surface sea waves interact in this way (it's the reason that beaches are often curved or crescent shaped, due to water waves passing through a natural gap into a bay and curving). But throughout modern scientific history, it was the nature of light that was the focus of a great debate, and drove the development of a mathematical theory that described interference.

Sir Isaac Newton thought that light was a particle, and developed the corpuscular theory of light in 1675 that relied upon the existence of the Ether (a hypothetical substance filling a vacuum and constituting the medium through which light travelled (and disproved by Michelson and Morley in 1887) and momentum conservation to try and predict how light behaved at a reflecting boundary, or described how light was bent when passing between different mediums. It was Dutch physicist Christiaan Huygen however, in 1678, who first managed to predict reflection and refraction using a wave theory of light, a theory that was not readily accepted (mainly due to the reputation of Newton) until 1801, when English scientist Thomas Young produced interference patterns through passing water waves through a pair of slits, and then extending this work to light, producing interference patterns and proving the wave nature of light.

Key Facts and Principles.

Hence for either two transverse or two longitudinal waves occupying the same region of space;

- When two wave crests meet they superimpose, and the displacements of the individual wave crests add together vectorially to form a '*super crest*'. This is **constructive interference**.

- When two wave troughs meet they superimpose to form a 'super trough' which has an amplitude equal to the sum of the two individual troughs. This is **constructive interference**.

- When a wave crest and wave trough meet they superimpose. Since the resulting amplitude is the vector sum of the individual amplitudes, it is either reduced, or cancelled out completely if the magnitude of the two amplitudes are equal. This is **destructive interference**.

- The waves pass through each other and then continue in their respective direction unaffected, at their original amplitudes.

Constructive interference occurs when two or more waves meet and add together constructively to form a new wave with larger amplitude

Destructive interference occurs when two or more waves meet and add together destructively (subtract) to form a new wave with smaller, or zero amplitude.

For destructive interference to occur and completely cancel the two propagating waves, the amplitudes of the two waves when superimposed must be *exactly* equal in magnitude, and opposite in sign.

For a stable interference pattern to occur, the two (or more) sources must be of the same frequency, wavelength, and equal phase (or constant phase difference). If this is the case, the sources are said to be ***coherent sources***. When sources are coherent, wave troughs and peaks overlap at fixed positions in space as the waves travel. For two or more coherent sources;

- Maximum constructive interference occurs when waves are in phase with each other (phase difference is an integer multiple of $360°$ (whole wavelength)).

- Maximum destructive interference occurs when waves are in anti-phase (phase difference is an odd integer multiple of $180°$ (half wavelength))

It is possible for transverse and longitudinal progressive waves to form a standing wave.

- Progressive waves are travelling waves, transferring energy from one region to another.

- Standing waves are stationary, they do not travel through space and thus do not transfer energy from one region to another.

Standing waves are formed from the superposition of two or more coherent progressive waves. For example, a wave on a string that is fixed at both ends may support a progressive wave that reflects at the fixed ends, resulting in two waves travelling in opposite directions that superimpose. Standing waves have regions of zero displacement called ***nodes (N)***, and regions of maximum amplitude called ***antinodes***.

- The lowest frequency standing wave that may be supported is known as the **fundamental** mode of oscillation. It has two regions of zero displacement and one region of maximum displacement.

- A **node** (N) is a point of **zero displacement**.

- An **antinode** is a point of **maximum displacement**.

Nodes occur where the two progressive waves forming the standing wave are 180° or ½ λ out of phase. Antinodes occur where the two progressive waves are in phase. Nodes are separated by ½ of the wavelength (180° of phase), with all points on the wave (except nodes) vibrating at the same frequency.

The wavelength of a standing wave is determined by the dimensions of the system in which it oscillates (the distance between the fixed ends). The lowest frequency and therefore longest wavelength standing wave is the ***fundemental***. It has a mode number $m=1$, and two nodes at the fixed ends. There is a ½ λ seperation between nodes, therefore

$$\frac{1}{2}\lambda_0 = L \Longrightarrow \lambda_0 = 2L$$

and

$$f_0 = \frac{v}{2L}$$

where λ_0 is the wavelength of the fundamental mode, v is wave velocity and L is the distance between end nodes.

The next allowed mode is the first harmonic, $m=2$, with 3 nodes and two antinodes. Each successive mode (m_2, m_3, m_4…… m_n) has one additional node and;

- The frequencies of allowed, successive standing wave modes increase as f_0, $2f_0$, $3f_0$, $4f_0$………..nf_0 where f_0 is the fundamental frequency.

- The wavelength is given by $\lambda = 2L/m$, where L is the length of the system and m is the mode number (for the fundamental mode $m = 1$, for the first harmonic $m = 2$ and so on).

When a wave is incident on a surface that it cannot cross, it is reflected. The incoming wave is called the *incident wave* and the outgoing the *reflected wave*.

When incident at a non-zero angle of incidence θ_i, it is reflected at an angle θ_r. Upon reflection, the incident angle is equal to the reflected angle. This is summarised by the law of reflection, which is true for all plane waves.

The Law of Reflection states that the angle of reflection θ_r is equal to the angle of incidence θ_i

$$\theta_r = \theta_i$$

where θ_r and *θ_I are measured from the normal to the reflecting surface*

The speed at which waves propagate depends upon the medium the wave is travelling in. Some waves travel faster in a dense medium and slower in a less dense medium (sound does this). Some waves travel slower in a dense medium (light does this). When a wave approaches a boundary between two media with different densities that it can cross, it is transmitted, from one medium to the other, and its speed must therefore change. This causes the wave direction to change. This change in direction (or bending of the wave) is known as ***refraction***.

In order to accommodate the change in speed directional change, the wavelength must also change. The frequency of the wave however DOES NOT change.

Diffraction describes waves passing through gaps or around objects. All types of waves diffract. When waves meet a gap in a barrier they pass through the opening. As the wave front passes through it curves, and spreads out. The curving and spreading out is diffraction, and the amount of diffraction that occurs depends on the size of the gap / obstacle compared to the size of the wavelength.

For gaps which are large compared to the wavelength, only a small amount of diffraction occurs. For gaps that are small compared to the wavelength, not much of the wave gets through. Maximum diffraction occurs when the gap is the same size as the wavelength. Note that increasing the wavelength of the wave would be equivalent to reducing the size of the gap.

Glossary

Amplitude – maximum displacement or distance moved by a point on a vibrating body or wave as measured from its rest position.

Coherent – waves of the same frequency with a fixed (constant) phase difference between them.

Equilibrium (vibrational) – the original 'at rest' position of the vibrating particle

Fundamental (mode) – Lowest frequency mode of oscillation (also known as the first harmonic).

Harmonic (wave) – a wave (usually in a series) with a frequency that is an integer multiple of the fundamental frequency.

Interference – the process in which two or more waves of the same frequency combine to reinforce or cancel each other.

Longitudinal - A wave that oscillates back and forth on an axis that is the same as the axis along which the wave propagates.

Normal (to a surface) – a vector that is at right angles to the plane of the surface

Phase - the position of a point in time (an instant) on a waveform cycle

Propagation – movement along a direction of travel

Refraction – the change in direction of a wave passing from one medium to another caused by its change in speed

Superimpose –to impose, place, or set over, above, or on something else

Transverse (waves)- displacement is perpendicular (at right angles to) the direction of propagation