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Unit 3.
Optics

Text

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F.3.1. Lenses

Lenses are made from material that do not absorb light, and therefore are optically transparent. This is because unlike the mirror discussed in Unit 2, lenses transmit light rather than reflect. The purpose of a lens is to change the direction of the light that it transmits through the process of refraction. The *radius of curvature* of the lens and the *refractive index* of the lens material dictates how much the lens changes the propagation direction of light.

Consider *Figure 3.1.1.* Here we see a lens that is made of a solid transparent material (glass for example) that is curved on both sides. Light rays from a point source P pass through air and are incident upon the surface of the lens.

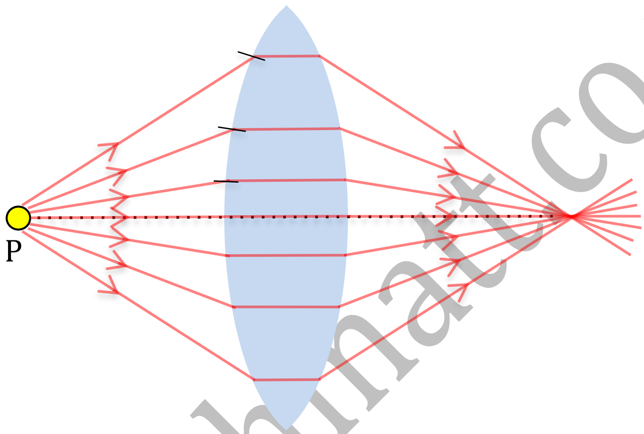


Figure 3.1.1.

The black dotted line passing through the center of the lens is normal to the air-lens interface and passes through the center of the lens. This line is known as the ***optic axis***. A ray travelling along the optic axis reaches the air – lens interface at an incident angle of 0° (measured from the normal). Since the wave fronts of this ray are normally incident, all of the wave front passes through the interface at the same time and therefore is not refracted.

> *Rays that travel along the optic axis pass through the lens without refraction. Their direction is therefore unchanged.*

As we move away from the optic axis the angle of incidence increases. This is because the angle of the ray from the point source increases, and in addition the radius of curvature of the lens relative to the ray effectively rotates the surface normal, increasing the angle of incidence further. Since the angle of refraction for each ray is given by;

$$\theta_r = sin^{-1} \frac{sin\theta_i}{n}$$

and both the angle of the ray and the curvature of the lens progressively increases as we move away from the optic axis, so does the amount of refraction the ray undergoes. Carefully choosing the refractive index and the curvature of the lens allows ***divergent*** (spreading out from a point) rays to be made parallel.

- *The process of converting converging or diverging rays to a set of parallel rays is known as **collimation**.*

Upon exiting the lens, the collimated rays pass through the lens – air interface. Since the rays are now moving from a high to low index medium, they are refracted away from normal, and the rays converge at a point on the other side of the lens.

*The point at which the rays intersect is the **focal point** of the lens.*

Points to note:

- The amount a light ray's direction is changed by a lens depends on the angle of incidence, the radius of curvature, and the refractive index of the lens material.

- Rays that spread out from a point source are ***divergent*** (they diverge).

- Rays that come together at a point are ***convergent*** (they converge).

- The process of converting divergent or convergent rays into parallel rays is ***collimation***.

- The point at which rays meet after converging from a converging lens is the ***focal point*** of the lens.

- A line normal to the surface of the lens passing through its center is the ***optic axis***.

For the main part, lenses can be divided into two categories.

The converging lens.

A converging lens is also known as a ***convex lens***, since the curvature is convex (curved outward) as shown by *Figure F.3.1.2*. (a). A convex lens with one side that is planar makes parallel rays converge at a focal point. The distance between the centre of the lens and the focal point *F* is the ***focal length.*** Notice that there is always a focus point marked on each side of the lens, and the two focal lengths are always equal. A convex lens is used in magnifying glasses, and also cameras to focus a distant image.

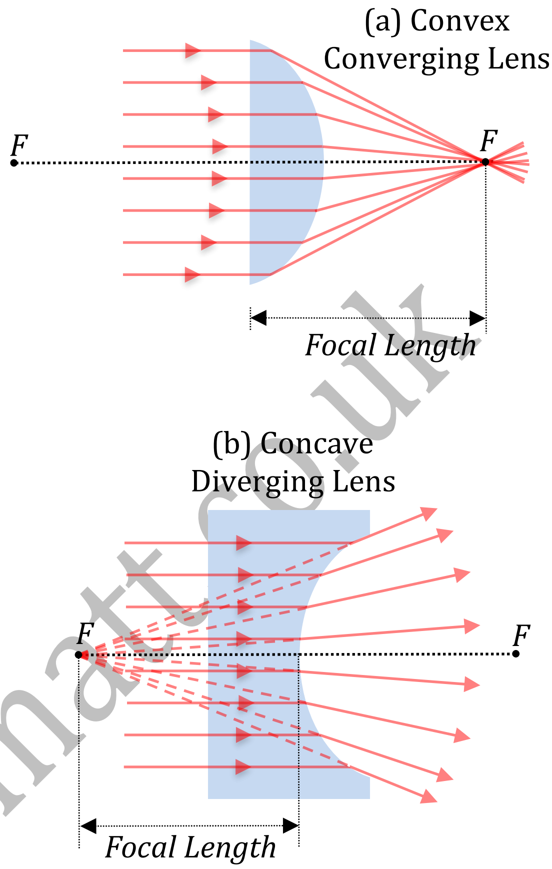


Figure 3.1.2

The diverging lens.

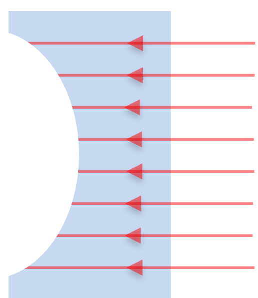
A diverging lens is also known as a ***concave lens***. This is because the surface of the lens is concave (curved inwards). A concave lens that is planar on one side makes parallel rays ***diverge*** (spread out) as shown by *Figure F.3.1.2.* (b). The focal point *F* and length for a concave lens is found by tracing the emerging rays back through the lens to the point at which they intersect (dotted red lines).

The lenses shown in *Figure 3.1.2* have one curved surface and one planar surface. However, lenses often have two curved surfaces. A diverging lens with an inward curvature on each side is called a double concave. Likewise, a converging lens with two convex surfaces as shown in *Figure 3.1.1* is a double convex. There are other combinations designed for specific purpose too, for example a concave on one side and a convex on the other, and graded index lenses which use a gradually changing refractive index instead of a curved surface.

Exercise F.3.1.1.

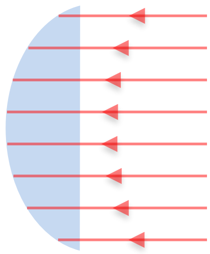
1. For the following diagram showing a glass lens;

(a) State the type of lens
(b) Copy the diagram and add the optic axis in the correct position. Through ray tracing or otherwise, mark the position of both focal points.



(c) Complete the ray diagram, including the ray that travels along the optic axis. Are the rays convergent or divergent?

2. For the following diagram showing a glass lens;



(a) State the type of lens
(b) Copy the diagram and add the optic axis in the correct position. Through ray tracing or otherwise, mark the position of both focal points.
(c) Complete the ray diagram including the ray that travels along the optic axis. Are the rays convergent or divergent?

3. By stating T for true or F for false, identify the correct statements from the list below.

A. A lens which is curved inwards is concave

B. A lens which is curved outward is convex

C. A lens which is curved outward is concave

D. A lens which is curved inwards is convex

4. State which type of lens brings rays together at a focal point. Which type of lens makes rays spread out? Which type of lens *always* creates a virtual image?

5. Rays that are parallel are also;

A. Convergent
B. Convex
C. Collimated
D. Divergent

F.3.2. Using Lenses

So far in our discussion of lenses we have only considered ***point objects*** such as a lamp, which act as a point source of spherical waves with diverging rays emanating radially outward from the source. To consider the forming an image of a real object using a lens, we have to consider the objects height. In optics, objects with height such as a building, person, or tree, are known as extended objects. To simplify ray diagrams, these extended objects are often represented by a slender upward pointing arrow, with a stated length equal to the length of the object being imaged. *Figure 3.2.1* shows the formation of an image from an extended object using a converging lens.

In order to locate the image, three rays are traced from the top of the extended object through the lens (technically only two are required to locate the image in this instance). For an extended object that is positioned ***outside the focal length*** of the lens;

Ray 1 leaves the top of the object and is parallel to the optic axis. It enters the lens and is refracted, passing through the focal point of the lens.

Ray 2 leaves the top of the lens and passes through the centre of the lens, and the optic axis simultaneously.

Ray 3 passes through the focal point on the incident side of the lens, enters the lens, and then emerges parallel to the optic axis. The three rays converge on the exit side of the lens, intersecting at the same point on the image that they left the object (the top of the arrow).

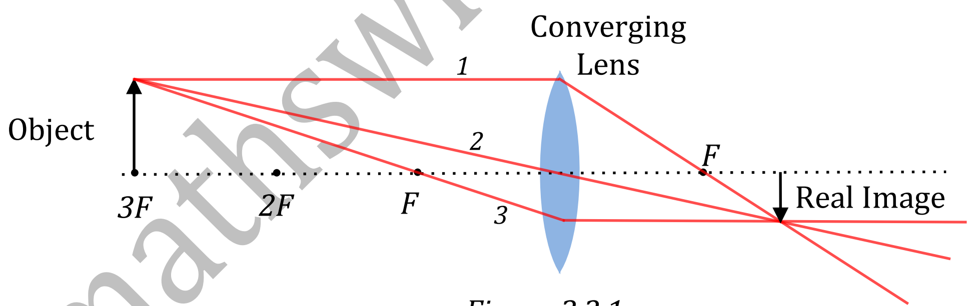


Figure 3.2.1.

The image formed is a *real image* since light rays pass through the lens to a point on the image. As such you can imagine that the image could be projected onto a screen. Light rays intersect at a position below the optic axis. This means that the head of the arrow is below the optic axis whilst the tail of the arrow must be on the optic axis. Therefore, the image is inverted (upside down). Also note that the image length is *less* than the object length. Therefore, the image is diminished (smaller than the object).

For a converging lens with the object at a distance greater than the focal length;

- *A real image is formed*
- *The image is inverted*
- *The image is diminished*

For an extended object that is positioned inside the focal length of the lens, two rays are sufficient to locate the image as shown by *Figure 3.2.2.*

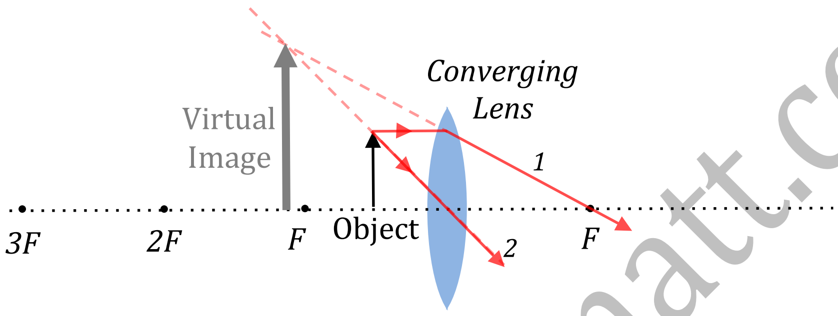


Figure 3.2.2

Again, we trace a ray from the top of the arrow that emerges parallel to the optic axis and is refracted through the focal point *F*. We then trace the second tray from the top of the arrow, which passes through the centre of the lens and the optic axis simultaneously. We immediately notice that the two rays diverge rather than converge. We must therefore locate the image by extending the rays on the incident side of the lens (red dashed lines) and find where they intersect, sine their point of intersection is the location of the image. The image formed has no rays that go to or come from the image, and is therefore virtual. In addition, the *virtual image* is the right way up and larger than the object. This is ***magnification***. Further, the image can *only* be viewed by looking through the lens.

For a converging lens where the object is inside the focal length;

- *A virtual image is formed*
- *The virtual image must be viewed by looking through the lens*
- *The virtual image is the right way up*
- *The virtual image is magnified*

In a modern digital camera, the image is focused by a converging lens at an array of light sensors called a charged coupled device (CCD). When the camera auto focuses, it is the lens – CCD distance, not focal length that is being adjusted. To adjust the focal length, the lens would have to change. To focus a distant object, the lens – CCD distance must be equal to the focal length of the lens. The nearer the object is to the camera, the greater the lens – CCD distance required.

The magnification produced by the lens is the ratio of the image height and the object height. Since the magnification is the ratio of two lengths, the magnification has no units.

- Magnification greater than 1 – image is larger than the object
- Magnification less than 1 – image is smaller than the object
- Magnification equal to 1 – image is the same size as the object

$$magnification = \frac{image\ height}{object\ height}$$

or in symbols

$$m = \frac{h_i}{h_o}$$

where m = magnification
h_i = image height
h_o = object height

The image formed from a concaved converging lens is virtual when the object is within a focal length and real when the object is outside the focal length. For a diverging lens however, the image is *always* virtual since on the real image side of the lens, the rays are always diverging. This is shown in Figure 3.2.3 for an extended object.

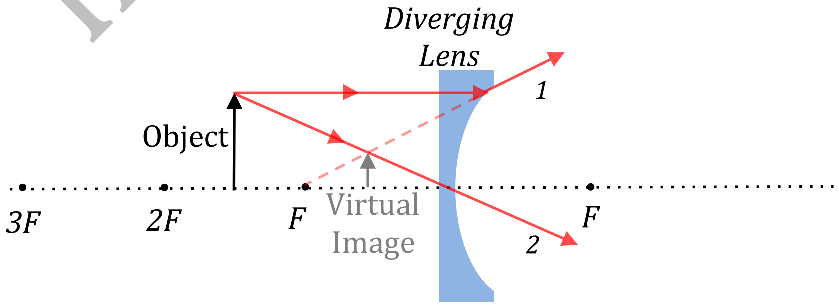


Figure 3.2.3

Ray 1 leaves the object parallel to the optic axis and is refracted away from normal at the curved surface of the lens.

Ray 2 crosses the optic axis at the center of the lens.

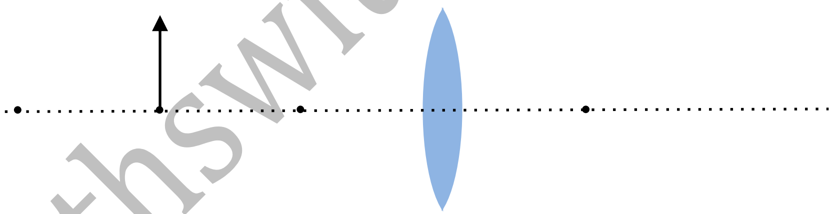
Since both rays are diverging to the right of the lens (which would be true for any object distance), we must trace the rays back through the lens to find the intersection in order to locate the image. As we see, the intersection is above the optic axis, and therefore the virtual image is upright, and smaller than the object.

For a diverging lens;

- *The image is always virtual*
- *The image is always upright*
- *The image is always smaller than the object*

Exercise F.3.2.

1. Copy and complete the following ray diagram to locate the image of the object.



(a) Label the diagram.
(b) Is the lens shown a concave or convex?
(c) Which orientation is the image? State whether the image is real, or virtual. Give your reasoning.
(d) Is the magnification greater than, equal to, or less than 1?

2. A 2cm long needle is viewed through a magnifying glass. What type of lens would this be? The image of the needle is 5cm long. Calculate the magnification of the lens.

3. The needle in question 2 is now viewed through another magnifying glass with m = 3.5. How long is the image?

4. What can be said about the images formed by a;

(a) Convex lens with the object distance greater than the focal length
(b) Converging lens with the object distance less than the focal length
(c) Diverging lens with the object distance greater than the focal length

Challenge Question

5. Draw a scaled ray diagram with an object height of 3.5cm and object distance of 2cm to show how a converging lens with a focal length of 4cm can be used as a magnifying glass. Use your diagram to locate the image. State the image distance, image height, and the magnification.

F.3.3. The Lens Makers Formula

In order to make lenses for reading glasses, cameras, telescopes or any other device that forms an image of an object, the distance from the lens to the screen at which the image is to be formed must be matched to the focal length of the lens. As a rule, (not in the case of the eye, as we will learn later) the focal length of the lens is fixed because the shape of the lens is fixed. It is no good putting a lens with a focal length of 10cm into a camera with a lens – CCD distance of 2cm! Lenses designed for specific purpose require a specific focal length, and to assess the suitability of the lens, the following lens maker's formula must be used:

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{f}$$

where
R_1 = the distance from the object to the lens
R_2 = the distance from the lens to the image
f = focal length

To use this formula, the '*real is positive*' convention must be followed;

- Objects distances are assigned positive values of R_1
- Real image distances are assigned positive values of R_2
- Virtual image distances are assigned negative values of R_2

- For converging (convex) lenses the focal length is positive
- For diverging (concave) lenses the focal length is negative

Worked Example

A camera with a converging lens requires a maximum lens – CCD distance of 2.5cm when it is focused on an object 30cm away from the lens if it is to focus a real image. What would be the focal length of a suitable lens for this camera? What would be the focal length if the lens was divergent (concaved)?

Answer

The object distance R_1 is 30cm = 0.30m.
The image distance must be R_2 must be 2.5cm = 0.025m. Since the image is real, R_2 is positive.

The focal length is given by the lens maker's formula;

$$\frac{1}{f} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{0.3} + \frac{1}{0.025} = 43.3$$

Therefore

$$\frac{1}{f} = 43.3 \; or \; f = \frac{1}{43.3} = 0.023m$$

Note: It would have been an easy mistake to calculate f as 43.3. Be careful to remember that the lens maker's formula calculates $1/f$, which then must be converted to f.

If the lens were concaved, the image would be virtual and therefore both f and R_2 are negative;

$$\frac{1}{f} = \frac{1}{R_1} + \frac{1}{-R_2} = \frac{1}{0.3} + \frac{1}{-0.025} = -36.6$$

$$\frac{1}{f} = -36.6 \; or \; f = \frac{1}{-36.6} = -0.027m$$

Exercise F.3.3

1. A converging lens is used to image a coin that is placed 4cm from the lens. The focal length of the lens is 0.09m. Calculate the distance from the lens to the image. Is the image real, or virtual?

2. A classroom projector is used to project a PowerPoint presentation onto the whiteboard. The object – lens distance is 0.05m and the lens has a focal length of 0.049m. How far from the lens is the whiteboard if the image is in focus?

3. A diverging lens is placed 35cm from an object. Calculate the focal length of the lens if the image distance is 3.3cm.

4. A converging lens has a focal length of 20cm. Find the image distances for the following object distances and in each case state whether the image is real, or virtual.

(a) R_1 = 45cm
(b) R_1 = 25cm
(c) R_1 = 15cm
(d) R_1 = 5cm

Challenge question

5. A lens is used to form an image of an object. The object is 14.0 cm from the lens. The image is 6.5 cm from the lens on the same side as the object.

(a) What is the focal length of the lens?

(b) Is the lens converging or diverging?

(c) Is the image erect or inverted?

F.3.4. The Eye

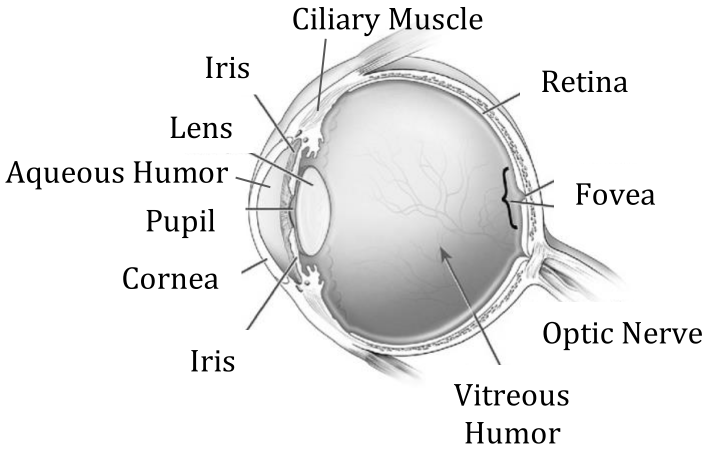


Figure 3.4.1.

The optical system of the eye, which behaves similarly to a camera, is shown in *Figure 3.4.1.* The human eye is roughly 2.5cm in diameter and nearly spherical, but the front is more sharply curved than the back. The front of the eye is covered by a tough transparent membrane called the *cornea*, which acts as a protective covering and holds in the *aqueous humor* (a fluid filled sack between the cornea and the lens). The *lens* itself is a sack of fibrous jelly that is harder in the middle and becomes progressively softer towards the outside. The lens is held in place by ligaments attached to the *ciliary muscle*. Behind the lens is a cavity filled with a thin jelly like fluid. This is the *vitreous humor*. At the back of the eye is the light sensitive *retina*, upon which the lens focuses the real, and upside down (inverted) image of the object. The retina acts like the film or the CCD of a camera, detecting light and transmitting electrical signals, which are proportional to the light detected, to the *optic nerve*.

The lens, vitreous humor, and the aqueous humor have very similar refractive index (the refractive index of the lens is approximately 1.47). This means that most of the refraction of the light that enters the eye occurs at the air-cornea interface. The *iris* is located in front of the lens and contains a variable opening (aperture) called the *pupil*. The pupil adjusts the amount of light entering the eye.

In order for an object to be viewed, the image of the object must be projected onto the retina. For the image to be *sharp*, it must be projected *exactly* onto the retina. Unlike a camera, the eye cannot adjust the position of the focal point by adjusting the lens – image distance. Instead, it copes with the varying distances to objects by adjusting the focal length of the lens. To do this, the ciliary muscles contract, changing the shape of the lens. For objects that are distant, the ciliary muscles are relaxed. For nearby objects, the muscles contract, pulling the lens and reshaping it, as shown by *Figure 3.4.2* (a) and (b). A summary comparison of the

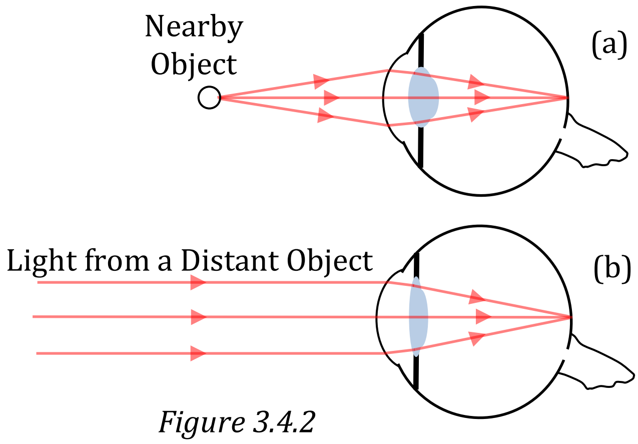


Figure 3.4.2

optical systems of the eye and the modern camera is shown in *Table 3.4.1.*

Table 3.4.1.

Property	**The Eye**	**The Camera**
Type of Lens	Variable focal length converging lens	Fixed focal length converging lens
Image	Real, inverted, magnification < 1	Real, inverted, magnification < 1
Focusing	Ciliary muscle alters lens shape / thickness	Adjustment of lens position relative to film / CCD
Image Detection	Light sensitive cone and rod like cells of the retina	Light sensitive photographic film or CCD
Brightness Control	Iris controls size of the pupil	Adjustment of the aperture (the *f*-stop of the camera)

Sight defects occur when the lens does not focus the image of the object directly onto the retina. In long sightedness, this is a problem with focusing the light from a nearby object onto the retina. Instead, the focal point of the lens is behind the retina, leaving a blurred and out of focus image on the retina as shown by *Figure 3.4.3 (a).* This occurs when the eyeball is to short or the cornea is too curved, and the lens cannot be made thick enough to focus the image at the retina. Long sightedness is corrected using a converging lens which essentially adds power to the eye's lens, making the rays from the object diverge less, as shown by *Figure 3.4.3.* (b). The corrected image is then formed at the retina.

Short sightedness is a problem with focusing far away objects. Here the parallel rays from the distant object is focused in front of the retina, because the ciliary muscles cannot make the lens thin enough to place the focal point at the retina (*Figure 3.4.3.* (c)). A diverging lens in front of the eye corrects short sightedness by diverging the rays slightly in front of the eye as shown by *Figure 3.4.3.* (d).

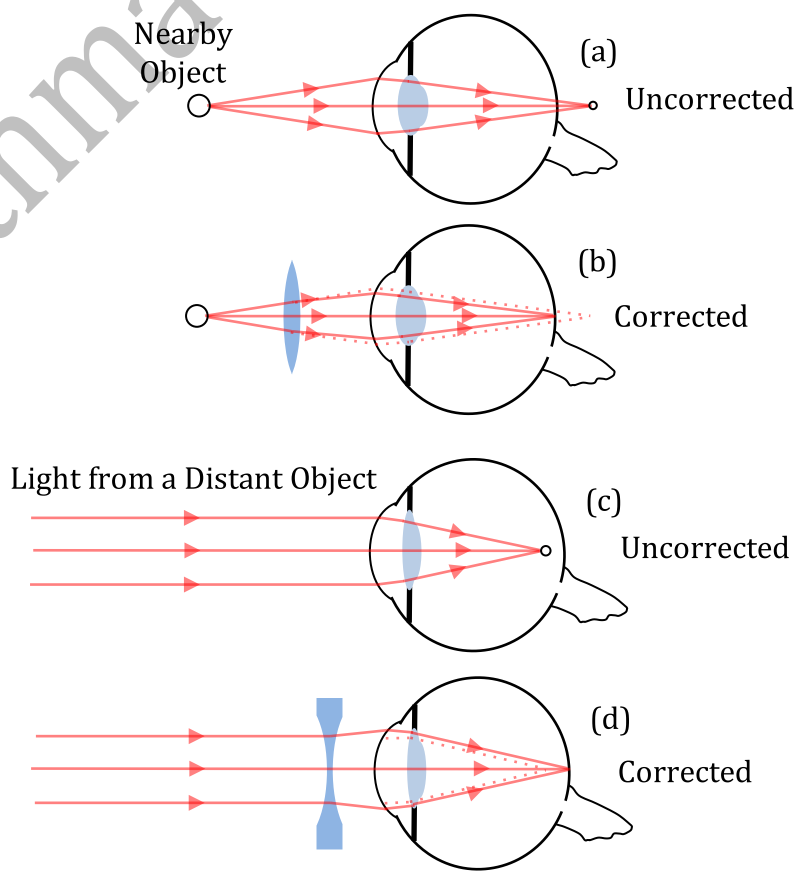


Figure 3.4.3

The power of the corrective lens is given by;

$$Power = \frac{1}{focal\ length}$$

or in symbols

$$P = \frac{1}{f}$$

where
P = Power of the lens
f = focal length

The unit of lens power is the *Dioptre* (D) or m^{-1}, and the type of lens used is signified as a positive number for a converging lens, and a negative number for a diverging lens. Thus +4D would be a converging lens with a focal length of 1/4 = 0.25m and a -6D would be a diverging lens with a focal length of -1/6 = -0.16m.

Exercise 3.4

1. Sketch a simple representation of the eye. Label the following:

(a) Cornea
(b) Iris
(c) Lens
(d) Retina
(e) Ciliary Muscles
(f) Vitreous Humor

2. State the function of each of the following;

(a) Cornea
(b) Iris
(c) Lens
(d) Retina
(e) Ciliary Muscles

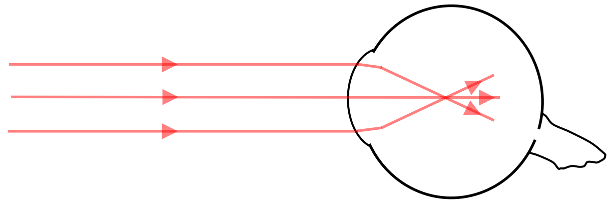
3. Explain the symptoms of long and short sightedness. Which type of lens would be used to correct each sight defect?

4. What is the power in Dioptres of the following lenses;

(a) A diverging lens of focal length 0.75m
(b) A converging lens of focal length 0.5m

Challenge Question

5. Consider the following diagram



(a) Which sight defect is this?

(b) Which correcting lens would be required to correct this defect?

(c) An optician prescribes a -2.8D lens. What is the focal length of the lens in centimeters?