CONSISTENCY OF EQUATIONS

Question 1 **(***)**

The system of simultaneous equations

$$x+2y+z=1$$
$$2x+3y+z=3$$
$$3x+4y+z=k$$

where k is a scalar constant, does not have a unique solution, but is consistent.

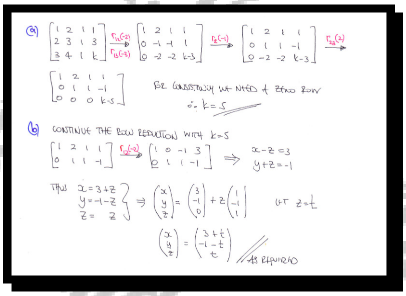
a) Determine the value of k.

b) Show that the general solution of the system can be written as

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} t+3 \\ -t-1 \\ t \end{pmatrix},$$

where t is a scalar parameter.

$\boxed{k=5}$



Question 2 (***)

$$\begin{pmatrix} 2 & 5 & 3 \\ 1 & 2 & 2 \\ 1 & 1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 10 \end{pmatrix}.$$

Show that the above simultaneous equations …

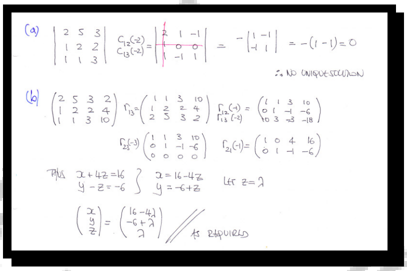
a) … do **not** have a unique solution.

b) … are **consistent** and their general solution can be written as

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 16-4\lambda \\ \lambda-6 \\ \lambda \end{pmatrix},$$

where λ is a scalar parameter.

proof



Question 3 (***)

The system of simultaneous equations

$$\begin{aligned} x + \ \ y + 2z &= 2 \\ x + 2y + \ \ z &= 2 \\ 2x + ay + 5z &= b \end{aligned}$$

where a and b are constants, does **not** have a unique solution, but it is **consistent**.

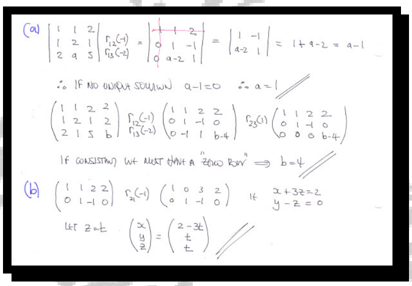
a) Determine the value of a and the value of b.

b) Show that the general solution of the system can be written as

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2-3t \\ t \\ t \end{pmatrix},$$

where t is a parameter.

$\boxed{a=1}$, $\boxed{b=4}$



Question 4 (***)

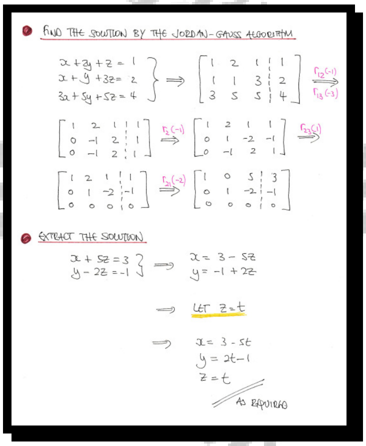
$$\begin{aligned} x+2y+\ \ z &= 1 \\ x+\ \ y+3z &= 2 \\ 3x+5y+5z &= 4 \end{aligned}$$

Show that the solution of the above simultaneous equations is

$$x = 3-5t, \quad y = 2t-1, \quad z = t$$

where t is a parameter.

[   ], [proof]



Question 5 (***)

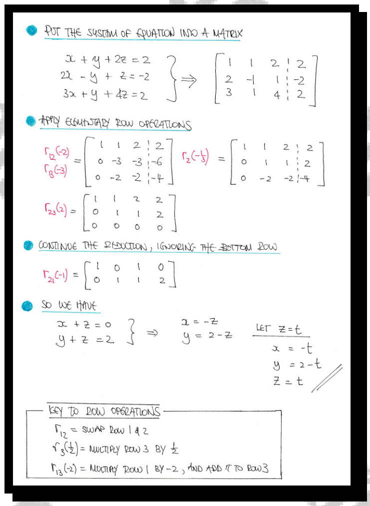
$$
\begin{aligned}
x + y + 2z &= 2 \\
2x - y + z &= -2 \\
3x + y + 4z &= 2
\end{aligned}
$$

Show, by reducing the augmented matrix of the above system of equations into row echelon form, that the solution can be written as

$$x = -t, \qquad y = 2 - t, \qquad z = t$$

where t is a scalar parameter.

proof



- PUT THE SYSTEM OF EQUATION INTO A MATRIX

$$\left.\begin{aligned} x + y + 2z &= 2 \\ 2x - y + z &= -2 \\ 3x + y + 4z &= 2 \end{aligned}\right\} \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & 2 \\ 2 & -1 & 1 & -2 \\ 3 & 1 & 4 & 2 \end{array}\right]$$

- APPLY ELEMENTARY ROW OPERATIONS

$$\begin{matrix} r_{12}(-2) \\ r_{13}(-3) \end{matrix} = \left[\begin{array}{ccc|c} 1 & 1 & 2 & 2 \\ 0 & -3 & -3 & -6 \\ 0 & -2 & -2 & -4 \end{array}\right] \quad r_2(-\tfrac{1}{3}) = \left[\begin{array}{ccc|c} 1 & 1 & 2 & 2 \\ 0 & 1 & 1 & 2 \\ 0 & -2 & -2 & -4 \end{array}\right]$$

$$r_{23}(2) = \begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

- CONTINUE THE REDUCTION, IGNORING THE BOTTOM ROW

$$r_{21}(-1) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 2 \end{bmatrix}$$

- SO WE HAVE

$$\left.\begin{aligned} x + z &= 0 \\ y + z &= 2 \end{aligned}\right\} \Rightarrow \begin{aligned} x &= -z \\ y &= 2 - z \end{aligned} \qquad \text{LET } z = t: \quad x = -t,\; y = 2 - t,\; z = t$$

KEY TO ROW OPERATIONS
r_{12} = SWAP ROW 1 & 2
$r_3(\tfrac{1}{2})$ = MULTIPLY ROW 3 BY $\tfrac{1}{2}$
$r_{13}(-2)$ = MULTIPLY ROW 1 BY −2, AND ADD IT TO ROW 3

Question 6 **(***)**

A system of equation is given in matrix form below

$$\begin{pmatrix} t & 2 & 3 \\ 2 & 3 & -t \\ 3 & 5 & t+1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix},$$

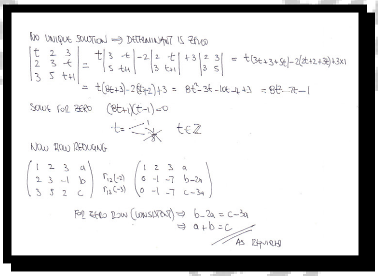
where t is an integer constant, and a, b and c are real constants.

The system of equations does not have a unique solution, but it is consistent.

Show clearly that

$$a+b=c.$$

proof


NO UNIQUE SOLUTION ⇒ DETERMINANT IS ZERO

$$\begin{vmatrix} t & 2 & 3 \\ 2 & 3 & -t \\ 3 & 5 & t+1 \end{vmatrix} = t\begin{vmatrix} 3 & -t \\ 5 & t+1 \end{vmatrix} - 2\begin{vmatrix} 2 & -t \\ 3 & t+1 \end{vmatrix} + 3\begin{vmatrix} 2 & 3 \\ 3 & 5 \end{vmatrix} = t(3t+3+5t) - 2(2t+2+3t) + 3\times 1$$

$$= t(8t+3) - 2(5t+2) + 3 = 8t^2 - 3t - 10t - 4 + 3 = 8t^2 - 7t - 1$$

SOLVE FOR ZERO $(8t+1)(t-1)=0$

$t = 1$ ~~$-\frac{1}{8}$~~ $\quad t \in \mathbb{Z}$

NOW ROW REDUCING

$$\begin{pmatrix} 1 & 2 & 3 & a \\ 2 & 3 & -1 & b \\ 3 & 5 & 2 & c \end{pmatrix} \begin{matrix} r_{12}(-2) \\ r_{13}(-3) \end{matrix} \begin{pmatrix} 1 & 2 & 3 & a \\ 0 & -1 & -7 & b-2a \\ 0 & -1 & -7 & c-3a \end{pmatrix}$$

FOR ZERO ROW (CONSISTENT) ⇒ $b-2a = c-3a$

⇒ $a+b=c$

AS REQUIRED

Question 7 (***)

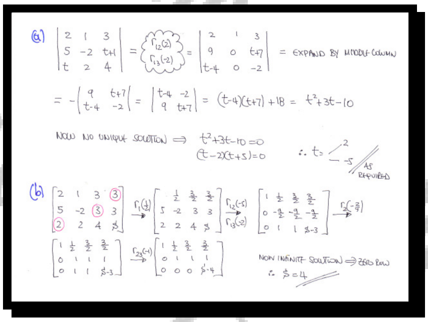
A system of equations is given below in terms of the scalar parameters t and s.

$$\begin{aligned} 2x \quad + y \qquad\quad + 3z &= t + 1 \\ 5x \; - 2y + (t+1)z &= 3 \\ tx \; + 2y \qquad\quad + 4z &= s \end{aligned}$$

a) Show that if $t = -5$ or $t = 2$, the system does not have a unique solution.

b) Determine the value of s is the system is to have infinite solutions with $t = 2$.

$\boxed{s = 4}$



Question 8 (***)

The three planes defined by the equations

$$\begin{aligned} x + 2y + \quad z &= 2 \\ 2x + a\,y + \quad z &= 2 \\ x + \quad y + 2z &= b \end{aligned}$$

where a and k are constants, intersect along a straight line L.

Determine an equation of L.

$\boxed{}$, $\boxed{\mathbf{r} = (6-3t)\mathbf{i} + (t-2)\mathbf{j} + t\mathbf{k}}$

- FIRSTLY IF THERE IS NO UNIQUE SOLUTION, THE DETERMINANT OF THE MATRIX $\begin{bmatrix} 1 & 2 & 1 \\ 2 & a & 1 \\ 1 & 1 & 2 \end{bmatrix}$ MUST BE ZERO

- EXPAND BY TOP ROW

$$1\begin{vmatrix} a & 1 \\ 1 & 2 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} + 1\begin{vmatrix} 2 & a \\ 1 & 1 \end{vmatrix} = 0$$

$$(2a-1) - (2\times 3) + (2-a) = 0$$

$$2a - 1 - 6 + 2 - a = 0$$

$$a - 5 = 0$$

$$a = 5$$

- START ROW REDUCING TO OBTAIN A "BOTTOM ZERO ROW" IF THE SYSTEM IS TO BE CONSISTENT

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 2 & 5 & 1 & 2 \\ 1 & 1 & 2 & b \end{array}\right] \begin{matrix} r_{12}(-2) \\ r_{13}(-1) \end{matrix} = \left[\begin{array}{ccc|c} 0 & 2 & 1 & 2 \\ 0 & 1 & -1 & -2 \\ 0 & -1 & 1 & b-2 \end{array}\right]$$

$$r_{23}(1) = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & b-4 \end{array}\right] \quad \therefore\ b = 4$$

- CONTINUING ROW REDUCING, IGNORING THE BOTTOM ROW

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 1 & -1 & -2 \end{array}\right] r_{21}(-2) = \left[\begin{array}{ccc|c} 1 & 0 & 3 & 6 \\ 0 & 1 & -1 & -2 \end{array}\right]$$

- EXTRACTING THE SOLUTION, WE HAVE

$$x + 3z = 6$$
$$y - z = -2$$

- LET $z = t$, SOME PARAMETER

$$x = 6 - 3t$$
$$y = -2 + t$$
$$z = t$$

IE $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ -2 \\ 0 \end{pmatrix} + t\begin{pmatrix} -3 \\ 1 \\ 1 \end{pmatrix}$

OR $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6-3t \\ t-2 \\ t \end{pmatrix}$ //

Question 9 (***)

$$\begin{aligned} x + y - 2z &= 2 \\ 3x - y + 6z &= 2 \\ 6x + 5y - 9z &= k \end{aligned}$$

a) Show that the system of equations does not have a unique solution.

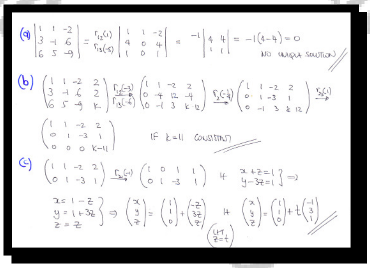
b) Show that there exists a value of k for which the system is consistent.

c) Show, by reducing the system into row echelon form, that the consistent solution of the system can be written as

$$x = 1 - t, \qquad y = 3t + 1, \qquad z = t$$

where t is a scalar parameter.

$\boxed{k = 11}$



Question 10 (***)

$$3x - y + 5z = 5$$
$$2x + y - 5z = 10$$
$$x + y + kz = 7$$

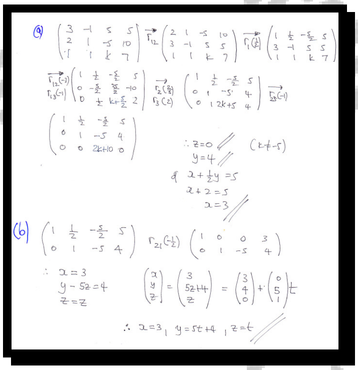
where k is a constant.

a) Given that $k \neq -5$ find the unique solution of the system of equations.

b) Given instead that $k = -5$ show, by reducing the system into row echelon form, that the consistent solution of the system can be written as

$$x = 3, \quad y = 5t + 4, \quad z = t.$$

$\boxed{x = 3, \quad y = 4, \quad z = 0}$



Question 11 (***)

A system of equations is given below

$$\begin{aligned} x + 2y - z &= 4 \\ 2x - y + z &= 5 \\ 4x - 7y + 5z &= 7 \end{aligned}$$

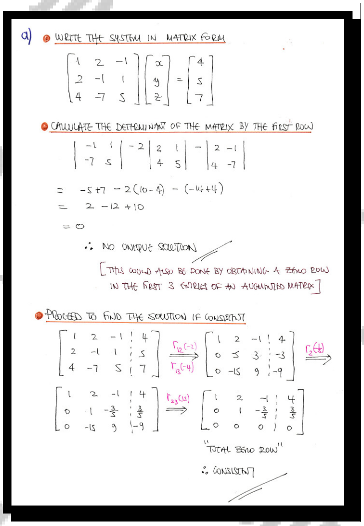
a) Show that the system does not have a unique solution but is consistent.

b) Show that the general solution of the system can be written as

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2-t \\ 3+3t \\ 4+5t \end{pmatrix},$$

where t is a parameter.

[ ] , [proof]



a) WRITE THE SYSTEM IN MATRIX FORM

$$\begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 1 \\ 4 & -7 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 7 \end{bmatrix}$$

CALCULATE THE DETERMINANT OF THE MATRIX BY THE FIRST ROW

$$\begin{vmatrix} -1 & 1 \\ -7 & 5 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ 4 & 5 \end{vmatrix} - \begin{vmatrix} 2 & -1 \\ 4 & -7 \end{vmatrix}$$

$= -5+7-2(10-4)-(-14+4)$

$= 2-12+10$

$= 0$

$\therefore$ NO UNIQUE SOLUTION

[THIS COULD ALSO BE DONE BY OBTAINING A ZERO ROW IN THE FIRST 3 ENTRIES OF AN AUGMENTED MATRIX]

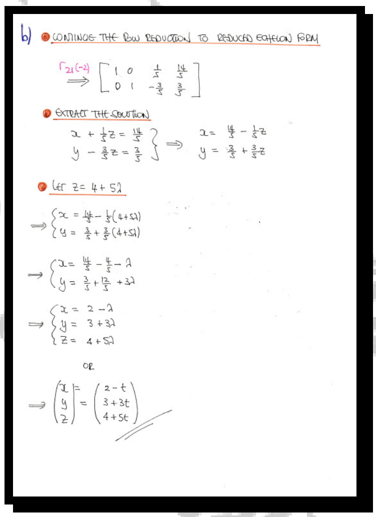
PROCEED TO FIND THE SOLUTION IF CONSISTENT

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 2 & -1 & 1 & 5 \\ 4 & -7 & 5 & 7 \end{array}\right] \xrightarrow[r_{13}(-4)]{r_{12}(-2)} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 0 & -5 & 3 & -3 \\ 0 & -15 & 9 & -9 \end{array}\right] \xrightarrow{r_2(-\frac{1}{5})}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 0 & 1 & -\frac{3}{5} & \frac{3}{5} \\ 0 & -15 & 9 & -9 \end{array}\right] \xrightarrow{r_{23}(15)} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 0 & 1 & -\frac{3}{5} & \frac{3}{5} \\ 0 & 0 & 0 & 0 \end{array}\right]$$

"TOTAL ZERO ROW"

$\therefore$ CONSISTENT



b) CONTINUE THE ROW REDUCTION TO REDUCED ECHELON FORM

$$\xrightarrow{r_{21}(-2)} \left[\begin{array}{cccc} 1 & 0 & \frac{1}{5} & \frac{14}{5} \\ 0 & 1 & -\frac{3}{5} & \frac{3}{5} \end{array}\right]$$

EXTRACT THE SOLUTION

$$\left.\begin{aligned} x + \tfrac{1}{5}z &= \tfrac{14}{5} \\ y - \tfrac{3}{5}z &= \tfrac{3}{5} \end{aligned}\right\} \Rightarrow \begin{aligned} x &= \tfrac{14}{5} - \tfrac{1}{5}z \\ y &= \tfrac{3}{5} + \tfrac{3}{5}z \end{aligned}$$

LET $z = 4 + 5\lambda$

$$\Rightarrow \begin{cases} x = \frac{14}{5} - \frac{1}{5}(4+5\lambda) \\ y = \frac{3}{5} + \frac{3}{5}(4+5\lambda) \end{cases}$$

$$\Rightarrow \begin{cases} x = \frac{14}{5} - \frac{4}{5} - \lambda \\ y = \frac{3}{5} + \frac{12}{5} + 3\lambda \end{cases}$$

$$\Rightarrow \begin{cases} x = 2 - \lambda \\ y = 3 + 3\lambda \\ z = 4 + 5\lambda \end{cases}$$

OR

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2-t \\ 3+3t \\ 4+5t \end{pmatrix}$$

Question 12 (***)

$$\mathbf{A} = \begin{pmatrix} 9 & 2 & k \\ 1 & -1 & -3 \\ k-1 & 1 & 3 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}.$$

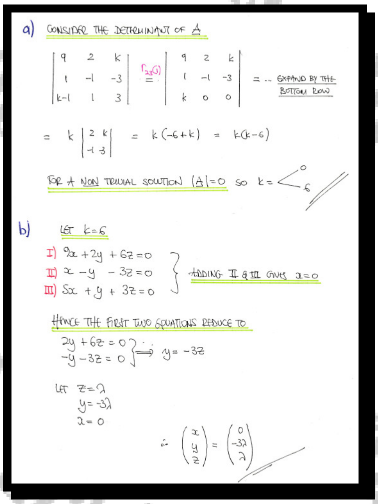
Consider the homogeneous system of simultaneous equations

$$\mathbf{Ax} = \mathbf{0}$$

a) Find the values of k for which the system has a **non trivial** solution.

b) If $k \neq 0$ find the general, non trivial solution of the system.

$k = 0,\ 6$, $x = 0,\ y = -3\lambda,\ z = \lambda$


a) CONSIDER THE DETERMINANT OF $\underline{A}$

$$\begin{vmatrix} 9 & 2 & k \\ 1 & -1 & -3 \\ k-1 & 1 & 3 \end{vmatrix} \overset{r_{23}(1)}{=} \begin{vmatrix} 9 & 2 & k \\ 1 & -1 & -3 \\ k & 0 & 0 \end{vmatrix} = \dots \text{ EXPAND BY THE BOTTOM ROW}$$

$$= k\begin{vmatrix} 2 & k \\ -1 & -3 \end{vmatrix} = k(-6+k) = k(k-6)$$

FOR A NON TRIVIAL SOLUTION $|\underline{A}| = 0$ SO $k = 0, 6$

b) LET $k = 6$

I) $9x + 2y + 6z = 0$
II) $x - y - 3z = 0$
III) $5x + y + 3z = 0$
ADDING II & III GIVES $x = 0$

HENCE THE FIRST TWO EQUATIONS REDUCE TO

$2y + 6z = 0$, $-y - 3z = 0$ $\Rightarrow$ $y = -3z$

LET $z = \lambda$
$y = -3\lambda$
$x = 0$

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ -3\lambda \\ \lambda \end{pmatrix}$$

Question 13 (***)

Consider the system of simultaneous equations

$$\begin{aligned} kx + ky - z &= -1 \\ ky + 2z &= 2k \\ x + 2y + z &= 1 \end{aligned}$$

where the constant k can **only** take the values 0, 1 and 2.

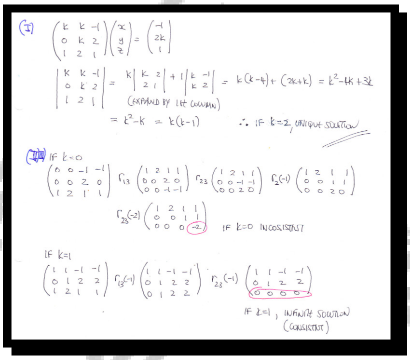
Determine for each of the possible values of k whether the system …

i. … has a unique solution

ii. … has no unique solution, but it is consistent.

iii. … is inconsistent.

$\boxed{k = 0 \Rightarrow \text{incosistent}}$, $\boxed{k = 1 \Rightarrow \text{no unique solution/consistent}}$, $\boxed{k = 2 \Rightarrow \text{unique solution}}$



Question 14 (***)

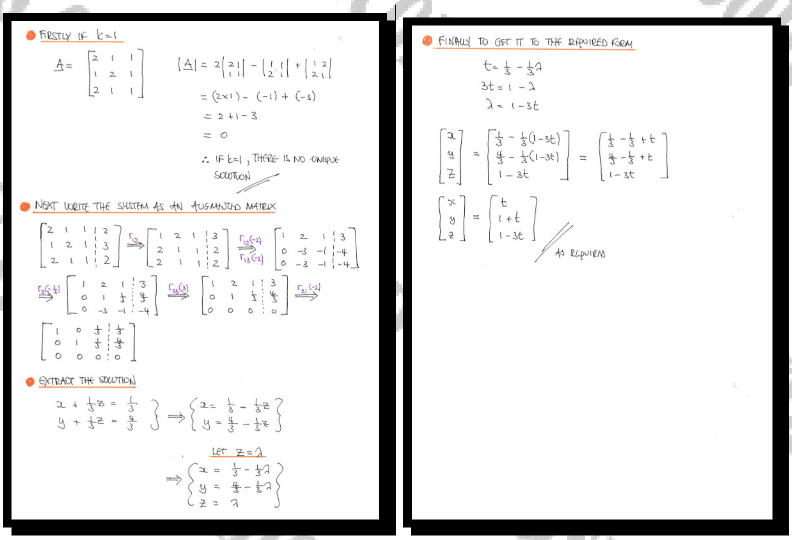
$$\mathbf{A} = \begin{pmatrix} k+1 & 1 & k \\ 1 & 2 & k \\ 2 & k & 1 \end{pmatrix}, \qquad \mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}.$$

Show that the system of equations $\mathbf{Ax} = \mathbf{b}$ does not have a unique solution, but is consistent if $k = 1$, and its general solution in this case can be written as

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} t \\ 1+t \\ 1-3t \end{pmatrix}.$$

where t is a scalar parameter.

[ ], [proof]



Question 15 (***)

A system of equations is given below

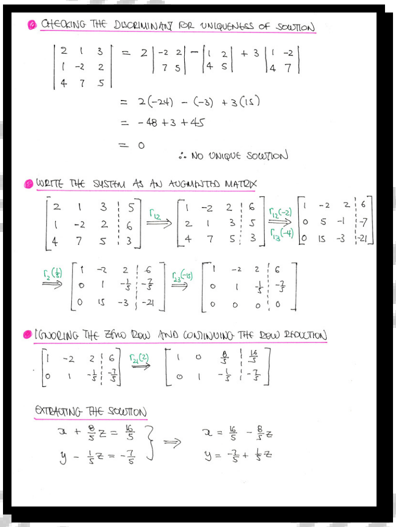
$$\begin{pmatrix} 2 & 1 & 3 \\ 1 & -2 & 2 \\ 4 & 7 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \\ 3 \end{pmatrix}.$$

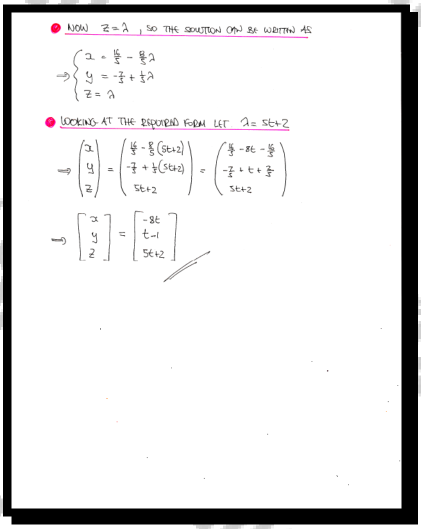
Show that the system does not have a unique solution but is consistent and its general solution can be written as

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -8t \\ t-1 \\ 5t+2 \end{pmatrix},$$

where t is a parameter.

[ ] , [proof]





Question 16 (***)

$$\begin{pmatrix} 3 & k & -1 \\ 2k & 1 & 0 \\ 1 & -1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$$

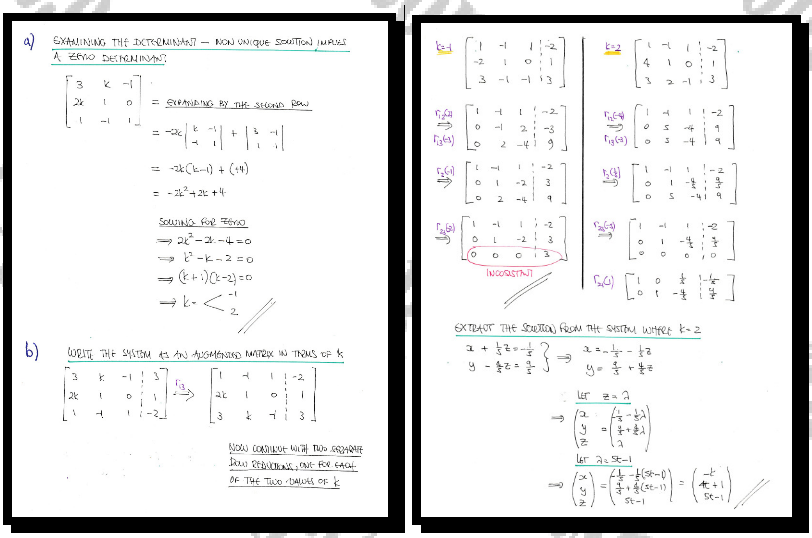
a) Determine the values of k for which the above system of equations does not have a unique solution.

b) Show, that one of these values of k leads to inconsistency and the other produces a general solution of the form

$$x = -t\,, \quad y = 4t+1\,, \quad z = 5t-1$$

where t is a scalar parameter.

[ ] , $k = -1, \ 2$



Question 17 (***)

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 3 \\ 4 & 5 & 7 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ 3 \\ b \end{pmatrix}$$

Show that the system of equations $\mathbf{Ax} = \mathbf{b}$ does not have a unique solution, but for a certain value of b is consistent and its general solution can be written as

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3+4\lambda \\ \lambda \\ -1-3\lambda \end{pmatrix}$$

where λ is a parameter.

[ ], proof

EXAMINING THE DETERMINANT OF $\underline{A}$

$$|A| = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 3 \\ 4 & 5 & 7 \end{vmatrix} = \begin{vmatrix} 1 & 3 \\ 5 & 7 \end{vmatrix} - 2\begin{vmatrix} 2 & 3 \\ 4 & 7 \end{vmatrix} + 2\begin{vmatrix} 2 & 1 \\ 4 & 5 \end{vmatrix}$$

$$= -8 - 2(2) + 2(6) = -8 - 4 + 12 = 0$$

$\therefore$ NO UNIQUE SOLUTION

WRITE THE SYSTEM AS AN AUGMENTED MATRIX

$$\left[\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 2 & 1 & 3 & 3 \\ 4 & 5 & 7 & b \end{array}\right] \xrightarrow[r_{13}(-4)]{r_{12}(-2)} \left[\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 0 & -3 & -1 & 1 \\ 0 & -3 & -1 & b-4 \end{array}\right]$$

FOR CONSISTENCY WE REQUIRE A ZERO ROW WHICH EVIDENTLY OCCURS WHEN $b-4=1$, IE $b=5$.

CONTINUING THE ROW REDUCTION WITHOUT THE BOTTOM ROW

$$\left[\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 0 & -3 & -1 & 1 \end{array}\right] \xrightarrow{r_2(-\frac{1}{3})} \left[\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 0 & 1 & \frac{1}{3} & -\frac{1}{3} \end{array}\right] \xrightarrow{r_{21}(-2)} \left[\begin{array}{ccc|c} 1 & 0 & \frac{4}{3} & \frac{5}{3} \\ 0 & 1 & \frac{1}{3} & -\frac{1}{3} \end{array}\right]$$

EXTRACTING THE SOLUTION

$$\left.\begin{array}{l} x + \frac{4}{3}z = \frac{5}{3} \\ y + \frac{1}{3}z = -\frac{1}{3} \end{array}\right\} \Rightarrow \begin{array}{l} x = \frac{5}{3} - \frac{4}{3}z \\ y = -\frac{1}{3} - \frac{1}{3}z \end{array}$$

$\Rightarrow$ LET $z = t$

$$\Rightarrow \begin{cases} x = \frac{5}{3} - \frac{4}{3}t \\ y = -\frac{1}{3} - \frac{1}{3}t \\ z = t \end{cases}$$

LOOKING AT THE REQUIRED FORM OF THE SOLUTION, WE LET $t = -1-3\lambda$ (BY LOOKING AT z)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{5}{3} - \frac{4}{3}t \\ -\frac{1}{3} - \frac{1}{3}t \\ t \end{bmatrix} = \begin{bmatrix} \frac{5}{3} - \frac{4}{3}(-1-3\lambda) \\ -\frac{1}{3} - \frac{1}{3}(-1-3\lambda) \\ -1-3\lambda \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{5}{3} + \frac{4}{3} + 4\lambda \\ -\frac{1}{3} + \frac{1}{3} + \lambda \\ -1-3\lambda \end{bmatrix} = \begin{bmatrix} 3+4\lambda \\ \lambda \\ -1-3\lambda \end{bmatrix}$$

AS REQUIRED

Question 18 (***+)

Consider the following matrix equation

$$\begin{pmatrix} k & 1 & 0 \\ 3 & -2 & k-3 \\ 10k & 3 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a \\ b \\ 15 \end{pmatrix},$$

where a, b and k are scalar constants.

a) Find the values of k for which the equation has a unique solution.

It is further asserted that $k = 2$.

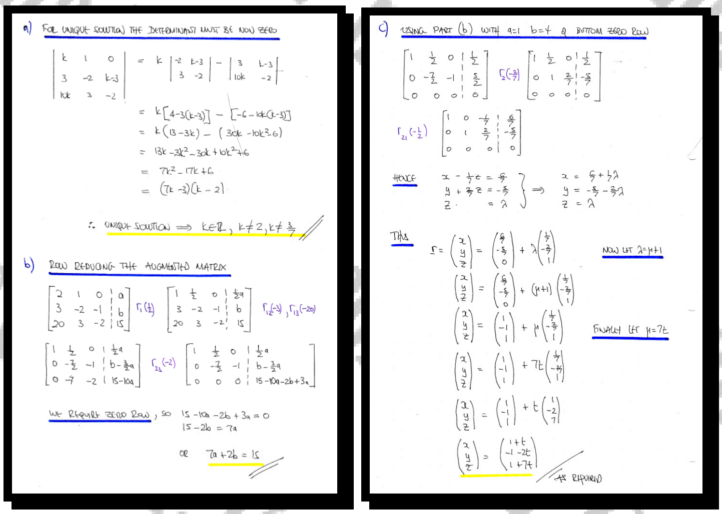
b) Express a in terms of b if the matrix equation is to be consistent.

c) Show that if $a = 1$ and $b = 4$, the solution of the matrix equation is

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} t+1 \\ -2t-1 \\ 7t+1 \end{pmatrix},$$

where t is a scalar parameter.

$\boxed{}$, $\boxed{k \neq 2 \ \cup \ k \neq \frac{3}{7}}$, $\boxed{2a + 7b = 15}$



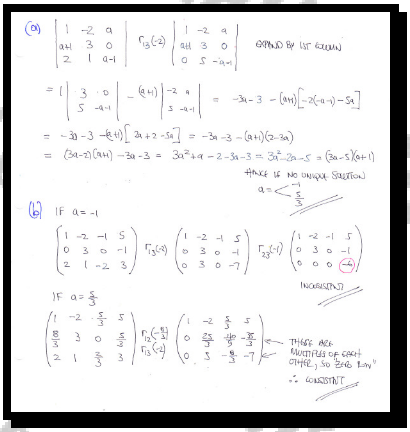
Question 19 (***+)

$$\begin{aligned} x \;\; -2y \;\; + \;\; az &= 5 \\ (a+1)x \;\; +3y \qquad &= a \\ 2x \;\; + \;\; y \;\; +(a-1)z &= 3 \end{aligned}$$

a) Determine the two values of the constant a for which the above system of equations does **not** have a unique solution.

b) Show clearly that the system is consistent for one of these values and inconsistent for the other.

$$\boxed{a=-1,\ \frac{5}{3}}$$



Question 20 (***+)

A system of equations is given below

$$\begin{aligned} 3x + 2y \quad - z &= 10 \\ 5x \quad - y - 4z &= 17 \\ x + 5y + pz &= q \end{aligned}$$

where p and q are constants.

a) Find the value of the constant p so that the system of equations does not have a unique solution.

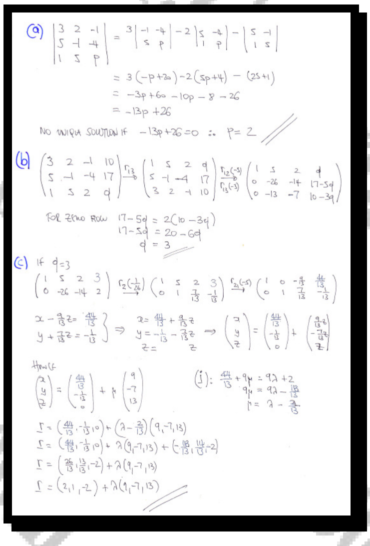
b) Show that for this value of p the system is consistent if $q = 3$.

c) Show that the general solution of the system can be written as

$$\mathbf{r} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k} + \lambda(9\mathbf{i} - 7\mathbf{j} + 13\mathbf{k}),$$

where λ is a scalar parameter.

$\boxed{p = 2}$



Question 21 (****)

A system of equation is given below

$$3x-2y-18z=6$$
$$2x+y-5z=25$$

a) Show, by reducing the system into row echelon form, that the solution of the system can be written as

$$\mathbf{r}=8\mathbf{i}+9\mathbf{j}+\lambda(4\mathbf{i}-3\mathbf{j}+\mathbf{k}),$$

where λ is a scalar parameter.

A new system is now given

$$3x-2y-18z=6$$
$$2x+y-5z=25$$
$$7x+ky+2z=20$$

where k is a constant.

b) Determine if the system has solutions for different values of k.

$k \neq 10 \Rightarrow$ unique, otherwise incosistent