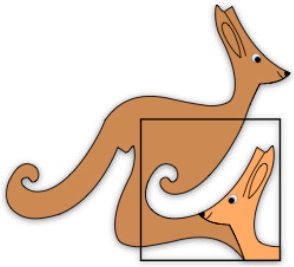
United Kingdom Mathematics Trust



Pink Kangaroo

Thursday 21 March 2019

Organised by the United Kingdom Mathematics Trust

a member of the Association Kangourou sans Frontières

Supported by Overleaf

England & Wales:	*Year* 11 *or below*
Scotland:	*S*4 *or below*
Northern Ireland:	*Year* 12 *or below*

Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **60 minutes**.
   No answers, or personal details, may be entered after the allowed time is over.
3. The use of blank or lined paper for rough working is allowed; **squared paper**, **calculators and measuring instruments are forbidden**.
4. **Use a B or an HB non-propelling pencil.** Mark at most one of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
5. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
6. **Scoring rules:**
   5 marks are awarded for each correct answer to Questions 1-15;
   6 marks are awarded for each correct answer to Questions 16-25;
   In this paper you will not lose marks for getting answers wrong.
7. Your Answer Sheet will be read by a machine. **Do not write or doodle on the sheet except to mark your chosen options.** The machine will read all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of eraser stuck to the page, the machine will interpret the mark in its own way.
8. **The questions on this paper are designed to challenge you to think, not to guess.** You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

Enquiries about the Pink Kangaroo should be sent to:

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1. What is the value of $20 \times 19 + 20 + 19$?

A 389 B 399 C 409 D 419 E 429

2. A model train takes exactly 1 minute and 11 seconds for one complete circuit of its track. How long does it take for six complete circuits?

A 6 minutes and 56 seconds B 7 minutes and 6 seconds C 7 minutes and 16 seconds
D 7 minutes and 26 seconds E 7 minutes and 36 seconds

3. A barber wants to write the word SHAVE on a board behind the client's seat in such a way that a client looking in the mirror reads the word correctly. Which of the following should the barber write on the board?

A SHAVE B SHAVƎ C ƎVAHS D EVAHS E ƎVAHS

4. How many different totals can be obtained by rolling three standard dice and adding up the scores?

A 14 B 15 C 16 D 17 E 18

5. A park has five gates. In how many ways can Monica choose a gate to enter the park and a different gate to leave the park?

A 25 B 20 C 16 D 15 E 10

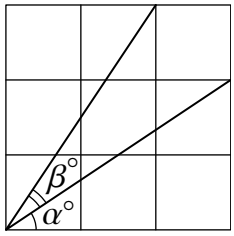
6. Pedro is asked to find three kangaroos whose weights are all whole numbers of kilograms and whose total weight is 97 kg. What is the largest possible weight of the lightest of the kangaroos Pedro could find?

A 1 kg B 30 kg C 31 kg D 32 kg E 33 kg

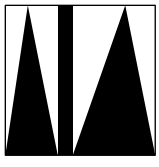
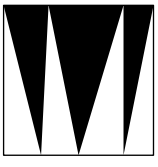
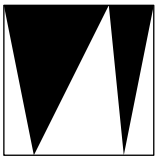
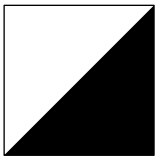
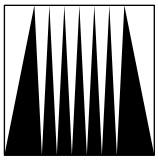
7. Two angles are marked on the 3×3 grid of squares.

Which of the following statements about the angles is correct?

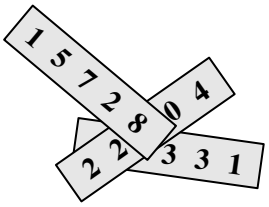
A $\alpha = \beta$ B $2\alpha + \beta = 90$ C $\alpha + \beta = 60$ D $2\beta + \alpha = 90$
E $\alpha + \beta = 45$



8. Inside each unit square a certain part has been shaded. In which square is the total shaded area the largest?

A  B  C  D  E 

9. On each of three pieces of paper a five-digit number is written as shown. Three of the digits are covered. The sum of the three numbers is 57263. What are the covered digits?



A 0, 2 and 2 B 1, 2 and 9 C 2, 4 and 9 D 2, 7 and 8
E 5, 7 and 8

10. A square has vertices P, Q, R, S labelled clockwise. An equilateral triangle is constructed with vertices P, T, R labelled clockwise. What is the size of angle RQT in degrees?

A 30 B 45 C 135 D 145 E 150

11. The numbers a, b, c and d are distinct positive integers chosen from 1 to 10 inclusive. What is the least possible value $\frac{a}{b} + \frac{c}{d}$ could have?

A $\frac{2}{10}$ B $\frac{3}{19}$ C $\frac{14}{45}$ D $\frac{29}{90}$ E $\frac{25}{72}$

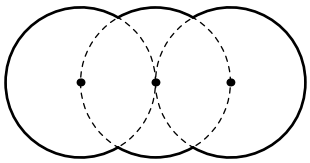
12. The flag of Kangaria is a rectangle with side-lengths in the ratio 3 : 5. The flag is divided into four rectangles of equal area as shown. What is the ratio of the length of the shorter sides of the white rectangle to the length of its longer sides?

A 1 : 3 B 1 : 4 C 2 : 7 D 3 : 10 E 4 : 15

13. The triathlon consists of swimming, cycling and running. The cycling accounts for three-quarters of the total distance, the running for one-fifth and the swimming for 2 km. What is the total distance of this triathlon?

A 10 km B 20 km C 38 km D 40 km E 60 km

14. The diagram shows a shape made of arcs of three circles, each with radius R. The centres of the circles lie on the same straight line, and the middle circle passes through the centres of the other two circles. What is the perimeter of the shape?



A $\frac{2\pi R\sqrt{3}}{3}$ B $\frac{5\pi R}{3}$ C $\frac{10\pi R}{3}$ D $2\pi R\sqrt{3}$ E $4\pi R$

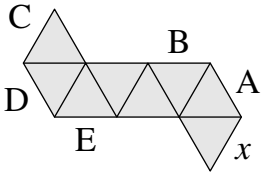
15. The sum of the seven digits of the number '$aaabbbb$' is equal to the two-digit number 'ab'. What is the value of $a + b$?

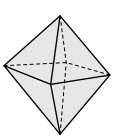
A 8 B 9 C 10 D 11 E 12

16. Sixty apples and sixty pears are to be packed into boxes so that each box contains the same number of apples, and no two boxes contain the same number of pears. What is the largest possible number of boxes that can be packed in this way?

A 20 B 15 C 12 D 10 E 6

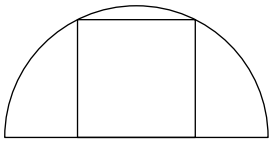
17. The diagram shows a net of an octahedron. When this is folded to form the octahedron, which of the labelled line segments will coincide with the line segment labelled x?





A B C D E

18. A square has two of its vertices on a semicircle and the other two on the diameter of the semicircle as shown. The radius of the circle is 1. What is the area of the square?



A $\frac{4}{5}$ B $\frac{\pi}{4}$ C 1 D $\frac{4}{3}$ E $\frac{2}{\sqrt{3}}$

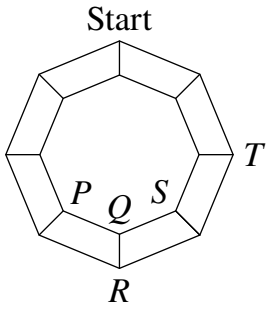
19. The integers from 1 to 99 are written in ascending order without spaces. The sequence of digits is then grouped into triples of digits:

$$123456789101112\ldots979899 \rightarrow (123)(456)(789)(101)(112)\ldots(979)(899).$$

Which of the following is not one of the triples?

A (222) B (434) C (464) D (777) E (888)

20. A network consists of 16 vertices and 24 edges that connect them, as shown. An ant begins at the vertex labelled Start. Every minute, it walks from one vertex to a neighbouring vertex, crawling along a connecting edge. At which of the vertices labelled P, Q, R, S, T can the ant be after 2019 minutes?



A only P, R or S, B not Q C only Q
D only T E all of the vertices are possible

21. Each of the positive integers a, b, and c has three digits, and for each of these integers the first digit is the same as its last digit. Also $b = 2a + 1$ and $c = 2b + 1$. How many possibilities are there for the integer a?

A 0 B 1 C 2 D 3 E more than 3

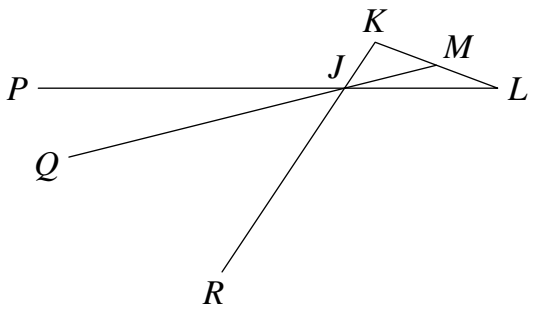
22. A positive integer is to be placed on each vertex of a square. For each pair of these integers joined by an edge, one should be a multiple of the other. However, for each pair of diagonally opposite integers, neither should be a multiple of the other. What is the smallest possible sum of the four integers?

A 12 B 24 C 30 D 35 E 60

23. Rhona wrote down a list of nine multiples of ten: 10, 20, 30, 40, 50, 60, 70, 80, 90. She then deleted some of the nine multiples so that the product of the remaining multiples was a square number. What is the least number of multiples that she could have deleted?

A 1 B 2 C 3 D 4 E 5

24. The diagram shows triangle JKL of area S. The point M is the midpoint of KL. The points P, Q, R lie on the extended lines LJ, MJ, KJ, respectively, such that $JP = 2 \times JL$, $JQ = 3 \times JM$ and $JR = 4 \times JK$.



What is the area of triangle PQR?

A S B $2S$ C $3S$ D $\frac{1}{2}S$ E $\frac{1}{3}S$

25. How many four-digit numbers have the following property? "For each of its digits, when this digit is deleted the resulting three-digit number is a factor of the original number."

A 5 B 9 C 14 D 19 E 23

