AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

I declare this is my own work.

# A-level MATHEMATICS

## Paper 3

Thursday 20 June 2024 Afternoon Time allowed: 2 hours

**Materials**
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do not write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked

**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

| For Examiner's Use | |
|---|---|
| Question | Mark |
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| 19 | |
| **TOTAL** | |

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## Section A

Answer **all** questions in the spaces provided.

**1** Each of the series below shows the first four terms of a geometric series.

Identify the only one of these geometric series that is convergent.

**[1 mark]**

Tick (✓) **one** box.

$0.1 + 0.2 + 0.4 + 0.8 + \ldots$ ☐

$1 - 1 + 1 - 1 + \ldots$ ☐

$128 - 64 + 32 - 16 + \ldots$ ☐

$1 + 2 + 4 + 8 + \ldots$ ☐

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**2** The quadratic equation

$$4x^2 + bx + 9 = 0$$

has one repeated real root.

Find $b$

Circle your answer.

**[1 mark]**

$b = 0$ $b = \pm 12$ $b = \pm 13$ $b = \pm 36$

**Turn over for the next question**

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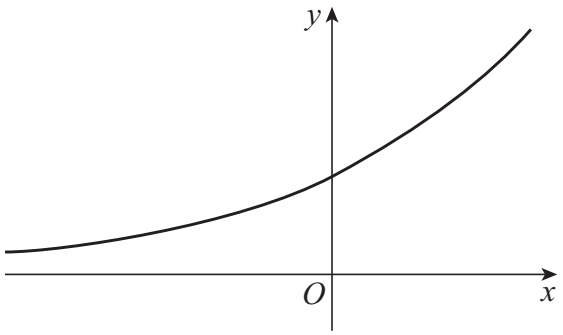
**3** One of the graphs shown below **cannot** have an equation of the form

$$y = a^x \qquad \text{where } a > 0$$

Identify this graph.

Tick (✓) **one** box.

**[1 mark]**

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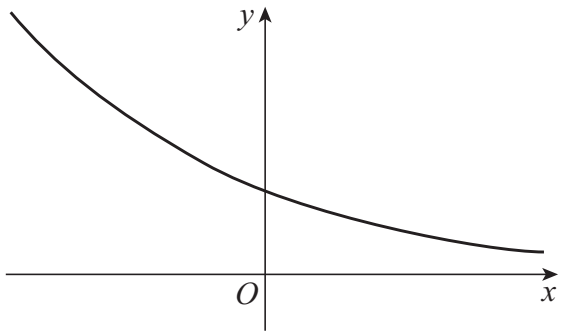


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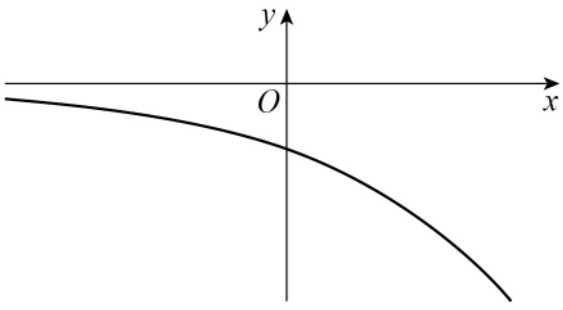


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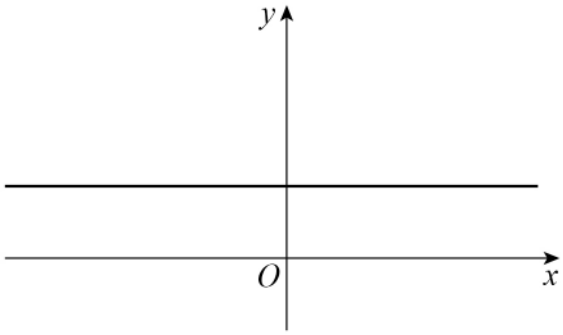


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**4** A curve has equation $y = x^4 + 2^x$

Find an expression for $\frac{dy}{dx}$

**[2 marks]**

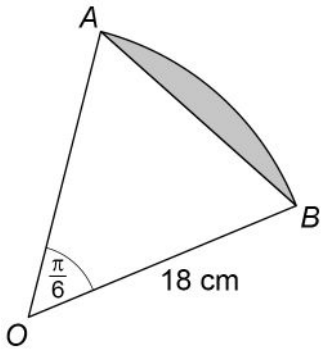
**Turn over for the next question**

**5** The diagram below shows a sector of a circle $OAB$.

The chord $AB$ divides the sector into a triangle and a shaded segment.

Angle $AOB$ is $\frac{\pi}{6}$ radians.

The radius of the sector is 18 cm.




Show that the area of the shaded segment is

$$k(\pi - 3) \text{ cm}^2$$

where $k$ is an integer to be found.

**[3 marks]**

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**Turn over for the next question**

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**ANSWER IN THE SPACES PROVIDED**

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**6 (a)** Find $\int \left(6x^2 - \frac{5}{\sqrt{x}}\right) \mathrm{d}x$

**[3 marks]**

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**6 (b)** The gradient of a curve is given by

$$\frac{dy}{dx} = 6x^2 - \frac{5}{\sqrt{x}}$$

The curve passes through the point (4, 90).

Find the equation of the curve.

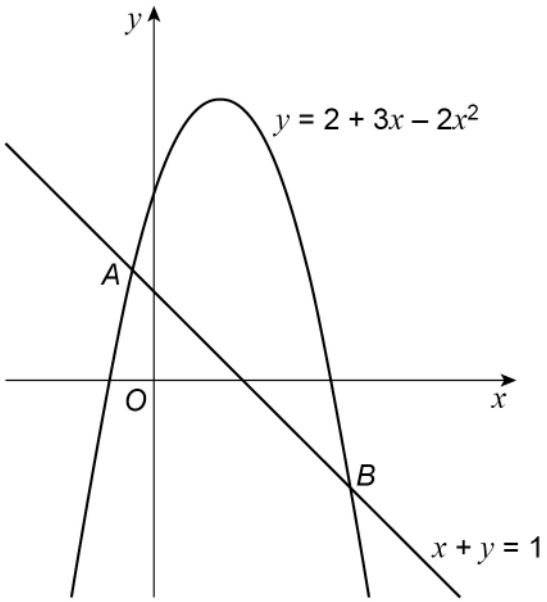
**[2 marks]**

**Turn over for the next question**

**7** The graphs with equations

$$y = 2 + 3x - 2x^2 \text{ and } x + y = 1$$

are shown in the diagram below.




The graphs intersect at the points *A* and *B*

**7 (a)** On the diagram above, shade and label the region, *R*, that is satisfied by the inequalities

$$0 \leq y \leq 2 + 3x - 2x^2$$

and

$$x + y \geq 1$$

**[2 marks]**

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**7 (b)** Find the exact coordinates of $A$

**[3 marks]**

**Turn over for the next question**

**8** The temperature $\theta$ °C of an oven $t$ minutes after it is switched on can be modelled by the equation

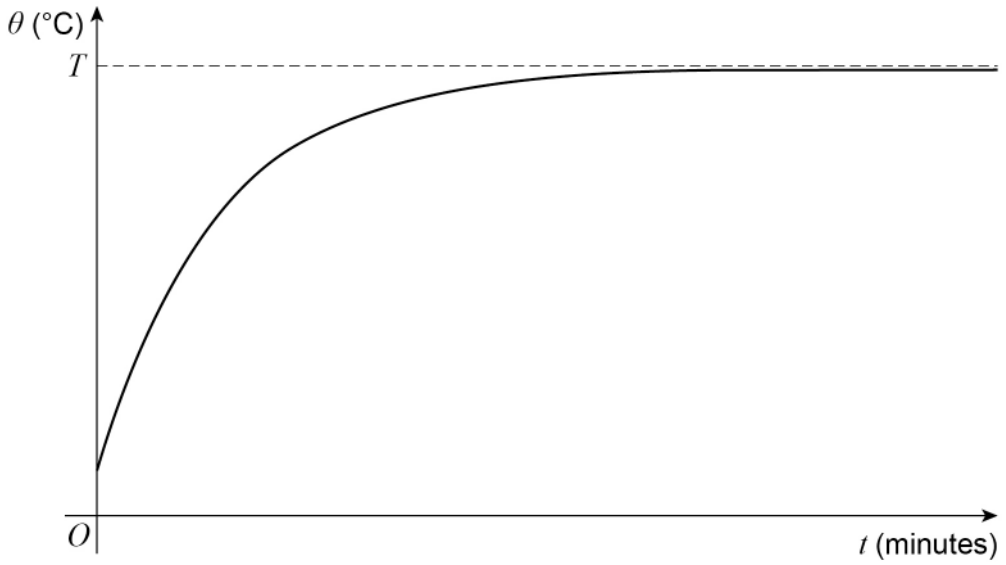
$$\theta = 20(11 - 10e^{-kt})$$

where $k$ is a positive constant.

Initially the oven is at room temperature.

The maximum temperature of the oven is $T$ °C

The temperature predicted by the model is shown in the graph below.




**8 (a)** Find the room temperature.

**[2 marks]**

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**8 (b)** Find the value of $T$

**[2 marks]**

**Question 8 continues on the next page**

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**8 (c)** The oven reaches a temperature of 86 °C one minute after it is switched on.

**8 (c) (i)** Find the value of $k$.

**[2 marks]**

**8 (c) (ii)** Find the time it takes for the temperature of the oven to be within 1°C of its maximum.

**[2 marks]**

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**Turn over for the next question**

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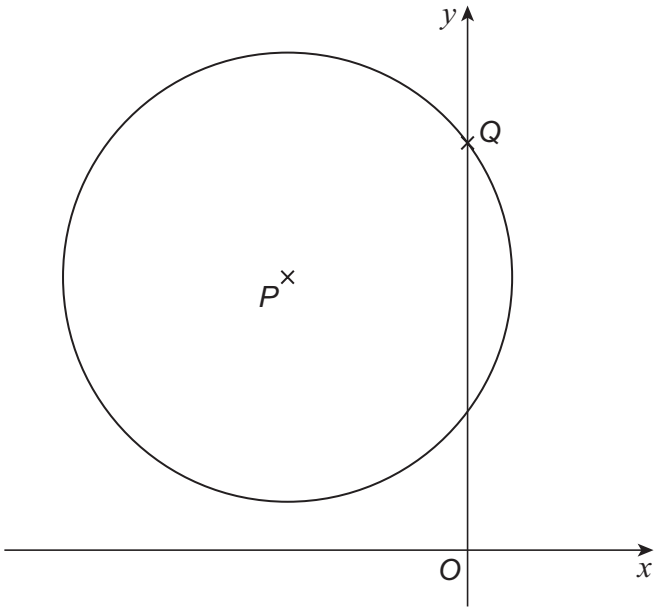
**9** **Figure 1** below shows a circle.

**Figure 1**




The centre of the circle is $P$ and the circle intersects the $y$-axis at $Q$ as shown in **Figure 1**.

The equation of the circle is

$$x^2 + y^2 = 12y - 8x - 27$$

**9 (a)** Express the equation of the circle in the form

$$(x - a)^2 + (y - b)^2 = k$$

where $a$, $b$ and $k$ are constants to be found.

**[3 marks]**

**9 (b)** State the coordinates of $P$

**[1 mark]**

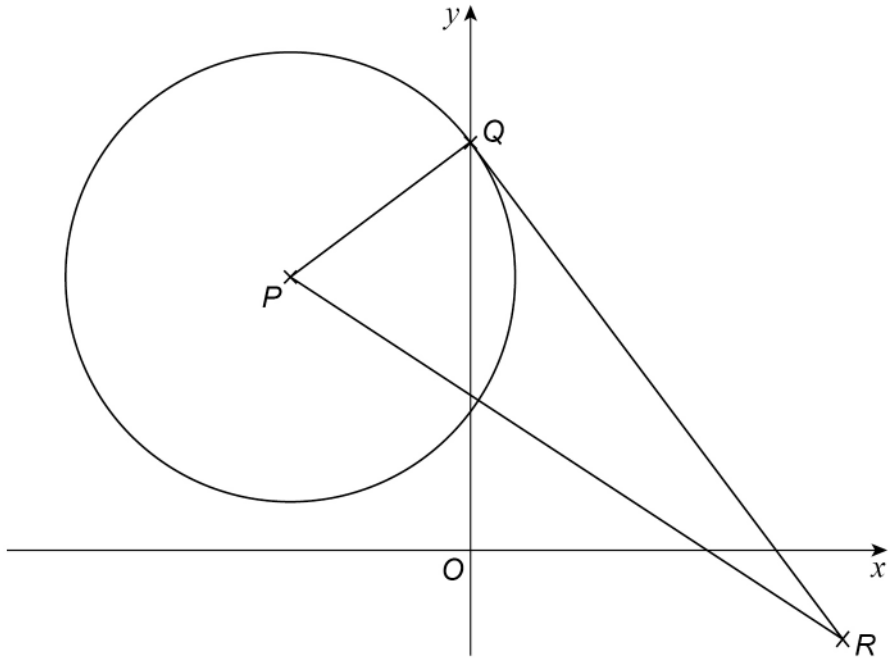
**9 (c)** Find the $y$-coordinate of $Q$

**[2 marks]**

**Question 9 continues on the next page**

**9 (d)** The line segment $QR$ is a tangent to the circle as shown in **Figure 2** below.

**Figure 2**




The point $R$ has coordinates $(9, -3)$.

Find the angle $QPR$

Give your answer in radians to three significant figures.

**[3 marks]**

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**10** It is given that

$$f(x) = 5x^3 + x$$

Use differentiation from first principles to prove that

$$f'(x) = 15x^2 + 1$$

**[5 marks]**

**Turn over for the next question**

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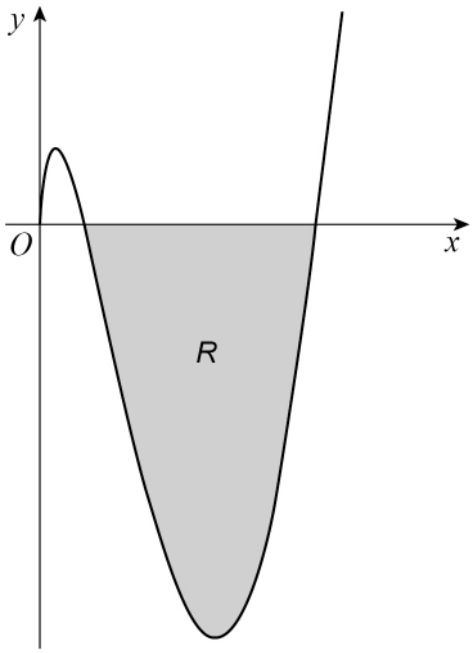
**11** The curve $C$ with equation

$$y = (x^2 - 8x) \ln x$$

is defined for $x > 0$ and is shown in the diagram below.




The shaded region, $R$, lies below the $x$-axis and is bounded by $C$ and the $x$-axis.

Show that the area of $R$ can be written as

$$p + q \ln 2$$

where $p$ and $q$ are rational numbers to be found.

**[10 marks]**

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**END OF SECTION A**

**TURN OVER FOR SECTION B**

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## Section B

Answer **all** questions in the spaces provided.

**12** A random sample of 84 students was asked how many revision websites they had visited in the past month.

The data is summarised in the table below.

| Number of websites | Frequency |
|---|---|
| 0 | 1 |
| 1 | 4 |
| 2 | 18 |
| 3 | 16 |
| 4 | 5 |
| 5 | 37 |
| 6 | 2 |
| 7 | 1 |

Find the interquartile range of the number of websites visited by these 84 students.

Circle your answer.

**[1 mark]**

3 4 19 42

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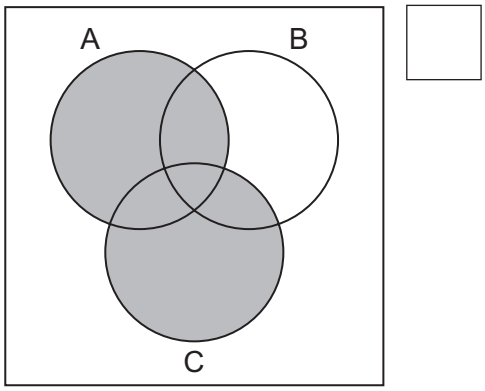
**13** The shaded region on one of the Venn diagrams below represents $(A \cup C) \cap B$

Identify this Venn diagram.

Tick (✓) **one** box.

**[1 mark]**

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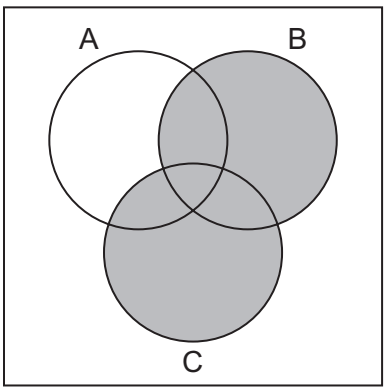


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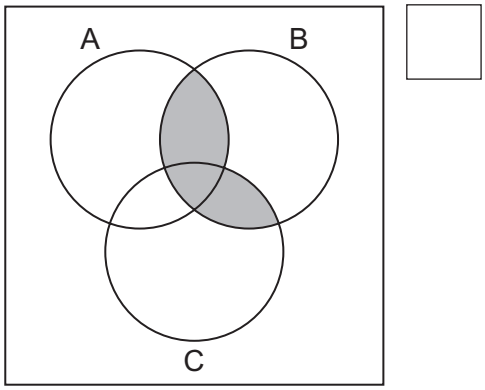


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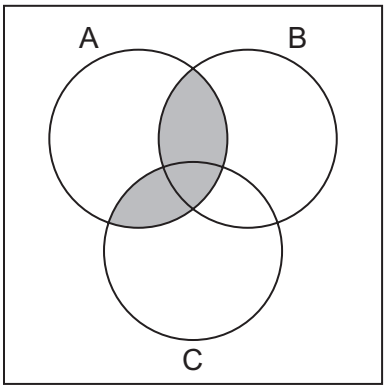


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**Turn over for the next question**

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**14** The annual cost of energy in 2021 for each of the 350 households in Village A can be modelled by a random variable £$X$

It is given that

$$\sum x = 945\,000 \qquad \sum x^2 = 2\,607\,500\,000$$

**14 (a)** Calculate the mean of $X$.

**[1 mark]**

**14 (b)** Calculate the standard deviation of $X$.

**[2 marks]**

**14 (c)** For households in Village B the annual cost of energy in 2021 has mean £3100 and standard deviation £325

Compare the annual cost of energy in 2021 for households in Village A and Village B.

**[2 marks]**

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**15** It is given that

$$X \sim \text{B}(48, 0.175)$$

**15 (a)** Find the mean of $X$

**[1 mark]**

**15 (b)** Show that the variance of $X$ is 6.93

**[1 mark]**

**15 (c)** Find $\text{P}(X < 10)$

**[1 mark]**

**15 (d)** Find $\text{P}(X \geq 6)$

**[2 marks]**

**15 (e)** Find $P(9 \leq X \leq 15)$

**[2 marks]**

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**15 (f)** The aeroplanes used on a particular route have 48 seats.

The proportion of passengers who use this route to travel for business is known to be 17.5%

Make **two** comments on whether it would be appropriate to use $X$ to model the number of passengers on an aeroplane who are travelling for business using this route.

**[2 marks]**

**Turn over for the next question**

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**16** A medical student believes that, in adults, there is a **negative correlation** between the amount of nicotine in their blood stream and their energy level.

The student collected data from a random sample of 50 adults.

The correlation coefficient between the amount of nicotine in their blood stream and their energy level was $-0.45$

Carry out a hypothesis test at the 2.5% significance level to determine if this sample provides evidence to support the student's belief.

For $n = 50$, the critical value for a one-tailed test at the 2.5% level for the population correlation coefficient is 0.2787

**[4 marks]**

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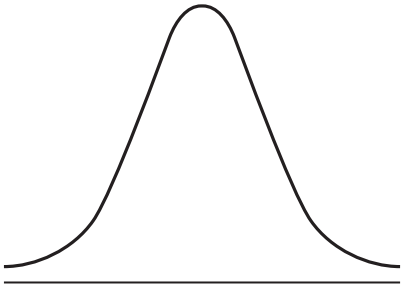
**17** In 2019, the lengths of new-born babies at a clinic can be modelled by a normal distribution with mean 50 cm and standard deviation 4 cm.

**17 (a)** This normal distribution is represented in the diagram below.

Label the values 50 and 54 on the horizontal axis.

**[2 marks]**




**17 (b)** State the probability that the length of a new-born baby is less than 50 cm.

**[1 mark]**

**17 (c)** Find the probability that the length of a new-born baby is more than 56 cm.

**[1 mark]**

**17 (d)** Find the probability that the length of a new-born baby is more than 40 cm but less than 60 cm.

**[1 mark]**

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**17 (e)** Determine the length exceeded by 95% of all new-born babies at the clinic.

**[2 marks]**

**17 (f)** In 2020, the lengths of 40 new-born babies at the clinic were selected at random.

The total length of the 40 new-born babies was 2060 cm.

Carry out a hypothesis test at the 10% significance level to investigate whether the mean length of a new-born baby at the clinic in 2020 has **increased** compared to 2019.

You may assume that the length of a new-born baby is still normally distributed with standard deviation 4 cm.

**[7 marks]**

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**Turn over for the next question**

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**18** The Human Resources director in a company is investigating the graduate status and salaries of its employees.

Event $G$ is defined as the employee is a graduate.

Event $H$ is defined as the employee earns at least £40 000 a year.

The director summarised the findings in the table of probabilities below.

| | $H$ | $H'$ |
|---|---|---|
| $G$ | 0.21 | 0.18 |
| $G'$ | 0.07 | 0.54 |

**18 (a)** An employee is selected at random.

**18 (a) (i)** Find $P(G)$

**[1 mark]**

**18 (a) (ii)** Find $P[(G \cap H)']$

**[2 marks]**

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**18 (a) (iii)** Find $P(H \mid G')$

**[2 marks]**

**18 (b)** Determine whether the events $G$ and $H$ are independent.

Fully justify your answer.

**[2 marks]**

**Turn over for the next question**

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**19** It is known that 80% of all diesel cars registered in 2017 had carbon monoxide (CO) emissions less than 0.3 g/km.

Talat decides to investigate whether the proportion of diesel cars registered in 2022 with CO emissions less than 0.3 g/km has **changed**.

Talat will carry out a hypothesis test at the 10% significance level on a random sample of 25 diesel cars registered in 2022.

**19 (a) (i)** State suitable null and alternative hypotheses for Talat's test.

**[1 mark]**

**19 (a) (ii)** Using a 10% level of significance, find the critical region for Talat's test.

**[5 marks]**

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**19 (a) (iii)** In his random sample, Talat finds 18 cars with CO emissions less than 0.3 g/km.

State Talat's conclusion in context.

**[1 mark]**

**19 (b)** Talat now wants to use his random sample of 25 diesel cars, registered in 2022, to investigate whether the proportion of diesel cars in England with CO emissions more than 0.5 g/km has changed from the proportion given by the Large Data Set.

Using your knowledge of the Large Data Set, give **two** reasons why it is not possible for Talat to do this.

**[2 marks]**

**END OF QUESTIONS**

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