BAAO
British Astronomy and Astrophysics Olympiad

British Astronomy and Astrophysics Olympiad 2020-21

Astronomy & Astrophysics Challenge Paper

September - December 2020

Instructions

Time: 1 hour (30 marks).

Questions: Answer all questions in Sections A and B, but only **one** question in Section C.

Marks: Marks allocated for each question are shown in brackets on the right. Working must be shown in order to get full credit, and you may find it useful to write down numerical values of any intermediate steps.

Solutions: Answers and calculations are to be written on loose paper or in examination booklets. Students should ensure their name and school is clearly written on all answer sheets and pages are numbered. A standard formula booklet with standard physical constants should be supplied.

Eligibility: All sixth form students (or younger) are eligible to sit any BAAO paper.

Further Information about the British Astronomy and Astrophysics Olympiad

This is the first paper of the British Astronomy and Astrophysics Olympiad in the 2020-2021 academic year. To progress to the next stage of the BAAO, you **must** *take the BPhO Round 1 in November 2020, which is a general physics problem paper. Those achieving at least a Gold will be invited to take the BAAO Competition paper on* **Monday 25th January 2021**.

To be awarded the highest grade (Distinction) in this paper, it should be sat under test conditions and marked papers achieving 60% or above should be sent in to the BPhO Office in Oxford by **Friday 23rd October 2020**. *All papers, including papers sat after that date, or below that mark (i.e. Merit or Participation), should have their results recorded using the online form by* **Friday 4th December 2020**.

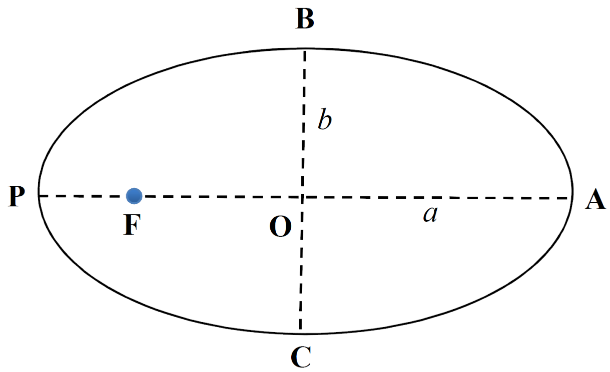
To solve some of the questions, you will need to write equations, draw diagrams and, in general, show your working. You are also encouraged to look at the clear sky and identify the brightest stars, a few days before sitting the paper.

This paper has more than an hour's worth of questions. You are encouraged to have a go at as many as you can and to follow up on those that you do not complete in the time allocated.

Important Constants

Constant	Symbol	Value
Speed of light	c	$3.00 \times 10^8\,\mathrm{m\,s^{-1}}$
Earth's rotation period	1 day	24 hours
Earth's orbital period	1 year	365.25 days
parsec	pc	$3.09 \times 10^{16}\,\mathrm{m}$
Astronomical Unit	au	$1.50 \times 10^{11}\,\mathrm{m}$
Radius of the Earth	R_E	$6.37 \times 10^6\,\mathrm{m}$
Semi-major axis of the Earth's orbit		1 au
Radius of the Sun	$R_\odot$	$6.96 \times 10^8\,\mathrm{m}$
Mass of the Sun	$M_\odot$	$1.99 \times 10^{30}\,\mathrm{kg}$
Mass of the Earth	M_E	$5.97 \times 10^{24}\,\mathrm{kg}$
Luminosity of the Sun	$L_\odot$	$3.85 \times 10^{26}\,\mathrm{W}$
Gravitational constant	G	$6.67 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$

You might find the diagram of an elliptical orbit below useful in solving some of the questions:



Elements of an elliptic orbit:

$a = \mathbf{OA}\ (= \mathbf{OP})$	semi-major axis
$b = \mathbf{OB}\ (= \mathbf{OC})$	semi-minor axis
$e = \sqrt{1 - \frac{b^2}{a^2}}$	eccentricity
$\mathbf{F}$	focus
$\mathbf{PF} = a(1-e)$	periapsis distance (shortest distance from $\mathbf{F}$)
$\mathbf{AF} = a(1+e)$	apoapsis distance (longest distance from $\mathbf{F}$)

Kepler's Third Law: For an elliptical orbit, the square of the period, T, of an object about the focus is proportional to the cube of the semi-major axis, a (as defined above), such that

$$T^2 = \frac{4\pi^2}{GM} a^3 ,$$

where M is the total mass of the system (typically dominated by the central object) and G is the universal gravitational constant.

Magnitudes: The apparent magnitudes of two objects, m_1 and m_0, are related to their apparent brightnesses, b_1 and b_0, via the formula:

$$\frac{b_1}{b_0} = 10^{-0.4(m_1 - m_0)}$$

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Section A: Multiple Choice

Write the correct answer to each question. Each question is worth 1 mark. There is only one correct answer to each question. **Total: 10 marks**.

1. The year 1990 held many exciting events in astronomy. Which of the following did **not** celebrate its 30^{th} anniversary this year?

   A. The launch of the Hubble Space Telescope, part of NASA's 'Great Observatories' programme
   B. The most distant photo ever taken of the Earth by the probe *Voyager 1* from 40.5 au, nicknamed the 'Pale Blue Dot' by Carl Sagan
   C. The arrival at Venus of the *Magellan* probe, where it used radar to create the highest resolution maps we have of the surface of the planet
   D. ESA's *Giotto* probe passing within 600 km of the nucleus of Halley's Comet, taking our best photos of this famous object

2. Which well-known star dropped in brightness by 40% between October 2019 and April 2020, leading to speculation it may be about to go supernova?

   A. Aldebaran
   B. Antares
   C. Arcturus
   D. Betelgeuse

3. A very strong argument for the existence of dark matter is:

   A. the existence of a supermassive black hole at the centre of most large galaxies
   B. the shape of the rotation curve of spiral galaxies
   C. the detection of neutrino emission from Type II supernovae
   D. that almost all galactic spectra are redshifted

4. The Ring Nebula, M57, is surrounded by thin shells of out-flowing material, the outermost (and faintest) of which is measured to have an angular diameter of 230 arcseconds (where 1 arcsecond $= 1/3600^{\text{th}}$ of a degree). Given the nebula is 787 pc from Earth, estimate its radius.

   A. 0.44 pc
   B. 0.88 pc
   C. 25 pc
   D. 50 pc

5. Which of these constellations is entirely south of the ecliptic?

   A. Orion
   B. Andromeda
   C. Aquila
   D. Ophiuchus

6. For an observer at a latitude of $52°$, which of these stars culminates with the highest altitude?

   A. Kochab (declination $= 74°$)
   B. Capella (declination $= 46°$)
   C. Vega (declination $= 38°$)
   D. Pollux (declination $= 28°$)

7. On 21st March the Sun is in the constellation of Pisces. In which of these constellations would it be possible to find Venus? You are given that Venus has a circular orbit of radius 0.723 au.

   A. Aries
   B. Leo
   C. Libra
   D. Gemini

8. When an exoplanet transits a star, the brightness of the star dims by 0.0557 magnitudes. Let R_P be the radius of the planet and R_S be the radius of the star. What is the ratio R_P/R_S?

   A. 0.00250
   B. 0.224
   C. 0.776
   D. 0.998

9. The event horizon of a non-spinning, uncharged black hole can be thought of as a sphere with a radius equal to the Schwarzschild radius, $r_S = 2GM/c^2$ where M is the mass of the black hole and c is the speed of light. If the black hole at the centre of the Milky Way has a mass of $4.15 \times 10^6\, M_\odot$, what is the approximate average density within the event horizon?

   A. ~ 1 kg m^{-3}
   B. $\sim 10^3$ kg m^{-3}
   C. $\sim 10^6$ kg m^{-3}
   D. $\sim 10^9$ kg m^{-3}

10. On 19th July 2020 the first Arab interplanetary mission was launched on its way to Mars, consisting of the *Hope* probe designed by the United Arab Emirates Space Agency. When it reaches Mars in February 2021 it will do a short burn at a distance of $49\,400$ km away from the surface to slow it down and put it into an elliptical capture orbit which will bring it as close as only 1000 km above the planet. Given the mass of Mars is 6.39×10^{23} kg and its radius is 3390 km, what will be the period of this orbit?

    A. 37.3 hours
    B. 40.9 hours
    C. 105 hours
    D. 116 hours

Section B: Short Answer

Each short question is worth 5 marks. **Total: 10 marks**.

Temperature on an Exoplanet

11. In January 2020, NASA's Transiting Exoplanet Survey Satellite (TESS) discovered an Earth-sized exoplanet, called TOI 700 d, in its star's habitable zone. This is the range of distances a planet can orbit a star so that liquid water can exist on the surface, given sufficient atmospheric pressure. It was discovered using the transit method, where the planet passes directly between the observer and the star, causing a drop in brightness.

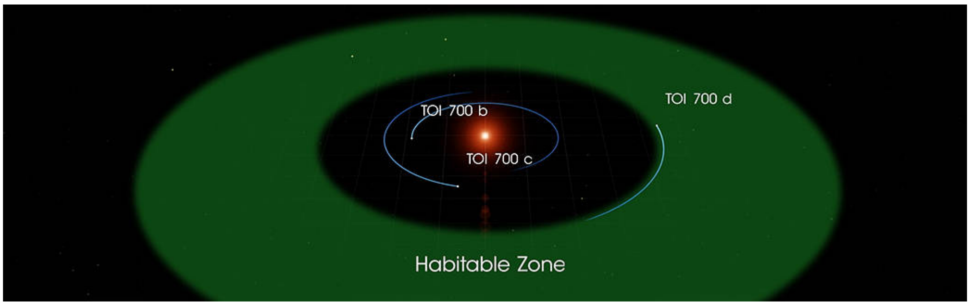

Figure 1: The three planets of the TOI 700 system, illustrated here, orbit a small, cool M dwarf star. TOI 700 d is the first Earth-size habitable-zone world discovered by TESS. Credit: NASA's Goddard Space Flight Center.

TOI 700 d has radius $R_P = 1.19\ R_E$ orbiting a star with luminosity $0.0233\,L_\odot$ at a distance of 0.163 au. Assume that the planet absorbs all the light that hits the surface, and that the orbit is circular.

(a) Determine the total power incident on the surface of the planet, L_{incident}.

[2]

Assuming the planet is in thermal equilibrium and a perfect blackbody emitter with temperature T_P, the total amount of energy emitted is given by the Stefan-Boltzmann Law,

$$L_{\text{emitted}} = 4\pi R_P^2 \sigma T_P^4 ,$$

where $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}.

(b) Find the value of T_P. Give your answer in °C.
[*Hint: you may find it is colder than expected.*]

[2]

(c) Suggest one reason to still justify the claim that the exoplanet is in the habitable zone.

[1]

Helicopter Drone on Mars

12. NASA's Mars 2020 mission involves a large rover called *Perseverance*, and an audacious experiment to see if motor driven flight is possible on the red planet. The helicopter drone is called *Ingenuity*, shown in Figure 2, and is the first attempt to fly on another world.

Figure 2: The *Ingenuity* drone with its helicopter rotors ready to take off, with the solar panel used to charge its battery on the very top. In the background you can see a part of the *Perseverance* rover. Credit: NASA/JPL-Caltech.

Although gravity is weaker on Mars, the Martian atmosphere is only about 1% the density of that on Earth, which makes it very hard to get lift. To overcome this, the probe has two counter-rotating blades (directly above each other) with a tip-to-tip length of 1.2 m that will spin at 2400 rpm (about 5 times faster than helicopters on Earth), and is incredibly light with a mass of 1.8 kg. A consequence of the weight restriction is that the battery is small (and charged by a solar panel on top of the drone, above the blades). Since flying in such a thin atmosphere is a very high power activity the maximum flight duration is therefore short, and given the time needed to recharge, they will be limited to only one flight per day.

(a) The energy stored in the battery available for flight is 10 Wh out of a 35 Wh capacity (the rest is mainly used to keep the drone warm on the cold Martian surface). During flight the drone uses an average of 390 W. Determine the maximum duration (in seconds) of one flight. [1]

(b) If the active region of the solar panel has an area of 544 cm^2 and Mars takes 687 days to orbit the Sun, determine the minimum amount of time (in minutes) the battery needs to charge for between flights. Assume the panel is 30% efficient and the Sun is directly overhead throughout. [4]

Section C: Long Answer

Each long question is worth 10 marks. Answer **either** *Qu 13* **or** *Qu 14.* **Total: 10 marks**.

History of the Universe

13. The Universe consists of three main components: radiation (including neutrinos), matter (both atoms and dark matter), and dark energy. The overall density of the universe has been dominated by the density of each of those in turn at different times in its history, leading to three different epochs (shown in Fig 3).

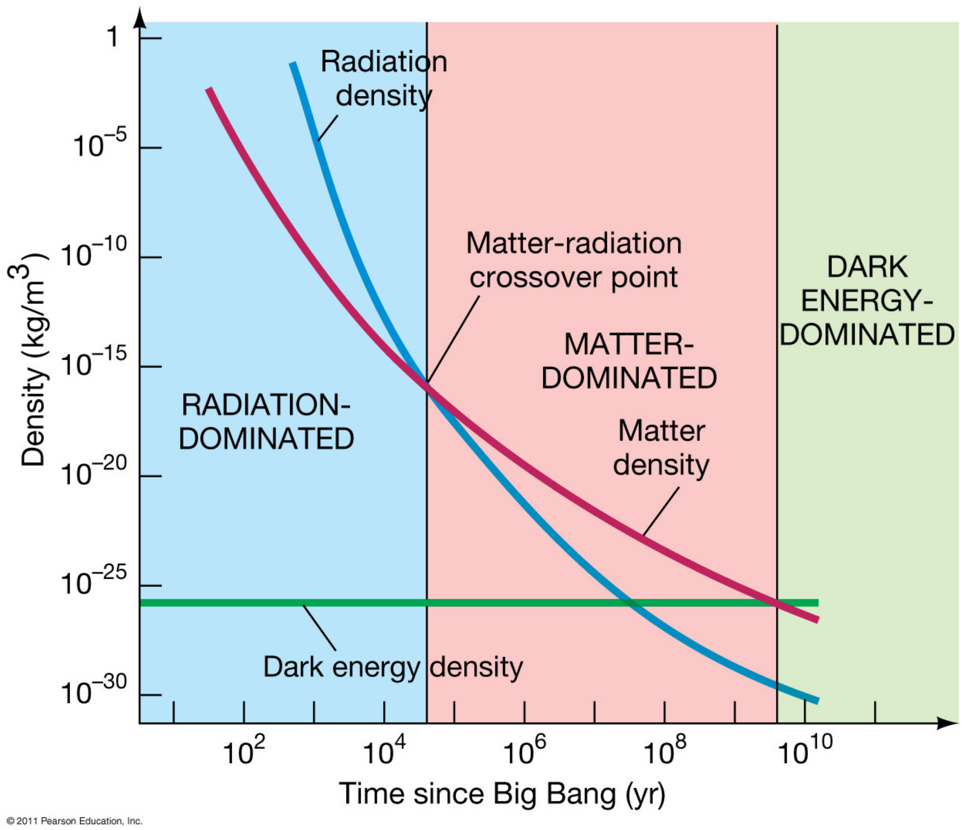

Figure 3: A outline of the three main epochs in the history of the Universe. You do not need to read any data off this graph to answer this question. Credit: Pearson Education, Inc.

The scale factor, a, describes how the Universe has expanded (i.e. a measure of the relative radius of the Universe), and the current value is defined as $a_0 \equiv 1$ where the subscript '0' indicates it is as measured today. At earlier times $a < 1$ and at the Big Bang $a = 0$.

The redshift of an object, z, is related to the scale factor as $a = (1 + z)^{-1}$ and so the redshift corresponding to now is $z = 0$, and far away objects have higher redshift ($z > 0$) since we observe them as they were long ago when the scale factor was smaller.

For a Universe to be flat (i.e. zero curvature), its average density must be equal to the critical density,

$$\rho_{\rm crit,0} = \frac{3H_0^2}{8\pi G} ,$$

where H_0 is the Hubble constant, measured in 2018 from the cosmic microwave background by the Planck spacecraft to be 67.36 km s^{-1} Mpc^{-1}.

The density of the i^{th} component of the Universe can be expressed relative to the critical density as the density parameter,

$$\Omega_i = \frac{\rho_i}{\rho_{\text{crit}}} \ .$$

Planck measured the current density parameters of dark energy and matter as $\Omega_{\Lambda,0} = 0.6847$ and $\Omega_{m,0} = 0.3153$ respectively.

In each epoch, the scale factor increases at a different rate with time, t, as the density also varies differently with scale factor.

- Radiation-dominated epoch: The Universe's early history, where $\rho \propto a^{-4}$ and so $a \propto t^{1/2}$
- Matter-dominated epoch: This represents much of the history of the Universe, where $\rho \propto a^{-3}$ and so $a \propto t^{2/3}$
- Dark-energy-dominated epoch: This is an era we have recently entered and will remain in for the rest of time, where ρ doesn't vary with scale factor (i.e. is a constant) and so $a \propto e^{H_0 t}$

(a) Assuming our Universe is flat, calculate the current average density of the Universe.

[1]

(b) Find the time, t_{DE}, when the current epoch began (i.e. when the densities of dark energy and matter were equal), given that the age of the Universe today is $t_0 = 13.80$ Gyr (where 1 Gyr $= 10^9$ years). Give you answer in Gyr. You do not need to read anything off the graph.

[4]

(c) Assuming the Universe has always been flat, find the time t_{eq}, corresponding to when the densities of matter and radiation were equal, given that data from Planck has allowed us to calculate the redshift of this to be $z_{eq} = 3402$, and find the average density of the Universe at t_{eq}. Again, do not try and read any data off the graph.

[5]

Comet NEOWISE

14. The NEOWISE telescope discovered a new comet on 27th March 2020, later given the official designation C/2020 F3 NEOWISE. Although when first discovered it only had an apparent magnitude of 18.0, it would become sufficiently bright that it could be seen with the naked eye by observers throughout the northern hemisphere, and was one of the brightest comets since Hale-Bopp in 1997.

Figure 4: The comet C/2020 F3 NEOWISE as seen from the UK in late July. Credit: Alex Calverley.

On its discovery date the comet was 1.702 au from the Earth and 2.089 au from the Sun, and at perihelion (when it was closest to the Sun) on 3rd July 2020 it was only 0.294649 au from the Sun.

(a) Given the comet's orbit has an eccentricity of 0.999188, estimate the year of its next perihelion.

[3]

For a spherical object, the luminosity of the reflected light is a function of how much of the lit surface of the object we can see, known as the phase (for example crescent and gibbous phases of the moon).

This correction is known as the phase factor, $p(\theta)$, such that the total power reflected is related to the total power incident as $P_{\text{ref}} = P_{\text{inc}} \times p(\theta)$. You are given that

$$p(\theta) = B\left[\left(1 - \frac{\theta}{\pi}\right)\cos\theta + \frac{1}{\pi}\sin\theta\right],$$

where B is a constant and θ is the phase angle, as defined in Fig 5, and is measured in radians (a measure of angle such that 2π radians $= 360°$).

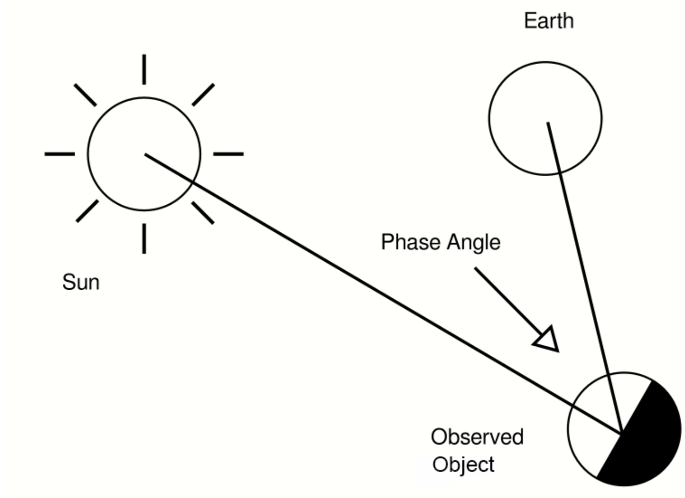

Figure 5: The phase angle is defined as the angle between the Earth and Sun as viewed from the object. Credit: Wikipedia.

(b) Treating the comet nucleus as a sphere of fixed radius as it orbits, estimate the comet's apparent magnitude when it is at aphelion (when it is furthest from the Sun), assuming the phase angle is zero when it is observed from the Earth. Comment on whether the Hubble Space Telescope (limiting magnitude ~ 30) would be able to see it.

[7]

END OF PAPER

Questions proposed by:
Dr Alex Calverley (Royal Grammar School, Guildford)
Josh Brown (University of Cambridge)
Tom Hillman (University of Cambridge)
Niam Vaishnav (University of Oxford)
Nick Koukoufilippas (University of Oxford)