# British Astronomy and Astrophysics Olympiad 2021-22

# Astronomy & Astrophysics Challenge Paper

# September - December 2021

## Instructions

**Time**: 1 hour (30 marks).

**Questions**: Answer all questions in Sections A and B, but only **one** question in Section C.

**Marks**: Marks allocated for each question are shown in brackets on the right. Working must be shown in order to get full credit, and you may find it useful to write down numerical values of any intermediate steps.

**Solutions**: Answers and calculations are to be written on loose paper or in examination booklets. Students should ensure their name and school is clearly written on all answer sheets and pages are numbered. A standard formula booklet with standard physical constants should be supplied.

**Eligibility**: All sixth form students (or younger) are eligible to sit any BAAO paper.

---

**Further Information about the British Astronomy and Astrophysics Olympiad**

*This is the first paper of the British Astronomy and Astrophysics Olympiad in the 2021-2022 academic year. To progress to the next stage of the BAAO, you* **must** *take the BPhO Round 1 in November 2021, which is a general physics problem paper. Those achieving a Top Gold will be invited to take the BAAO Competition paper on* **Monday 24th January 2022***, as will those achieving a Distinction on this paper.*

*To be awarded the highest grade (Distinction) in this paper, it should be sat under test conditions and marked papers achieving 60% or above should be sent in to the BPhO Office in Oxford by* **Friday 22nd October 2021**. *All papers, including papers sat after that date, or below that mark (i.e. Merit or Participation), should have their results recorded using the online form by* **Friday 3rd December 2021**.

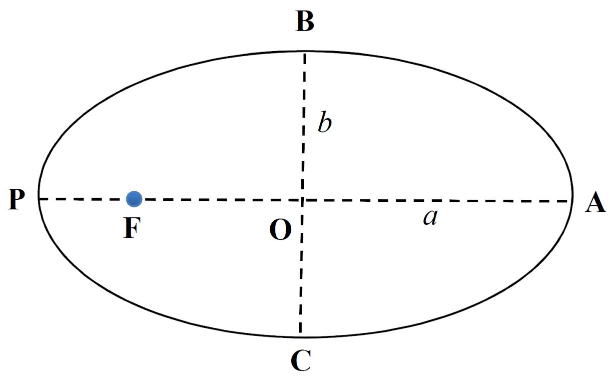
*To solve some of the questions, you will need to write equations, draw diagrams and, in general, show your working. You are also encouraged to look at the clear sky and identify the brightest stars, a few days before sitting the paper.*

*This paper has more than an hour's worth of questions. You are encouraged to have a go at as many as you can and to follow up on those that you do not complete in the time allocated.*

# Important Constants

| Constant | Symbol | Value |
|---|---|---|
| Speed of light | $c$ | $3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$ |
| Earth's rotation period | 1 day | 24 hours |
| Earth's orbital period | 1 year | 365.25 days |
| parsec | pc | $3.09 \times 10^{16}\,\mathrm{m}$ |
| Astronomical Unit | au | $1.50 \times 10^{11}\,\mathrm{m}$ |
| Radius of the Earth | $R_E$ | $6.37 \times 10^{6}\,\mathrm{m}$ |
| Semi-major axis of the Earth's orbit | | 1 au |
| Radius of the Sun | $R_\odot$ | $6.96 \times 10^{8}\,\mathrm{m}$ |
| Mass of the Sun | $M_\odot$ | $1.99 \times 10^{30}\,\mathrm{kg}$ |
| Mass of the Earth | $M_E$ | $5.97 \times 10^{24}\,\mathrm{kg}$ |
| Luminosity of the Sun | $L_\odot$ | $3.85 \times 10^{26}\,\mathrm{W}$ |
| Gravitational constant | $G$ | $6.67 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$ |

You might find the diagram of an elliptical orbit below useful in solving some of the questions:




**Elements of an elliptic orbit**:

| | |
|---|---|
| $a = \mathbf{OA}\ (= \mathbf{OP})$ | semi-major axis |
| $b = \mathbf{OB}\ (= \mathbf{OC})$ | semi-minor axis |
| $e = \sqrt{1 - \frac{b^2}{a^2}}$ | eccentricity |
| **F** | focus |
| $\mathbf{PF} = a(1-e)$ | periapsis distance (shortest distance from **F**) |
| $\mathbf{AF} = a(1+e)$ | apoapsis distance (longest distance from **F**) |
| $\pi ab$ | area of the ellipse |

**Kepler's Third Law**: For an elliptical orbit, the square of the period, $T$, of an object about the focus is proportional to the cube of the semi-major axis, $a$ (as defined above), such that

$$T^2 = \frac{4\pi^2}{GM} a^3 \,,$$

where $M$ is the total mass of the system (typically dominated by the central object) and $G$ is the universal gravitational constant.

**Magnitudes**: The apparent magnitudes of two objects, $m_1$ and $m_0$, are related to their apparent brightnesses, $b_1$ and $b_0$, via the formula:

$$\frac{b_1}{b_0} = 10^{-0.4(m_1 - m_0)}$$

NPL
National Physical Laboratory

Maplesoft
Mathematics • Modeling • Simulation

UNIVERSITY OF OXFORD

UNIVERSITY OF CAMBRIDGE

OXFORD UNIVERSITY PRESS

TRINITY COLLEGE CAMBRIDGE

Worshipful Company of Scientific Instrument Makers

ROYAL ASTRONOMICAL SOCIETY
QUICQUID NITET NOTANDUM
1820

ONEWORLD

PRINCETON UNIVERSITY PRESS

Springer

World Scientific
Connecting Great Minds

THE RANDOM HOUSE GROUP

## Section A: Multiple Choice

*Write the correct answer to each question. Each question is worth 1 mark. There is only one correct answer to each question.* **Total: 10 marks**.

1. As of September 2021, which of the following billionaires has **not** been into space (defined as at least 80 km above the surface of the Earth)?

   A. Jeff Bezos with the company *Blue Origin*
   B. Richard Branson with the company *Virgin Galactic*
   C. Elon Musk with the company *SpaceX*
   D. Dennis Tito with the company *Space Adventures*

2. The website *what3words* splits up the Earth's surface into 3 m $\times$ 3 m squares and gives a co-ordinate of three randomly chosen words (for example the entrance to the Oxford University Physics Department is engage.proud.police). If each of the words is taken from the same list of $n$ words, what value of $n$ is needed?

   A. $\sim 10\,000$
   B. $\sim 20\,000$
   C. $\sim 30\,000$
   D. $\sim 40\,000$

3. After many delays, the James Webb Space Telescope (JWST) is due to be launched in December 2021. This will be the successor to the Hubble Space Telescope (HST) and has a much bigger primary mirror (6.5 m in diameter against 2.4 m for the HST) so will be able to study the Universe in unprecedented detail. Given the faintest objects the HST can see have a magnitude of $\sim 31$ (known as the limiting magnitude), what limiting magnitude might we expect for JWST?

   A. $\sim 29$
   B. $\sim 31$
   C. $\sim 33$
   D. $\sim 36$

4. For which of these lines of latitude will a vertical stick in the ground have no shadow at local midday on 21$^{\text{st}}$ December 2021?

   A. Tropic of Cancer
   B. Equator
   C. Tropic of Capricorn
   D. Antarctic Circle

5. When Mars has its opposition in 2022, an observer in the UK will see it in Taurus. In roughly which month will this opposition take place? [Mars' opposition corresponds to when it is closest in its orbit to the Earth.]

   A. March
   B. June
   C. September
   D. December

6. In which constellation is Saturn visible during Autumn 2021? [Hint: it is close to Jupiter, which spent most of Summer 2021 in Aquarius]

   A. Capricorn
   B. Gemini
   C. Leo
   D. Libra

7. Which of these stars are closest to the Sun in October 2021?

   A. Rigel (Right ascension $= 05^{\mathrm{h}}\ 15^{\mathrm{m}}$, declination $= -8.20°$)
   B. Regulus (Right ascension $= 10^{\mathrm{h}}\ 08^{\mathrm{m}}$, declination $= +11.97°$)
   C. Spica (Right ascension $= 13^{\mathrm{h}}\ 25^{\mathrm{m}}$, declination $= -11.16°$)
   D. Vega (Right ascension $= 18^{\mathrm{h}}\ 37^{\mathrm{m}}$, declination $= +38.78°$)

8. Estimate the number of photons incident on a human pupil (of radius 2mm) from the Sun per second when it is at the zenith on a clear day.

   A. $\sim 10^{10}$
   B. $\sim 10^{13}$
   C. $\sim 10^{16}$
   D. $\sim 10^{19}$

9. A comet orbits the Sun with a period of 172 years and eccentricity 0.94. It is currently at a distance of 60 au away from the Sun. After which of these times will the comet be moving the fastest?

   A. 43 years
   B. 86 years
   C. 129 years
   D. 172 years

10. In the rotating reference frame where the Earth is stationary, an asteroid orbits the Sun in 3.5 years. What is the distance between the asteroid and the Sun?

   A. 1.25 au
   B. 2.08 au
   C. 3.95 au
   D. 6.54 au

## Section B: Short Answer

*Each short question is worth 5 marks.* **Total: 10 marks**.

### Distance to a Cepheid Variable

11. In the early 1900s, Henrietta Leavitt made several observations of variable stars from the Harvard College Observatory that led her to propose that the period of variation was related to the intrinsic brightness of the star. In particular, for a class of variable stars known as Cepheids, there was a strong power law relating luminosity to period, and hence a straight line on a log-log graph. Since magnitudes are $\propto \log L$, this can be described by the empirical relation

$$\langle \mathcal{M} \rangle = -2.43(\log P - 1) - 4.05$$

where $P$ is the period measured in days and $\langle \mathcal{M} \rangle$ is the mean absolute magnitude of the star, defined as the apparent magnitude measured from a distance of 10 pc. The apparent and absolute magnitudes are related as

$$m - \mathcal{M} = 5 \log d - 5$$

where $d$ is the distance measured in pc.

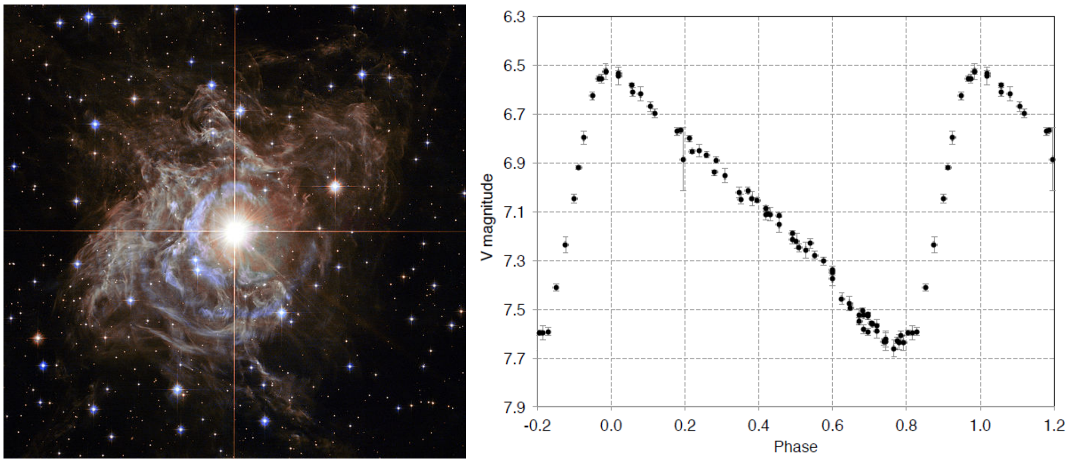



**Figure 1:** *Left:* Colour composite view of the circumstellar nebula around RS Puppis assembled from HST images. Credit: NASA, ESA, and the Hubble Heritage Team (STScI/AURA)-Hubble/Europe Collaboration. *Right:* The light curve of RS Puppis. Phase 0 corresponds to the when the star is at its brightest, and the data has been folded over a known period of 41.5 days. Credit: AAVSO / P. Kervella et al. (2017).

In the past, distances to stars could only be determined using something called the parallax method, where the angular shift of a star relative to the background stars was measured at two points in the Earth's orbit six months apart. The parallax angle is the same as the angle subtended by 1 au at the distance of the star. In Henrietta's time, parallax measurements only reached about 100 pc, however her method with Cepheid variables extended this range to more than $10^6$ pc, enabling much better determination of distances across the Milky Way and into nearby galaxies.

The variable star RS Puppis (see Figure 1) is one of the biggest and brightest known Cepheids in the Milky Way galaxy and has one of the longest periods for this class of star at 41.5 days.

(a) Measure the mean apparent magnitude from Figure 1, and find the distance to the star, given that absorption of the light by interstellar dust means the star appears 1.42 magnitudes fainter. [3]

(b) In 2018 the Gaia satellite measured a parallax angle of 0.5844 milliarcseconds (where there are 3600 arcseconds in a degree). What does this suggest is the distance to the star? [2]

## Precession of the Earth

12. Currently, Polaris is very close to the north celestial pole (the projection of the Earth's rotational axis on the sky) and so all other stars appear to rotate around it. However, this axis is drawing out a large circle in the sky with an angular radius of $23.44°$ so Polaris will only temporarily be the pole star (see Figure 2). This precession of the rotational axis is mainly driven by the gravitational pull of the Moon and the Sun.

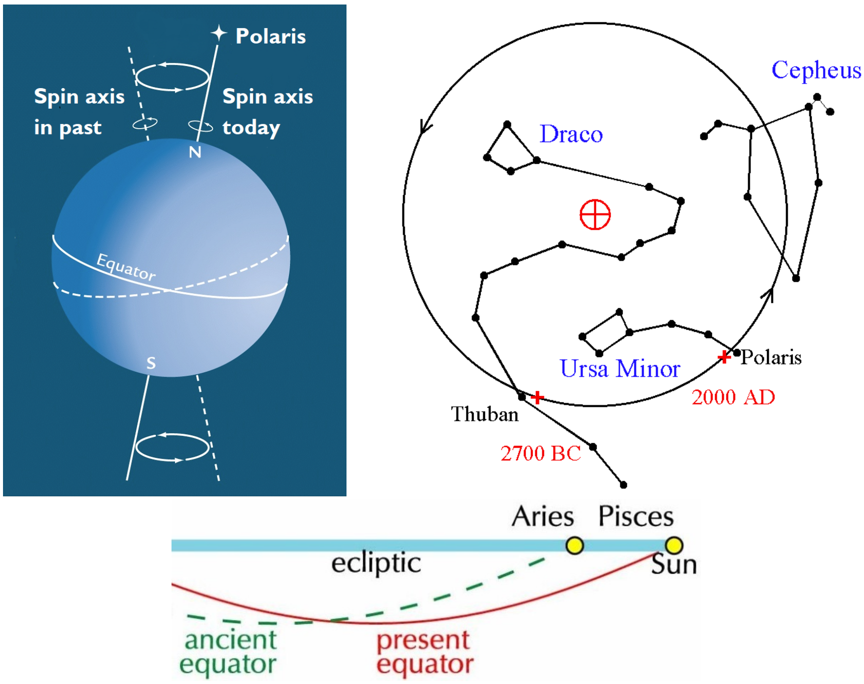



**Figure 2:** *Top left:* The Earth's rotational axis itself rotates slowly (white circle), in what is known as axial precession. Credit: David Battisti / University of Washington.
*Top right:* Due to precession, the pole star has changed over time. About 5000 years ago, the star Thuban in the constellation of Draco was the pole star. Credit: Richard W. Pogge / Ohio State University.
*Bottom:* The position of the Sun at the spring equinox (where the celestial equator meets the ecliptic) has also changed over the same period, moving from Aries to Pisces. Credit: Guy Ottewell / Universal Workshop.

Another consequence is that the position of the Sun at the equinoxes varies slightly, moving slowly westwards. This gives rise to two definitions of a year:

- a sidereal year (the time taken for the Earth to orbit the Sun once with respect to the background stars) = 365.256363 days
- a tropical year (the time taken for the Sun to return to the same position in the cycle of the seasons) = 365.242190 days

The Gregorian calendar is a 400-year cycle with a system of leap years. The rule is: "every year that is exactly divisible by four is a leap year, except for years that are exactly divisible by 100, but these centurial years are leap years if they are exactly divisible by 400."

(a) By working out the average length of a year in the Gregorian calendar, is it closer to the sidereal or the tropical year?

[2]

(b) Taking one day to be exactly 24 hours, what is the difference (in minutes and seconds) between a sidereal year and a tropical year?

[1]

(c) Assuming the rate of axial precession remains constant, work out the time (in sidereal years) to complete a whole precessional cycle.

[2]

**BLANK PAGE**

## Section C: Long Answer

*Each long question is worth 10 marks. Answer* **either** *Qu 13* **or** *Qu 14.* **Total: 10 marks**.

### Islamic Lunar Calendar

13. In the UK we use the Gregorian calendar; it is a solar calendar so that a year corresponds to the time to orbit the Sun once, where 1 solar year is $\approx 365.25$ days. Several cultures use a lunar calendar, where each month is determined by the time it takes to go from New Moon to New Moon, and have a lunar year that is exactly 12 lunar months. An example of this is the Islamic calendar. Since the length of a lunar month (29.53 days) is a little shorter than the average month length in our solar calendar (see Figure 3), it means the start date of each month in the Islamic calendar is not tied to the seasons and gradually moves earlier in the solar year.

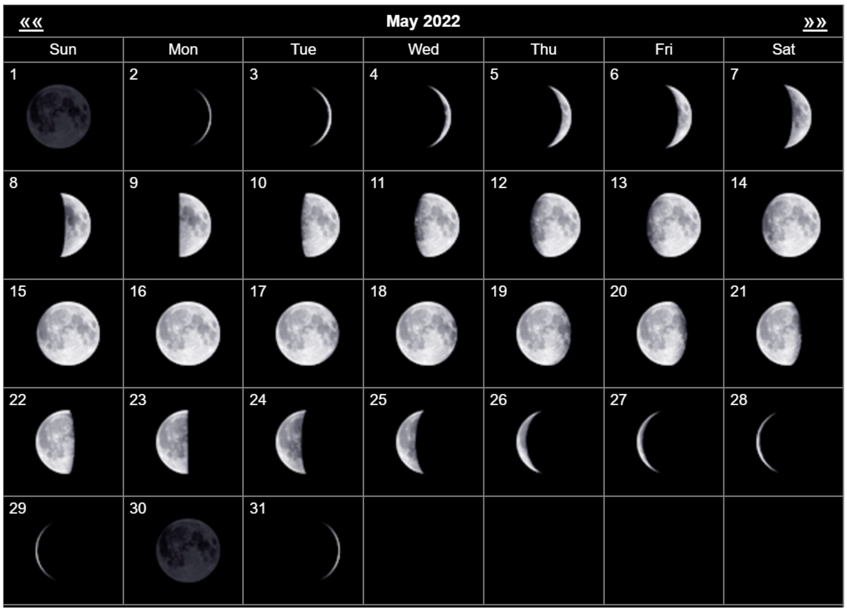



**Figure 3:** All the moon phases in May 2022, showing that the time measured from New Moon to New Moon (a lunar month) is shorter than a month in the Gregorian calendar. Credit: *MoonConnection.com*

(a) Calculate (to the nearest day) how much earlier in the Gregorian calendar (on average) a given month in the Islamic calendar will start compared to the previous year.

[1]

(b) Given that the Islamic year 1429 AH fell completely within the Gregorian year 2008 CE, calculate the Gregorian year in which the Islamic calendar started, and predict the Gregorian year when (at least in part of it) they will both have the same numerical value of year.

[3]

(c) In some Islamic countries, odd-numbered months have 30 days and even-numbered months have 29 days. How often will leap years be in this system (where the twelfth month is also 30 days)? [This is analogous to the Gregorian system where there is a leap year roughly once every 4 years.]

[1]

(d) In other Islamic countries, the start of a new month is determined through direct telescopic observations of the Moon, looking for a very thin crescent. This is a very hard measurement to make and the human eye struggles to recognise the presence of a crescent until about $0.6\%$ of the lunar disc is illuminated. Calculate how many hours this is after the end of the astronomical New Moon. Assume the lunar orbit is circular.

[5]

## Radio Emission from the Crab Nebula

14. The Crab Nebula is the expanding remnant of a supernova that was observed by Chinese astronomers about 1000 years ago. Although faint in the visible, it is one of the brightest objects in the sky both in the radio and in X-ray. Figure 4 shows it as observed with the Very Large Array (a radio telescope) operating at a frequency of 3.0 GHz. The whole image has an angular size of 530 by 530 arcseconds (where there are 3600 arcseconds in a degree).

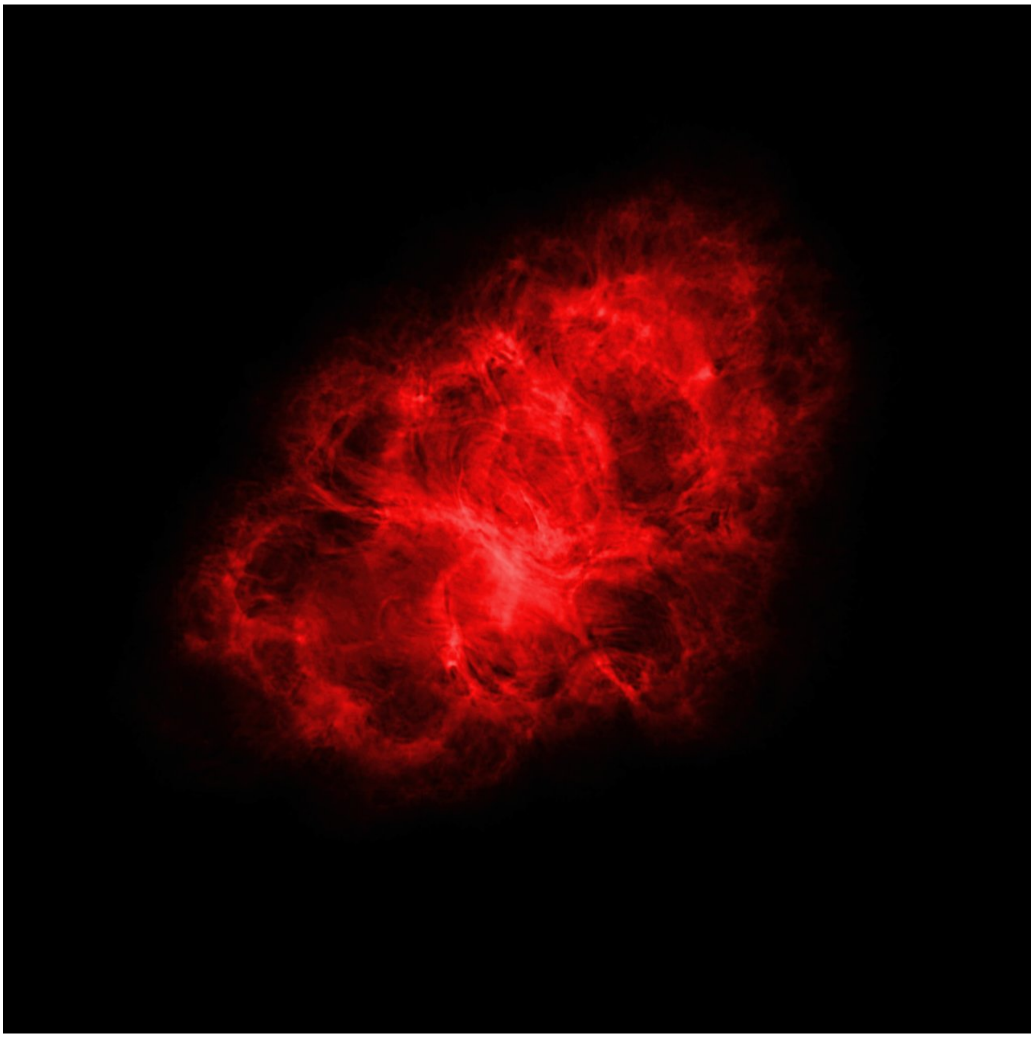


**Figure 4:** The Crab Nebula as seen at the radio frequency of 3.0 GHz, imaged by the Very Large Array in 2017. Credit: NRAO/AUI/NSF.

For long wavelengths (such as those found in the radio), Planck's Law for black-body radiation simplifies to the Rayleigh-Jeans Law,

$$B_\nu = \frac{2\nu^2 k_B T}{c^2} \, ,$$

where $B$ is the emitted intensity at frequency $\nu$ per unit frequency per steradian (a unit of solid angle, for which there are $4\pi$ steradians in a sphere), $k_B$ is the Boltzmann constant, $T$ is the temperature of the source, and $c$ is the speed of light. Typically what is measured is the emitted intensity per unit frequency, $I_\nu$, which is related to $B_\nu$ as

$$I_\nu = B_\nu \Omega \, ,$$

where $\Omega$ is the solid angle in steradians subtended by the emitting source as seen by the detector. A small angular ellipse subtends approximately $\Omega = \pi ab$ steradians, where $a$ is the semi-major axis and $b$ is the semi-minor axis of the ellipse, both specified in radians.

(a) The total measured intensity in Figure 4 is $I_\nu = 7.43 \times 10^{-24}$ W m$^{-2}$ Hz$^{-1}$. By taking suitable measurements from Figure 4, calculate the temperature of the nebula.

[6]

(b) The speed of the expansion of the nebula along its longest axis is about $1500$ km s$^{-1}$, although it was ejected at a higher speed during the supernova. Assuming an average deceleration of $15$ μm s$^{-2}$, taking the distance to the nebula as 2.0 kpc, and given the image was taken in 2017, estimate the year of the supernova.

[4]

## END OF PAPER

*Questions proposed by:*
*Dr Alex Calverley (Royal Grammar School, Guildford)*
*Josh Brown (University of Cambridge)*
*Niam Vaishnav (University of Oxford)*