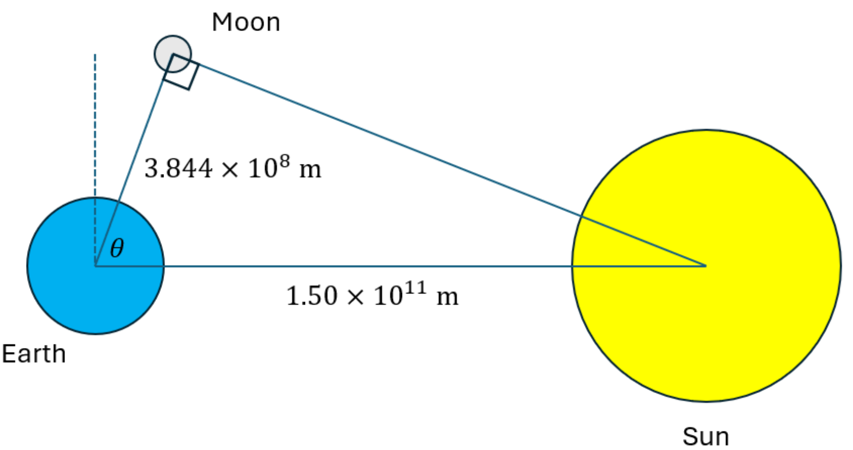
BAAO
British Astronomy and Astrophysics Olympiad

BAAO Astro Challenge

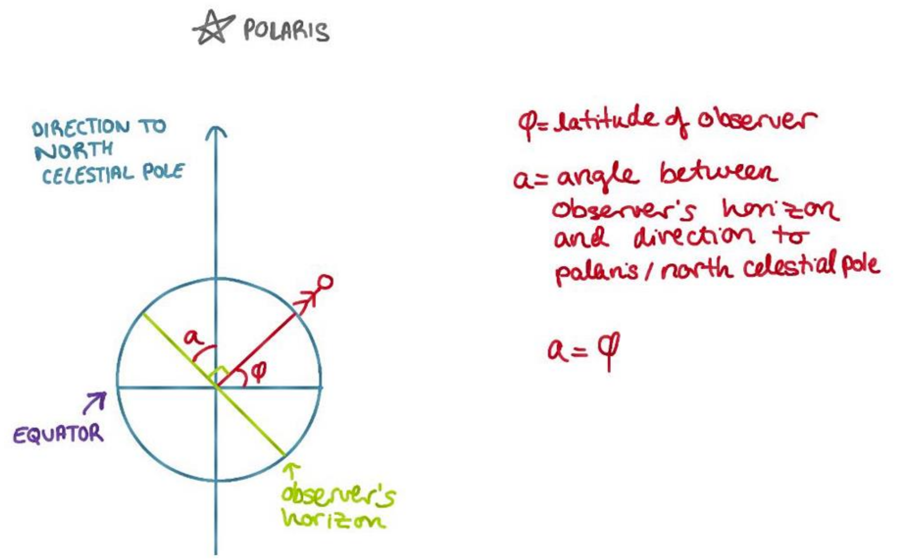
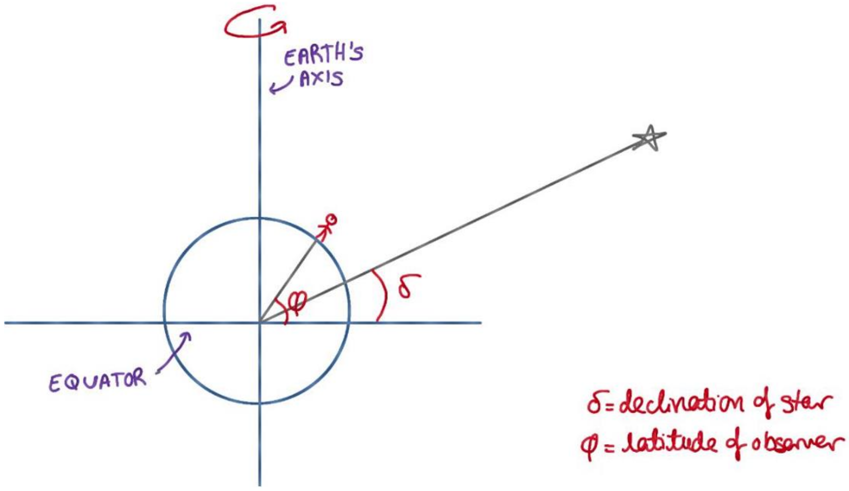
September to December 2024

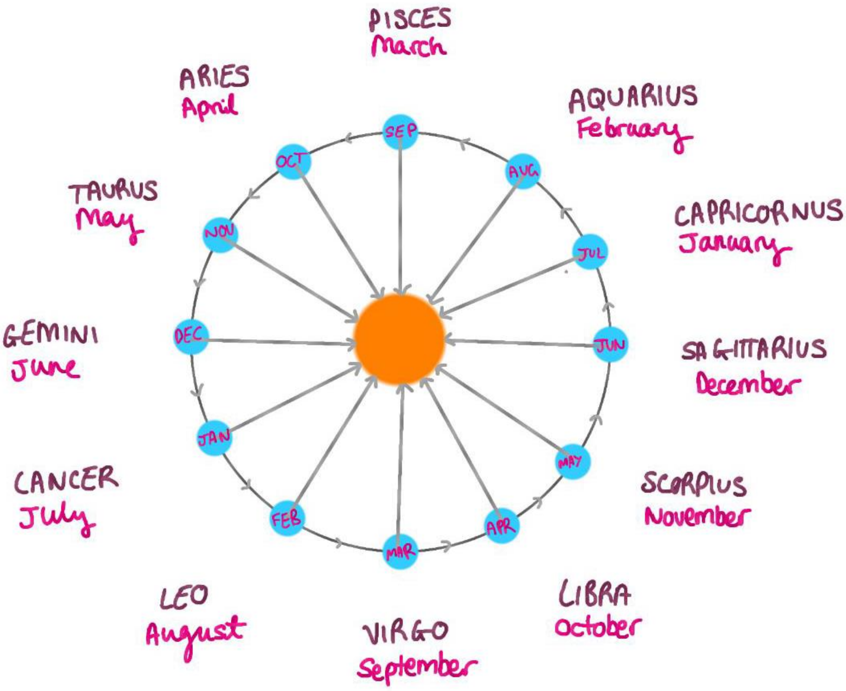
Solutions and marking guidelines

- The total mark for each question is in **bold** on the right-hand side of the table. The breakdown of the mark is below it.
- There is an explanation for each correct answer for the multiple-choice questions. However, the students are only required to write the letter corresponding to the right answer.
- In Section C, students should attempt **either** Qu 13 **or** Qu 14. If both are attempted, consider the question with the higher mark.
- Answers to two or three significant figures are generally acceptable. The solution may give more than that, especially for intermediate stages, to make the calculation clear.
- There are multiple ways to solve some of the questions; please accept all good solutions that arrive at the correct answer. Students getting the answer in a $\boxed{\text{box}}$ will get all the marks available for that calculation / part of the question (students may not explicitly calculate the intermediate stages, and should not be penalised for this so long as their argument is clear)

Question	Answer	Mark
Section A		**10**
1.	D Philae landed on the comet on 12th November 2014, having been launched with the Rosetta mission 10 years earlier. It landed out of direct sunlight next to a cliff and so could only operate for as long as its battery lasted (a couple of days) – it wasn't spotted in images until 2016!	1
2.	B 	1

	With the Moon being exactly half lit, the Sun-Moon-Earth angle must be 90° and so this is not the same time as quadrature (when the Sun-Earth-Moon angle is 90°) as the Sun is not infinitely far away from the Earth. Considering the geometry $$\theta = \cos^{-1}\left(\frac{3.844 \times 10^{8}}{1.5 \times 10^{11}}\right) = 89.85°$$ Aristarchus of Samos claimed that the angle between Sun and Moon at first quarter was 87° and used that to get the distance to the Sun as a multiple of the distance to the Moon – it is a very hard measurement to do with the naked eye, so it is no surprise he underestimated it	
3.	A This answer could be identified quickly through dimensional analysis since H_0 has units of s^{-1}, so only that option would have units $\mathrm{m\ s^{-2}}$ Alternatively, we could have also started with Hubble's law and differentiated it to identify the same option: $$v = H_0 d \ \ \therefore a = \dot{v} = H_0 \dot{d} = H_0 v = H_0(H_0 d) = H_0^2 d$$ (Differentiation is not required in the Astro Challenge, so dimensional analysis is what we expect most students to use) [Note: in our solution, we have assumed that the Hubble constant does not vary with time. In reality, this is not the case, but the term introduced by its variation is of approximately the same magnitude as the one we have found, so our solution still produces a reasonable estimate]	1
4.	C Using the standard formula connecting brightness and magnitude (given on the formula sheet) $$\frac{b_{2.5}}{b_{10.8}} = 10^{-0.4(2.5-10.8)} = 2090$$	1
5.	A Considering the area of sky in one photo in square degrees $$(6 \times 4096) \times (6 \times 4132) \times \left(\frac{0.101}{3600}\right)^2 = 0.48\ \mathrm{deg}^2$$ The surface area of a sphere is $4\pi r^2$, so making $r = 1$ radian then the total area of the sky in square degrees is $$4\pi\left(\frac{360}{2\pi}\right)^2 = 41253\ \mathrm{deg}^2$$ Consequently, the minimum number of photos needed to cover 36% of the sky $$\frac{41253 \times 0.36}{0.48} = 30967 \approx 31000\ \mathrm{photos}$$ In practice, far more of these 600-megapixel images will be taken over the course of its 6-year mission, sending about 100 GB of data back each day to be analysed – storage and processing of such a vast amount of data proves a technical challenge	1
6.	B Polaris is only visible in the northern hemisphere, so it had to be a northern latitude. As described in the observational astronomy guide, since Polaris is (effectively) directly above the North Pole the angle it makes with the horizon must be the same as your latitude (see diagram on the next page)	1

<table>
<tr><td></td><td>
</td><td></td></tr>
<tr><td>7.</td><td>B
Again, remembering the table given in the observational astronomy guide, Canis Major is the only one of the (non-zodiacal) constellations from the list that is south of the celestial equator
<table>
<tr><th>Constellation</th><th>Celestial equator?</th><th>Ecliptic?</th></tr>
<tr><td>Orion</td><td>On celestial equator</td><td>South</td></tr>
<tr><td>Taurus</td><td>North</td><td>On ecliptic</td></tr>
<tr><td>Gemini</td><td>North</td><td>On ecliptic</td></tr>
<tr><td>Canis Major</td><td>South</td><td>South</td></tr>
<tr><td>Cassiopeia</td><td>North</td><td>North</td></tr>
<tr><td>Perseus</td><td>North</td><td>North</td></tr>
<tr><td>Andromeda</td><td>North</td><td>North</td></tr>
<tr><td>Pegasus</td><td>North</td><td>North</td></tr>
</table></td><td>1</td></tr>
<tr><td>8.</td><td>A
Aldebaran is in Taurus. Both M1, the Crab Nebula, and M45, the Pleiades, are in Taurus, but the Pleiades is an open cluster, while the Crab Nebula is a supernova remnant and nebula, as required</td><td>1</td></tr>
<tr><td>9.</td><td>C
The maximum altitude of culmination will happen for the star which has a declination closest to the latitude (since $h_{max} = 90° - |\delta - \phi|$), and for Rio $\phi = 22.91°$ S $= -22.91°$ so the closest δ is that of Nunki ($-26.30°$)

</td><td>1</td></tr>
</table>

10.	C If a planet is in opposition, then the Sun must be on the opposite side of the sky. Regarding the position of the Sun, Aquarius roughly corresponds to February, Pisces to March, Taurus to May, and Gemini to June. Therefore, the oppositions of these planets in these constellations occur roughly in August, September, November, and December respectively, and as such they are ordered, Saturn (08/09), Neptune (21/09), Uranus (17/11), Mars (16/01). (Note: since the correspondences between months and constellations are only approximate, the opposition of Saturn actually occurs in early September, not in August as we might expect) 	1
Section B		**10**
11.	a) Increase in brightness over the 13 years from 2011 to 2024 $$\frac{b_1}{b_0} = 1.0960^{13} = 3.29$$ Rearranging the relationship between brightness ratio and magnitude given on the formula sheet $$m_1 = m_0 - 2.5\log_{10}\left(\frac{b_1}{b_0}\right) = 5.0 - 2.5\log_{10} 3.29 = 5.0 - 1.29 = \boxed{3.7}$$ This change in brightness means that the number of stars visible over this period (compared to going from childhood to adulthood) has approximately halved. In the Kyba et al. (2023) study, they found that at a typical site, over a period of 18 years, the number of visible stars dropped from 250 to about 100, meaning the sky today looks markedly different from the sky when you were born.	**[2]** 1 1

b) **[3]**

Working out the necessary increase in brightness

$$\frac{b_1}{b_0} = 10^{-0.4(m_1 - m_0)} = 10^{-0.4(-1.46-5.0)} = 383.7$$ 1

Hence, the number of years, n, is

$$383.7 = 1.0960^n \therefore n = \frac{\log 383.7}{\log 1.0960} = 64.9 \approx 65 \text{ years}$$ 1

[Accept log to any base e.g. ln, $\log_{1.096}$ etc.]

Consequently, the year that this happens is

$$2011 + 65 = \boxed{2076}$$ 1

[Accept (late) 2075]

[An alternative method for the first mark is to work out the limiting magnitude change per year $\Delta m = 2.5 \log_{10} 1.0960 = 0.0995$, and hence the second mark becomes $n = \frac{5.0-(-1.46)}{0.0995} = 64.9$]

This year is close enough that it will likely fall within the lifetimes of students sitting this paper! In practice the rate of increase in sky brightness varied with continent (it was only 6.5% in Europe, so it would take somewhat longer to eliminate all stars from the sky) and is unlikely to be able to be maintained for that many decades – even so it is a stark reminder of the impact of light pollution, particularly in urban areas.

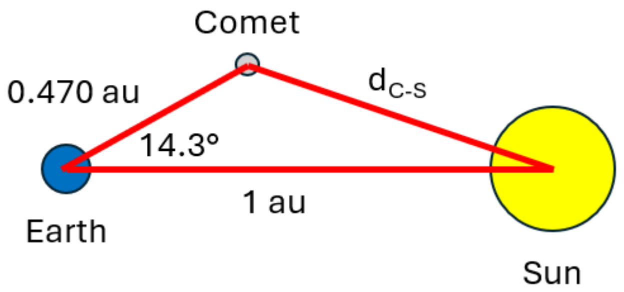
12. a) **[3]**

Finding the distance of closest approach to the Earth by the comet

$$g = \frac{GM_{\oplus}}{d_{C-E}^2} \therefore d_{C-E} = \sqrt{\frac{GM_{\oplus}}{g}} = \sqrt{\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{8.0 \times 10^{-8}}}$$

$$= 7.055 \times 10^{10} \text{ m} = 0.470 \text{ au}$$ 1

Using the cosine rule to then get the distance between the Sun and comet at that point

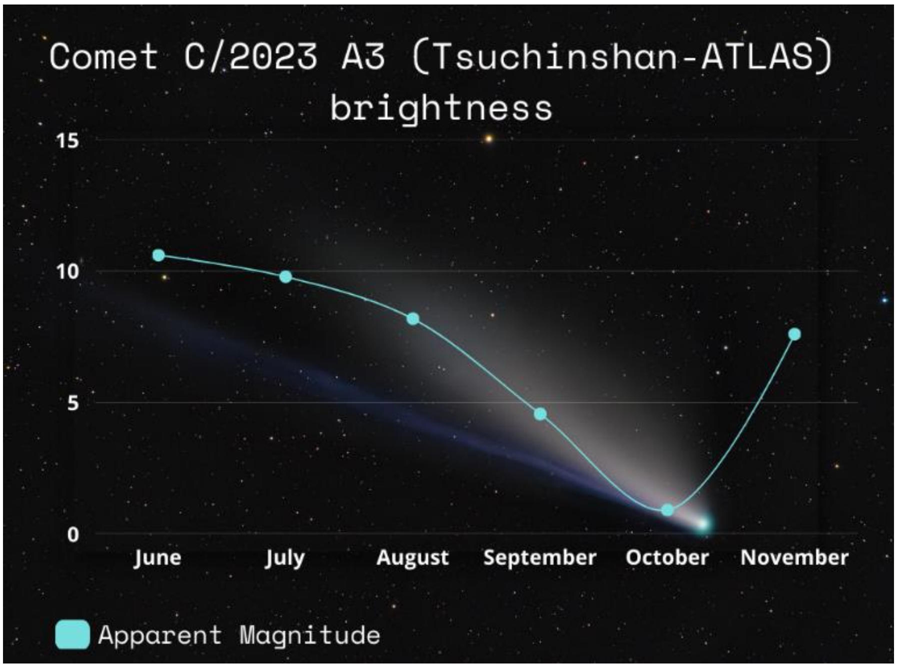


$$d_{C-S} = \sqrt{0.470^2 + 1^2 - 2 \times 0.470 \times 1 \times \cos 14.3°} = 0.556 \text{ au}$$ 1

Finally, putting it all into the given formula

$$m = 4.3 + \frac{5}{2} \times 4 \log_{10} 0.556 + 5 \log_{10} 0.470 = \boxed{0.12}$$ 1

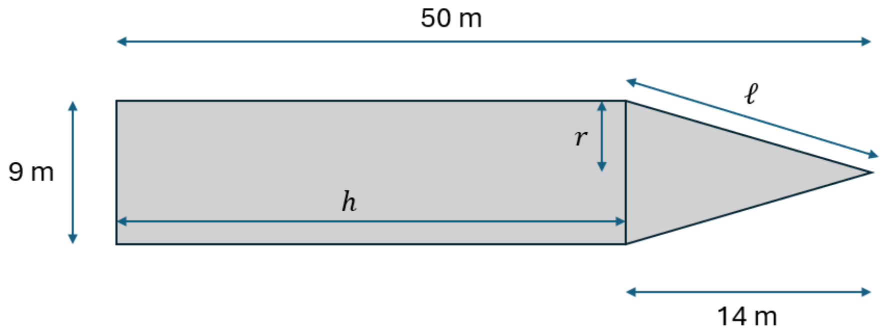
[Accept 0.1 so long as a value pre-rounding is also visible in the working]

	This apparent magnitude would make it an exceptionally bright comet – up there with some of the 'great comets' of the past. Of course, it is very dependent on whether the comet survives perihelion on 27th September and then on what happens to the value of K, which is notoriously unpredictable for a comet – conservative estimates though predict it should at least reach around magnitude 3.0. Since comets are more extended objects (rather than stars which have more concentrated light) this is just about within the naked eye limit and so looking towards Virgo and later Ophiucus just after sunset (ideally with binoculars) in the second half of October, you should see it. 	
	b) We are told the comet is very far from the Sun and hence $d_{C-S} \approx d_{C-E}$ Consequently, the formula becomes (with $K = 4$) $$m \approx \mathcal{M} + 15 \log_{10} d_{C-S}$$ $$\therefore d_{C-S} = 10^{\frac{m-\mathcal{M}}{15}} = 10^{\frac{18.1-4.3}{15}} = 10^{0.92} = \boxed{8.3 \text{ au}}$$ [Answer must be in au for the second mark. Accept 8 au (as a 1 s.f. answer) too] This places it between the orbits of Saturn and Jupiter when it was discovered, back in January 2023. Its eccentricity is very high (possibly slightly greater than 1) so after its visit to the inner Solar System it may never return.	**[2]** 1 1

Section C		**10**
13.	a) First, we determine the circular orbit velocity at an altitude of 231 km, and use it to determine the apoapsis velocity $$v_{circ} = \sqrt{\frac{GM_\oplus}{r}} = \sqrt{\frac{6.67\times10^{-11}\times5.97\times10^{24}}{(6370+213)\times1000}} = 7770\ \mathrm{m\ s^{-1}}$$ $$\therefore v_{apo} = 0.94 v_{circ} = 7310\ \mathrm{m\ s^{-1}}$$ The total energy of Starship is hence $$E_{tot} = GPE + KE = -\frac{GM_\oplus m}{r_{apo}} + \frac{1}{2} m v_{apo}^2$$ $$= -\frac{6.67\times10^{-11}\times5.97\times10^{24}\times85000}{(6370+213)\times1000} + \frac{1}{2}85000\times7310^2$$ $$= \boxed{-2.87\times10^{12}\ \mathrm{J}}$$ [The third mark is for realising that both GPE and KE need to be considered in the formula. Some students may try and use $E_{tot} = -\frac{1}{2}\frac{GMm}{a}$ for the third and fourth mark, in which case the first and second mark are for approaches to find the semi-major axis of the orbit. This requires formulae not expected for the Astro Challenge, but one approach is $v_{apo} = \sqrt{\frac{(1-e)GM}{r_{apo}}} = \sqrt{1-e}\times v_{circ} \therefore 0.94 = \sqrt{1-e} \therefore e = 0.1164$ [1] $r_{apo} = (1+e)a \therefore a = \frac{r_{apo}}{1+e} = \frac{6370+213}{1+0.1164} = 5897\ \mathrm{km}$ [1] If a student uses $E_{tot} = -\frac{1}{2}\frac{GMm}{a}$ with $a = r_{apo}$ leading to an answer of $E_{tot} = -2.57\times10^{12}$ J then they only get the third and fourth (as ecf) marks]	**[4]** 1 1 1 1 1
	b) Once Starship lands, its total energy will just be the same as the GPE at the Earth's surface (since the KE term will be gone as $v = 0$) $$E_{landed} = -\frac{GM_\oplus m}{R_\oplus} = -\frac{6.67\times10^{-11}\times5.97\times10^{24}\times85000}{6370\times1000}$$ $$= -5.31\times10^{12}\ \mathrm{J}$$ Hence, by conservation of energy, the work done by the atmosphere must be equal to the difference between these two energies $$\Delta E = E_{tot} - E_{landed} = \left(-2.87 - (-5.31)\right)\times10^{12} = \boxed{2.44\times10^{12}\ \mathrm{J}}$$ [Allow ecf from part a). If they have made the mistake mentioned above, then they will get $\Delta E = 2.74\times10^{12}$ J. The work done should be a positive number – if they get a negative number subtract 0.5 marks]	**[2]** 1 1

c) **[3]**

First, we determine the area of the forward-facing side of the steel skin (**half** of the total curved area) – a diagram can be very helpful



Remembering the formulae (from GCSE) for the curved surface of a cylinder and a cone

$$A = \frac{1}{2} \times (2\pi r h + \pi r \ell)$$

$$\therefore A = \frac{1}{2} \times \left(2\pi \times 4.5 \times (50 - 14) + \pi \times 4.5 \times \sqrt{14^2 + 4.5^2}\right) = 613 \text{ m}^2$$ 1

[Allow ecf on all following marks if they correctly use their area. If they only forget the factor of ½ then only remove half a mark]

We can now calculate the mass of steel used in this part of the skin, given the thickness x and density ρ

$$m = \rho V = \rho A x = 8000 \times 613 \times 4 \times 10^{-3} = 1.96 \times 10^4 \text{ kg}$$ 1

Hence, the maximum energy that can be safely transferred to the steel is

$$\Delta E_{max} = mc\Delta T = 1.96 \times 10^4 \times 500 \times 900 = \boxed{8.83 \times 10^9 \text{ J}}$$ 1

d) **[1]**

By comparing our answer to part c) to part b) we find

$$\text{Max } \% = \frac{\Delta E_{max}}{\Delta E} = \frac{8.83 \times 10^9}{2.44 \times 10^{12}} \times 100\% = \boxed{0.36\%} \ (\therefore < 1\%)$$ 1

[Answer must be given as a %, or a comparison must be made with 0.01]

This goes to show the incredible engineering feat involved in protecting the body of the spaceship from getting too hot during re-entry. Note that even without the thermal protection system much of the total energy released would be transferred to the atmosphere instead of the spacecraft, but not nearly enough to reach the value we have just calculated and protect the steel skin.

<table>
<tr><td>14.</td><td>a)

Given that $f_R \propto p$ and on Earth $p = 100\ \mathrm{kPa}$ and $f_R = 40\ \mathrm{kHz}$ then
$$f_{R,Mars} = \frac{p_{Mars}}{p_{Earth}} \times f_{R,Earth} = \frac{0.6}{100} \times 40 = 0.24\ \mathrm{kHz} = \boxed{240\ \mathrm{Hz}}$$
Since the human hearing range is 20 Hz to 20 kHz, this frequency sits within it and so humans would experience both sound speed regimes.</td><td>[1]

1</td></tr>
<tr><td></td><td>b)

Using the formula given, being careful to covert M into kg mol^{-1}
$$c_{low\ f} = \sqrt{\frac{\frac{9}{7} \times 8.314 \times 230}{43.34 \times 10^{-3}}} = 238.175\ \mathrm{m\ s^{-1}}$$
$$c_{high\ f} = \sqrt{\frac{\frac{7}{5} \times 8.314 \times 230}{43.34 \times 10^{-3}}} = 248.536\ \mathrm{m\ s^{-1}}$$
$$\therefore \Delta c = 248.536 - 238.175 = \boxed{10.4\ \mathrm{m\ s^{-1}}}$$
[Since it was a ‘show that’ question, expect Δc to be given to at least 3 s.f]

This is a big enough difference in sound speed that humans would notice it when considering distant sound sources (such as a dust storm, or a rocket taking off).</td><td>[2]

0.5

0.5

1</td></tr>
<tr><td></td><td>c)

Calculating the acoustic impedance on both planets in the low frequency regime
$$Z_{Earth} = \rho_{Earth} c = 1.217 \times 340 = 413.78\ (\mathrm{kg\ m^{-2}\ s^{-1}})$$
$$Z_{Mars} = \rho_{Mars} c = 0.02 \times 238.175 = 4.76\ (\mathrm{kg\ m^{-2}\ s^{-1}})$$
[Do not insist on units for Z to award these marks]

$$\therefore \text{intensity level difference} = 10 \log_{10}\left(\frac{413.78}{4.76}\right) = \boxed{19.4\ \mathrm{dB}}$$
[Accept 20 dB if clearly rounded to a 1 s.f. value, given ρ_{Mars} is 1 s.f.]

This drop in loudness is similar to the difference between a shout and quiet conversation, so would be something any humans spending time on Mars would have to get used to when outside their living quarters (which will be much closer to Earth pressure and temperature, so these effects are limited to outside). Another effect is that in CO_2 rich atmospheres specifically (like Mars and Venus), there is a strong attenuation of all sound waves due to the atmosphere’s viscosity and thermal conductivity, but particularly at high frequencies; an 8 kHz sound on Mars at a distance of 8 m would have attenuated as much as the same sound 65 m away on Earth – and due to the acoustic impedance is already quieter to begin with, so the Martian soundscape is rather local.</td><td>[2]

0.5

0.5

1</td></tr>
</table>

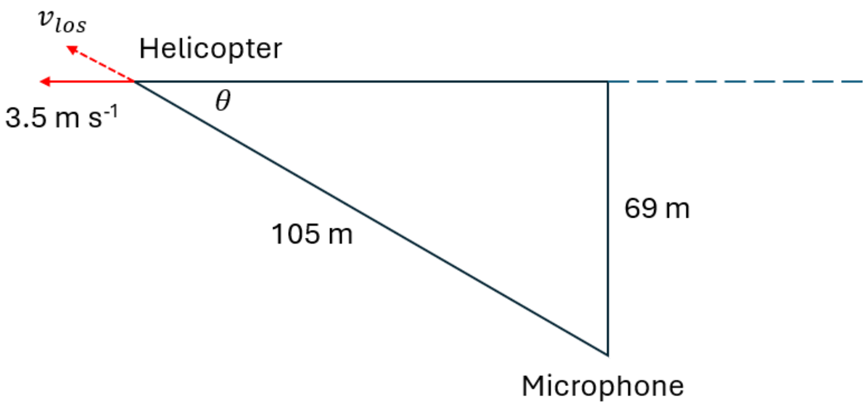
d) **[5]**

Ignoring the data from the first 60 seconds, the two obvious places to read are from where the Doppler effect was largest and so % uncertainty smallest – this corresponds to when the helicopter is travelling fastest with the largest line-of-sight velocity (although students can make reasonable estimates from any point on the solid line). Choosing the first trough around 70 seconds:

Doppler shift $= -1.1\%$; Speed $= 3.5\ \mathrm{m\ s^{-1}}$; Range $= 105\ \mathrm{m}$ 1

[This mark is for reading values off both graphs for the Doppler shift, helicopter speed and range – it should be from the solid line although allow **any** successful reading off the graph. For their read values allow tolerances of $\pm 0.1\%$ for shift, $\pm 0.1\ \mathrm{m\ s^{-1}}$ for speed and $\pm 2\ \mathrm{m}$ for range. If using the peak around 95 seconds, then they should get a shift $=$ $+1.1\%$; speed $= 3.5\ \mathrm{m\ s^{-1}}$; range $= 107\ \mathrm{m}$]

Constructing the relevant triangle to get the line-of-sight speed of the helicopter e.g. for the first trough the diagram is 1



[Accept any diagram that could be used to calculate θ based upon their numbers read from the graph, even if it also doesn't show the speed]

Hence the line-of-sight speed is

$$v_{los} = 3.5\cos\theta = 3.5\cos\left(\sin^{-1}\frac{69}{105}\right) = 3.5\cos 41.1° = 2.64\ \mathrm{m\ s^{-1}}$$ 1

For a source moving away from an observer at speed v_{los} then from the classical Doppler effect

$$f_{obs} = f_{emit}\left(\frac{c}{c + v_{los}}\right)$$

$$\therefore \frac{f_{emit}}{f_{obs}} = 1 + \frac{v_{los}}{c}$$

$$\therefore c = \frac{v_{los}}{\frac{f_{emit}}{f_{obs}} - 1}$$ 1

Putting in the numbers, given $f_{obs} = (1 - 0.011)f_{emit}$

$$c = \frac{2.64}{\frac{1}{1 - 0.011} - 1} = \boxed{237\ \mathrm{m\ s^{-1}}}$$ 1

This number is very close to our sound speed calculated in part (b) for the low frequency regime, $c_{low\ f}$, and acts as independent confirmation that temperature-derived sound speed measurements are reasonable
[No mark for comment]

[The fourth mark is for some method to get the speed of sound from v_{los} and the fifth is for calculating a value of c. The sound speed derived from this method is very sensitive to accurate reading off the graph – for example, if the student measures -1.2% for the shift instead, they get $c = 217\ \mathrm{m\ s^{-1}}$. Allow this final mark for any value that is consistent with their data read from the graph, so long as it is within the given tolerances, and they have a sensible method. If they use the data from the largest peak, then using the given values you get $v_{los} = 2.68\ \mathrm{m\ s^{-1}}$, the classical Doppler is $f_{obs} = f_{emit}\left(\frac{c}{c - v_{los}}\right)$ leading to $c = \frac{v_{los}}{1 - \frac{f_{emit}}{f_{obs}}} = \frac{2.68}{1 - \frac{1}{1+0.011}} =$ $\boxed{246\ \mathrm{m\ s^{-1}}}$, which is also close to expectations. As an alternative to the classical Doppler effect (since not all students will be aware of it) allow the fourth mark for those that make the approximation that the situation is like classical redshift (since $v_{los} \ll c$) and hence calculate that $c = \frac{v_{los}}{|z|}$ where z is the fractional shift. For the largest trough data this gives $c = \frac{2.64}{0.011} = \boxed{240\ \mathrm{m\ s^{-1}}}$ and for the largest peak data this gives $c = \frac{2.68}{0.011} =$ $\boxed{243\ \mathrm{m\ s^{-1}}}$, both of which are close enough to the correct Doppler application to justify our simplifying assumption]

Due to the sensitivity of the calculation to the % shift, the researchers fitted a model over all the data points and not just one or two, calculating $c = (237.7 \pm 3.0)\ \mathrm{m\ s^{-1}}$. Sensor data from Perseverance showed there was wind at a speed of 2.5 m s^{-1} in the direction of the helicopter, so the real sound speed from this audacious experiment is close to 240 m s^{-1}. During the flight the temperatures ranged from 232 K to 240 K suggesting temperature-derived speeds ranging from 238.8 m s^{-1} to 242.9 m s^{-1}, consistent with the Doppler measurements, so this was a very successful in-situ verification of the way sound travels through the Martian atmosphere.