0 7 . 1 Derive an expression to show that for satellites in a circular orbit

$$T^2 \propto r^3$$

where T is the period of orbit and r is the radius of the orbit.

[2 marks]

0 7 . 2 Pluto is a dwarf planet. The mean orbital radius of Pluto around the Sun is 5.91×10^9 km compared to a mean orbital radius of 1.50×10^8 km for the Earth.

Calculate in years the orbital period of Pluto.

[2 marks]

orbital period of Pluto = _____ yr

0 7 . 3 A small mass released from rest just above the surface of Pluto has an acceleration of $0.617\ \text{m s}^{-2}$.

Assume Pluto has no atmosphere that could provide any resistance to motion.

Calculate the mass of Pluto.

Give your answer to an appropriate number of significant figures.

radius of Pluto = $1.19 \times 10^{6}\ \text{m}$

[3 marks]

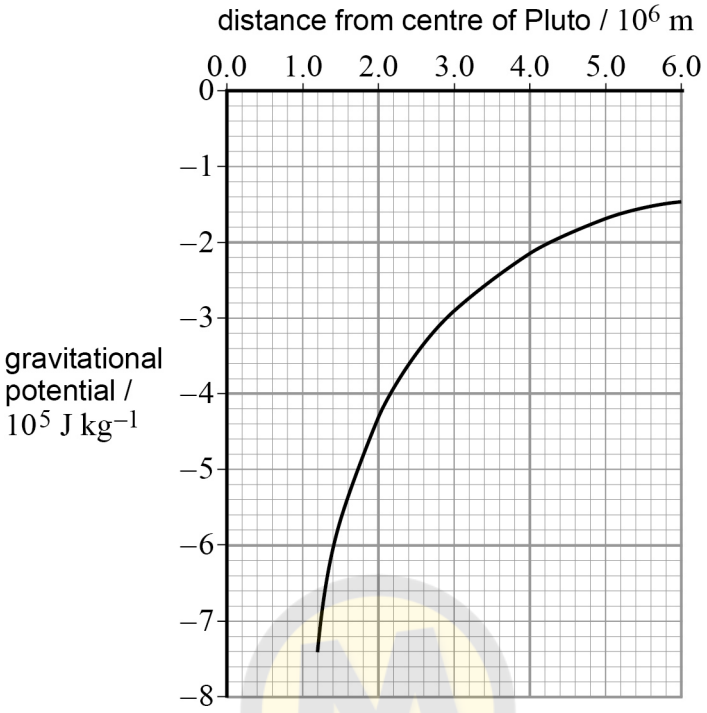
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mass of Pluto = _____ kg

Question 7 continues on the next page

0 7 . 4 **Figure 8** shows the variation in gravitational potential with distance from the centre of Pluto for points at and above its surface.

Figure 8



A meteorite hits Pluto and ejects a lump of ice from the surface that travels vertically at an initial speed of $1400\ \text{m s}^{-1}$.

Determine whether this lump of ice can escape from Pluto.

[3 marks]

10

END OF SECTION A

Question	Answers	Additional Comments/Guidelines	Mark
07.1	(centripetal) force = $m\,r\,(2\,\pi / T)^2$ Or $m\,r\,(\omega)^2$ (is given by the gravitational) force = $G\,m\,M / r^2$ ✓ (mark for both equations) (equating both expressions and substituting for ω if required) $T^2 = (4\pi^2 / GM)\,r^3$ ✓ ($4\pi^2 / GM$ is constant, the constants may be on either side of equation but T and r must be numerators)	First mark is for two equations (gravitational and centripetal) The second mark is for combining.	2
07.2	(use of $T^2 \propto r^3$ so $(T_P / T_E)^2 = (r_P / r_E)^3$) $(T_P / 1.00)^2 = (5.91 \times 10^9 / 1.50 \times 10^8)^3$ ✓ (mark is for substitution of given data into any equation that corresponds to the proportional equation given above) (T_P^2 = 61163) T_P = 250 (yr) ✓ (247 yr)	Answer only gains both marks The calculation may be performed using data for the Sun in $T^2 = (4\pi^2 / GM)\,r^3$ easily spotted from M_s = 1.99 x 10^{30} kg giving a similar answer 247 – 252 yr.	2
07.3	using M (= $g\,r^2 / G$) = $0.617 \times (1.19 \times 10^6)^2 / 6.67 \times 10^{-11}$ ✓ $M = 1.31 \times 10^{22}$ kg ✓ answer to 3 sig fig ✓ (this mark stands alone)	The last mark may be given from an incorrect calculation but not lone wrong answer.	3

07.4	Initial KE = ½ (m) 1400^2 = 9.8 x 10^5 (m) J✓ Energy needed to escape = 7.4 x 10^5 (m) J✓ So sufficient energy to escape. ✓ ***OR*** For object on surface escape speed given by 7.4 x10^5 = ½ v^2✓ escape speed = 1200 m s^{-1}✓(if correct equation is shown the previous mark is awarded without substitution) So sufficient (initial) speed to escape. ✓ ***OR*** escape velocity = $\sqrt{\frac{2GM}{R}}$ substituting *M* from 07.3 ✓ escape speed = 1200 m s^{-1}✓ (1210 m s^{-1}) So sufficient (initial) speed to escape. ✓ ***OR*** escape velocity = $\sqrt{2Rg}$ substituting from data in 07.3 ✓	Third alternative may come from a CE from 07.3 (1.06 x 10^{-8} x $\sqrt{\text{answer } 0.73}$) Conclusion must be explicit for third mark and cannot be awarded from a CE	3
Total			**10**

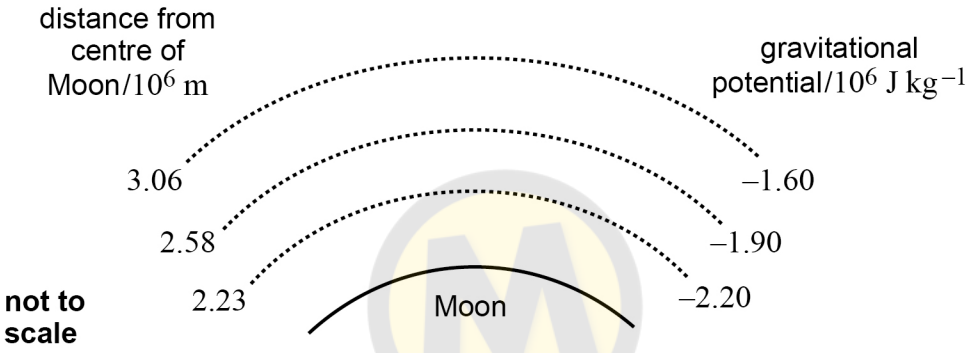
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0 3 . 1 Define gravitational potential at a point.

[1 mark]

0 3 . 2 **Figure 2** shows the positions of equipotential surfaces at different distances from the centre of the Moon.

Figure 2



Explain how the equipotential surfaces in **Figure 2** show that the gravitational field is **not** uniform.

[1 mark]

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0 3 . 3 Calculate, using **Figure 2**, the escape velocity at the surface of the Moon.

radius of Moon = 1.74×10^6 m

[4 marks]

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escape velocity = _____ m s^{-1}

6

Turn over for the next question

Question	Answers	Additional Comments/Guidance	Mark	ID details
03.1	the work done/energy <u>required</u> in bringing <u>1 kg/unit</u> mass from infinity to the point ✓	A test mass should not be taken to be a unit mass. Ignore extra comments eg about charge.	1	AO1-1a-1
03.2	The potential difference between the lines is constant but the distances are not Or $\frac{\Delta V}{\Delta r}$ is changing ✓ (potentials $\frac{\Delta V}{\Delta r}$, inner 0.857, outer = 0.625) Or The equipotential surfaces are not straight / not parallel / are curved ✓	The mark is given for the idea that the separation should be uniform or that equipotential lines should not be curved. Owtte Discussions should not imply there is a correct curvature. Errors can come from references to the moon to first equipotential distance. Or by saying the potential gaps are not uniform or by saying the distance from the centre of the Moon is not proportional to the potential.	1	AO3-1a-1
03.3	M = mass of Moon $= \frac{-V \times r}{G} = \frac{(-1) \times -1.60 \times 10^6 \times 3.06 \times 10^6}{6.67 \times 10^{-11}}$ ✓$_1$ M = 7.3 or 7.4 $\times 10^{22}$ (kg) ✓$_2$ (Use of $\frac{GMm}{r} = \frac{1}{2}mv^2$) $v = \sqrt{\frac{2GM}{r}}$ ✓$_3$	✓$_1$ The mark is given for use and rearranging the equation so errors may be seen in the data and any equipotential may be used. Condone the misuse of a negative sign ✓$_2$ Note All equipotential lines produce the same mass. The answer my be seen in the equations that follow. ✓$_2$ An attempt to use this re-arranged formula gains this 3rd mark.	4	AO3-1b-1 AO2-1f-3 Math2-2

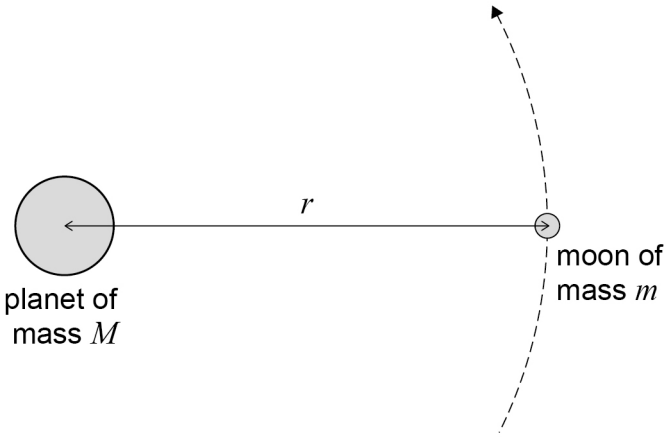
	$v = 2.3$ or 2.4×10^3 (m s^{-1}) ✓$_4$ (2370 m s^{-1})			

	Alternative (Use of $V = \frac{-GM}{r}$) $Vr = -MG$ = constant $Vr = -1.60 \times 10^6 \times 3.06 \times 10^6$ $= -4.9 \times 10^{12}$ (J Kg^{-1} m) ✓$_{1Alt}$ (at the surface of the Moon) $V_{surface} = \frac{-4.9 \times 10^{12}}{1.74 \times 10^6}$ $V_{surface} = -2.8(2) \times 10^6$ (J kg^{-1}) ✓$_{2Alt}$ (Use of $mV = \frac{1}{2}mv^2$) $v = \sqrt{2V_{surface}} = \sqrt{2 \times 2.82 \times 10^6}$ ✓$_{3Alt}$ $v = 2.4 \times 10^3$ (m s^{-1}) ✓	✓$_{1Alt}$ Any attempt to calculate Vr OR to indicate that it is constant gains this mark. ✓$_{3Alt}$ An attempt to use this re-arranged formula gains this 3rd mark. ✓$_{4Alt}$ A correct answer shows the previous work has been done correctly and gains all 4 marks.		

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0 2 **Figure 2** shows a moon of mass m in a circular orbit of radius r around a planet of mass M, where $m << M$.

Figure 2



The moon has an orbital period T.
T is related to r by

$$T^2 = kr^3$$

where k is a constant for this planet.

0 2 . 1 Show that $k = \dfrac{4\pi^2}{GM}$

[3 marks]

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Table 2 gives data for two of the moons of the planet Uranus.

Table 2

Name	T / days	r / m
Miranda	1.41	1.29×10^8
Umbriel	4.14	**X**

0 2 . 2 Calculate the orbital radius **X** of Umbriel.

[2 marks]

orbital radius = _____ m

0 2 . 3 Calculate the mass of Uranus.

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[3 marks]

mass = _____ kg

Question 2 continues on the next page

Table 3 gives data for three more moons of Uranus.

Table 3

Name	Mass / kg	Diameter / m
Ariel	1.27×10^{21}	1.16×10^{6}
Oberon	3.03×10^{21}	1.52×10^{6}
Titania	3.49×10^{21}	1.58×10^{6}

0 2 . 4 Deduce which moon in **Table 3** has the greatest escape velocity for an object on its surface.
Assume the effect of Uranus is negligible.

[3 marks]

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0 2 . 5 A spring mechanism can project an object vertically to a maximum height of $1.0\ \text{m}$ from the surface of the Earth.

Determine whether the same mechanism could project the same object vertically to a maximum height greater than $100\ \text{m}$ when placed on the surface of Ariel.

[3 marks]

14

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Turn over for the next question

Question	Answers	Additional comments/Guidelines	Mark	AO
02.1	$\frac{GmM}{r^2}(=ma)=mr\omega^2$ OR $\frac{GMm}{r^2}=\frac{mv^2}{r}$ ✓ $\omega^2=\left(\frac{2\pi}{T}\right)^2$ OR $v=\frac{s}{t}=\frac{2\pi r}{T}$ ✓ $\frac{GM}{r^2}=r\frac{4\pi^2}{T^2}$ results in $T^2=\left(\frac{4\pi^2}{GM}\right)r^3$ ✓	The starting point must be from relating forces or accelerations – not a remembered equation produced at a later stage in the manipulation of the equation. Middle mark may be given when seen as a substitution and can be a stand-alone mark. Last mark is only given if accompanied with the working and must eventually be in the form with T^2 and r^3 .	3	AO1.1b AO1.1a ×2 MS 2.1 ×2
02.2	$\left(\frac{T_U^2}{T_M^2}=\frac{r_U^3}{r_M^3}\right)$ $r_U=r_M\left(\frac{T_U}{T_M}\right)^{2/3}$ OR Substitution of data in equation in any of its forms ✓ (r_U=) radius = $2.6(5)\times10^8$ (m) ✓	The first mark is for converting the equation into a proportion with or without substitution of data. Eg $r_U=1.29\times10^8\left(\frac{4.14}{1.41}\right)^{2/3}$ Last mark is for evaluating the correct answer Answer only gains both marks	2	AO1.1b AO2.1d M2 ×2

Question	Answers	Additional comments/Guidelines	Mark	AO
02.3	Rearranging $T^2 = \frac{4\pi^2}{GM} r^3$ To give $M = \frac{4\pi^2}{T^2 G} r^3$ or $M = \frac{4\pi^2}{Gk}$ Or substitution of data ✓(allow use of their calculated *k* from **02.2**) Conversion of T to seconds for Miranda $T = 1.41 \times 24 \times 3600 = 1.22 \times 10^5$ s ✓ (or $T^2 = 2.48 \times 10^{10}$) Substitution into equation and determination of *M* $M = 8.5(6) \times 10^{25}$ kg ✓	Give full credit for use of Umbriel period and their answer to 02.2 Alternative: Conversion of T to seconds for Umbriel $T = 4.14 \times 24 \times 3600 = 3.58 \times 10^5$ s ✓ Determine k from $T^2 = kr^3$ for Miranda (or Umbriel) ✓ $k = 6.9 \times 10^{-15}$ Converting to seconds mark stands alone use of $k = \frac{4\pi^2}{GM}$ to find $M = 8.56 \times 10^{25}$ kg✓ No ecf for final answer mark.	3	AO2.1b ×3

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02.4	$\frac{GM\cancel{m}}{(d/2)} = \frac{1}{2}\cancel{m}v^2$ OR $v = \left(\frac{4GM}{d}\right)^{1/2}$ OR stating the escape velocity depends on M/d ✓ Any correct calculation of an escape velocity or ratio M/d or $\left(M/d\right)^{1/2}$ ✓ Last mark only given with three correct relevant calculations Correct conclusion = Titania ✓	Full credit may be given for answers that use *r* rather than *d*. Table to help identify 2nd mark **Name** / **m/d** / **$(m/d)^{1/2}$** / ***v*** Ariel / 1.09×10^{15} / 3.31×10^{7} / 540 Oberon / 1.99×10^{15} / 4.46×10^{7} / 729 Titania / 2.21×10^{15} / 4.70×10^{7} / 768	3	AO2.1a AO3.1a ×2 M2 ×2

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Question	Answers	Additional comments/Guidelines	Mark	AO
02.5	(On Ariel surface $g = \frac{GM}{r^2} = \frac{6.67\times10^{-11}\times1.27\times10^{21}}{(1.16\times10^{6}/2)^2}$) $g = 0.25$ (m s^{-2}) ✓ (spring supplies the same potential energy so $(\cancel{m}gh)_{\text{Earth}} = (\cancel{m}gh)_{\text{Ariel}}$) $9.8 \times 1.0 = 0.25 \times h$ OR (comparing $s = \frac{v^2-u^2}{2a}$) $h = \frac{0-20}{-2\times0.25}$ ✓ $h = 39$ or 40 m, so it could not ✓ {allow ecf for arithmetic errors only}	The second mark can be given for the idea that the gravitational potential energy gained on the Earth is the same as that on Ariel with or without data. Or that the initial kinetic energy is the same in both cases. Last mark allow ecf for arithmetic errors only. Condone incorrect use of signs.	3	AO2.1f ×3 M2 ×2
Total			***14***	

0 2 The Global Positioning System (GPS) uses satellites to support navigation on Earth.

0 2 . 1 One GPS satellite is in a circular orbit at a height h above the surface of the Earth. The Earth has mass M and radius R.

Show that the angular speed ω of the satellite is given by

$$\omega = \sqrt{\frac{GM}{(R+h)^3}}$$

[2 marks]

0 2 . 2 Calculate the orbital period of the satellite when h equals 2.02×10^7 m.

[2 marks]

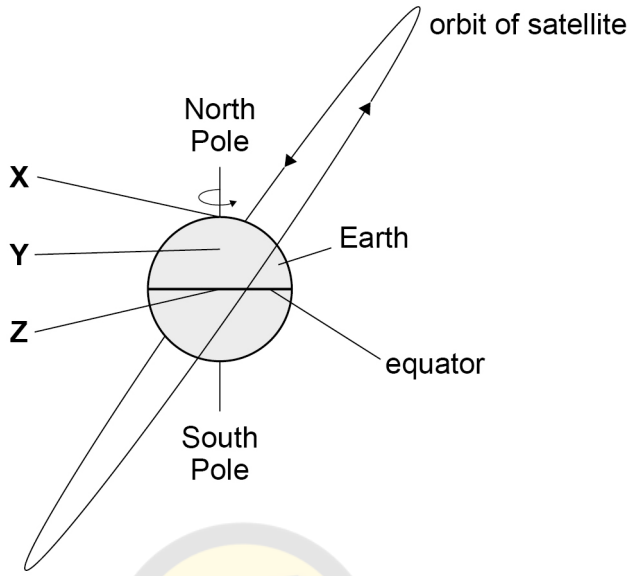
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orbital period = _____ s

0 2 . 3 **Figure 3** shows the orbital plane of the satellite inclined at an angle to the equator. **X**, **Y** and **Z** are locations on the Earth.

X is at the North Pole, **Y** is on a high mountain and **Z** is on the equator.

Figure 3



The satellite is to be launched from one of the locations.

State and explain which launch site **X**, **Y** or **Z** minimises the amount of fuel required to send the satellite into its orbit.

[2 marks]

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Question 2 continues on the next page

0 2 . 4 The satellite has a mass of $1630\ \text{kg}$.

Calculate the gravitational potential energy of the satellite when in the orbit in Question **02.2**.

[2 marks]

gravitational potential energy = _____ J

0 2 . 5 A different satellite is in a higher circular orbit.

Explain how the linear speed of this satellite compares with the linear speed of the satellite in Question **02.1**.

[2 marks]

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Question	Answers	Additional comments/Guidelines	Mark	AO
02.1	Relating the gravitational force to mass × acceleration with the acceleration being centripetal in any form ✓1 show the working leading to answer $\omega = \sqrt{\frac{GM}{(R+h)^3}}$ ✓2	✓1 $\frac{GMm}{r^2} = (ma) = \frac{mv^2}{r}$ or $mr\omega^2$ or $mv\omega$ and r can be replaced by $R + h$	2	3.7.2.4 AO2 1e ×2

Question	Answers	Additional comments/Guidelines	Mark	AO
02.2	$\left(\omega = \sqrt{\frac{GM}{(R+h)^3}} = \sqrt{\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{\left(2.02 \times 10^{7} + 6.37 \times 10^{6}\right)^3}}\right)$ $= 1.45 \times 10^{-4}$ (rad s^{-1}) ✓1 $T = \frac{2\pi}{\omega} = \frac{2\pi}{1.45 \times 10^{-4}} = 4.3(1) \times 10^{4}$ (s) ✓2	✓1 A full substitution in the equation can gain this mark even if ω is incorrectly calculated A final correct answer for *T* gains both marks	2	3.7.2.4 AO2 1b ×2

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Question	Answers	Additional comments/Guidelines	Mark	AO
02.3	Launch from **Z** with some speed or energy argument ✓1 At this position the satellite has the largest initial speed/kinetic energy from the Earth's rotation. ✓2	✓1 Condone ref. to equator rather than Z Consider answer Y only if extremely well explained in terms of different potentials and fuel use.	2	3.7.2.4 AO1 1b AO3 1b

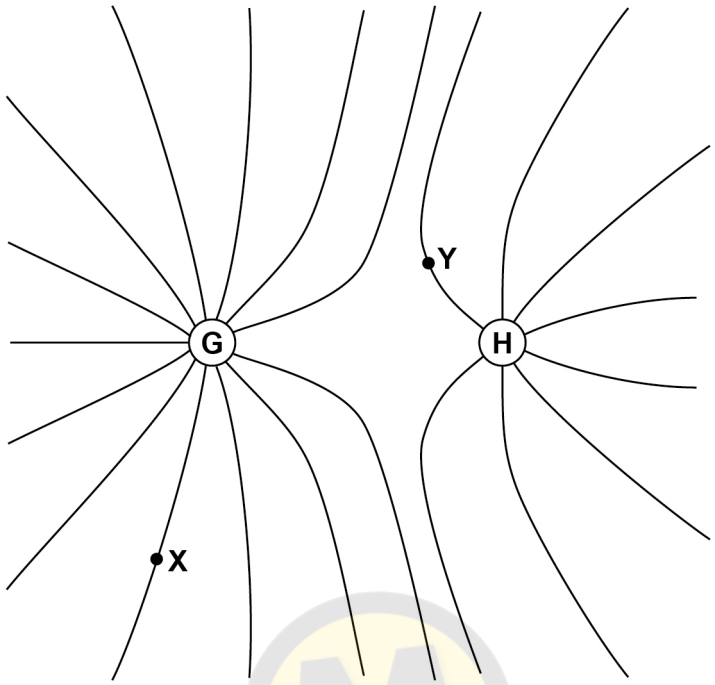
Question	Answers	Additional comments/Guidelines	Mark	AO
02.4	gravitational potential energy $= (-)\frac{GMm}{r}$ OR $(-)\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 1630}{2.66 \times 10^{7}}$ ✓1 gravitational potential energy $= -2.44 \times 10^{10}$ (J) ✓2	✓1 for the equation or substitution without the need for the negative sign only allow ecf for $r = 6.37 \times 10^{6}$ giving gpe = -1.02×10^{11}J ✓2 the minus is necessary for the mark Correct answer gains both marks	2	3.7.2.3 AO1 1b ×2

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Question	Answers	Additional comments/Guidelines	Mark	AO
02.5	In a higher orbit the linear speed is smaller using $v = \sqrt{\frac{GM}{r}}$ or $v \propto \frac{1}{\sqrt{r}}$ ✓1 Suitable justification of $v \propto \frac{1}{\sqrt{r}}$ ✓2	✓2 example $v = r\omega = r\sqrt{\frac{GM}{(R+h)^3}} = r\sqrt{\frac{GM}{(r)^3}} = \sqrt{\frac{GM}{r}}$ Or derivation from basics	2	3.7.2.4 AO2 1e ×2
Total			**10**	

0 4 The lines in **Figure 4** show the shape of the gravitational field around two stars **G** and **H**.

Figure 4



0 4 . 1 Compare, with reference to **Figure 4**, the masses of **G** and **H**.

[2 marks]

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0 4 . 2 **X** and **Y** are two points in the field.

Annotate **Figure 4** to show the field direction at **X** and the field direction at **Y**.

[1 mark]

Question 4 continues on the next page

0 4 . 3 A spherical asteroid **P** has a mass of 2.0×10^{20} kg.

The gravitational field strength at its surface is 0.40 N kg^{-1}.

Calculate the radius R of **P**.

[1 mark]

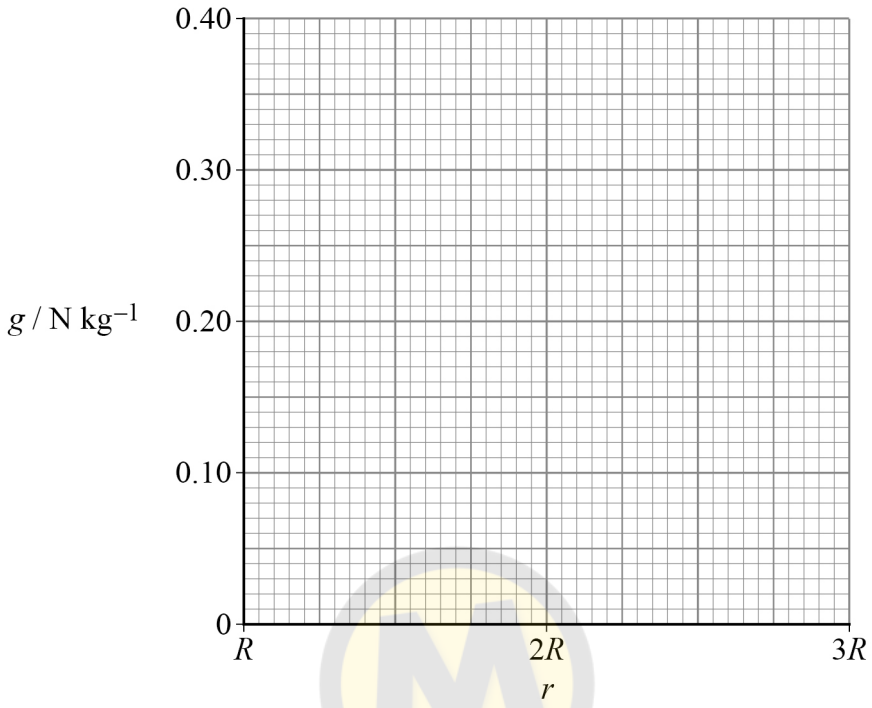
$R =$ _____ m

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0 4 . 4 Sketch, on **Figure 5**, the variation of the gravitational field strength g with distance r. The distance r is measured from the centre of **P**.

[1 mark]

Figure 5



0 4 . 5 Explain what is represented by the area under the graph between $r = R$ and $r = 2R$ on **Figure 5**.

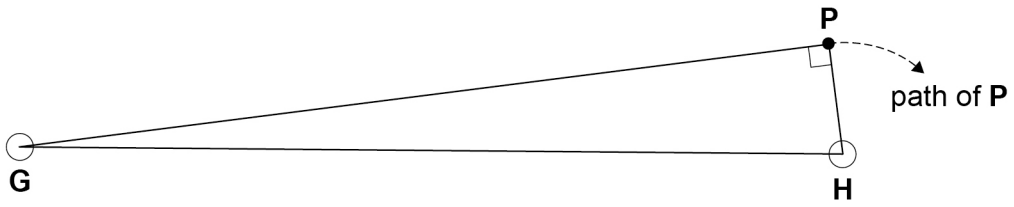
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[2 marks]

Question 4 continues on the next page

Asteroid **P** approaches the two stars **G** and **H**.
Figure 6 shows one position of **P** close to **H**.

Figure 6



0 4 . 6 The gravitational force on **P** from **G** is 6.38×10^{12} N.
The mass of **H** is 3.00×10^{25} kg and the mass of **P** is 2.00×10^{20} kg.
The distance **HP** is 1.50×10^{11} m.

Calculate the magnitude of the acceleration of **P**.

[4 marks]

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magnitude of acceleration = _____ m s^{-2}

0 4 . 7 Explain why **P** cannot have a circular orbit around **H**.

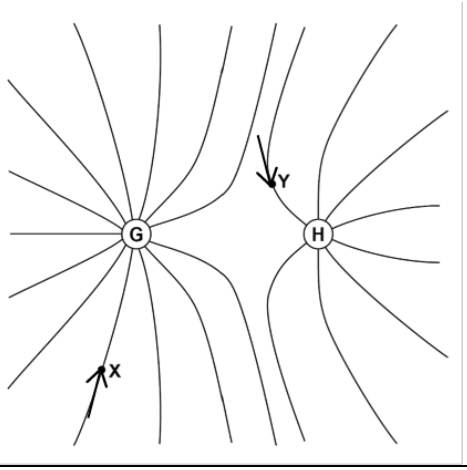
[1 mark]

12

Turn over for the next question

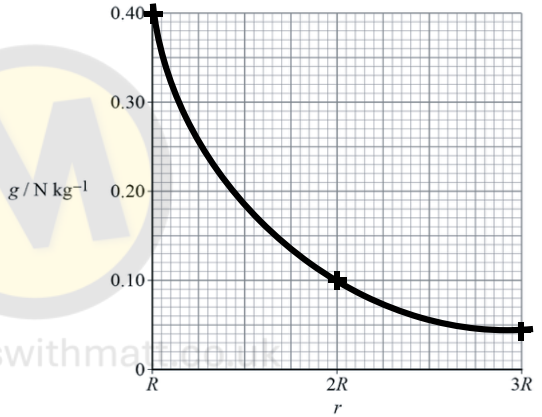
Question	Answers	Additional comments/Guidelines	Mark	AO
04.1	G has greater mass with evidence from diagram ✓1 Explanation based on gravitational field strength or gravitational potential ✓2	E.g. ALTERNATIVE A G has greater mass because null point is closer to H ✓1 G field equals that of H at a greater distance from null than H ✓2 ATERNATIVE B G has greater mass because the density of field lines is greater ✓1 Density of field lines depends on mass ✓2 Allow arguments based on potential maximum at null point.	2	2 x AO3 3.7.2.2

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Question	Answers	Additional comments/Guidelines	Mark	AO
04.2	The lines given tangential arrows at X and Y that flow towards **G** and **H** respectively. ✓  	Condone arrow heads only but if arrows are drawn in full they must not follow a curved line. Arrows are acceptable if drawn alongside X and Y but must not be further away than the **X** or **Y** label.	1	AO1 3.7.2.2

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Question	Answers	Additional comments/Guidelines	Mark	AO
04.3	$R = \left(\left\{ \frac{GM}{g} \right\}^{1/2} = \left\{ \frac{6.67\times10^{-11}\times2.0\times10^{20}}{0.40} \right\}^{1/2} \right) = 1.8 \times 10^{5}$ (m) ✓		1	AO2 3.7.2.2

Question	Answers	Additional comments/Guidelines	Mark	AO
04.4	Sketch must pass through coordinates $(R, 0.40)$, $(2R, 0.10)$ and $(3R, 0.044)$ ✓	Must be within one ½ small division of coordinates requested. If plotted points differ a little from the line drawn then plotted points take priority. 	1	AO2 3.7.2.3

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Question	Answers	Additional comments/Guidelines	Mark	AO
04.5	ALTERNATIVE A (The area underneath) represents the energy transferred/work done (for an object) of $1\ \text{kg}$ / unit mass ✓1a OR (area is) energy transferred to/work done on the object per unit mass when it is moving from *R* to 2*R* ✓1a✓2a ALTERNATIVE B change in gravitational potential ✓1b OR increase in gravitational potential when moving from *R* to 2*R* ✓1b✓2b	Accept reverse direction 2*R* to *R* with appropriate direction of energy transfer/gravitational potential. In each alternative, the first answer is only awarded MP1. The second, fuller answer scores MP1 and MP2. *R* may be given as 1.8×10^5	2	2 x AO2 3.7.2.3

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Question	Answers	Additional comments/Guidelines	Mark	AO
04.6	Use of $F = \frac{GMm}{r^2}$ to find the force between **P** and **H** ✓1 $(F_{(PH)} = 1.8 \times 10^{13}$ N) (Calculation of the resultant force Use of $F_{total} = (F_{(PH)}{}^2 + F_{(PG)}{}^2)^{1/2}$) ✓2 $\left(F_{total} = \left[\left(1.8 \times 10^{13}\right)^2 + \left(6.4 \times 10^{12}\right)^2\right]^{1/2}\right)$ $F_{total} = 1.9 \times 10^{13}$ ✓3 Use of $a = \frac{F}{m} = \frac{1.9\times10^{13}}{2.0\times10^{20}} = 9.4$ to 9.5×10^{-8} (m s^{-2}) ✓4	✓1 $F_{(PH)} = \frac{6.67\times10^{-11}\times3.0\times10^{25}\times2.0\times10^{20}}{\left(1.5\times10^{11}\right)^2}$ ✓2 Mark is for the use of the equation allowing for ecf from candidate's force calculation. ✓3 Correct answer only, no ecf this interim calculation may be subsumed in the next mark. ✓4 Allow ecf from F_{total} Condone the vector addition of the acceleration to obtain the answer acc^n due to G = 3.91x10^{-8} m s^{-2} acc^n due to H = 8.9 x10^{-8} m s^{-2}	4	2 x AO1 2 x AO2 3.7.2.1 (synop)

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Question	Answers	Additional comments/Guidelines	Mark	AO
04.7	The resultant force is not (centripetal and continually) directed towards the centre of **H**. OR A circular orbit does not follow a gravitational equipotential (owtte) ✓	Condone lack of "resultant" The answer can focus on the conditions necessary for circular motion eg the need for a centripetal force. Or At different locations on a circular path the total gravitational potential energy is different which requires energy which is not provided. NB stating that the force is not perpendicular to the motion does not count as a full explanation as the motion has not been established.	1	AO3 3.7.2.3 (synop)
Total			**12**	

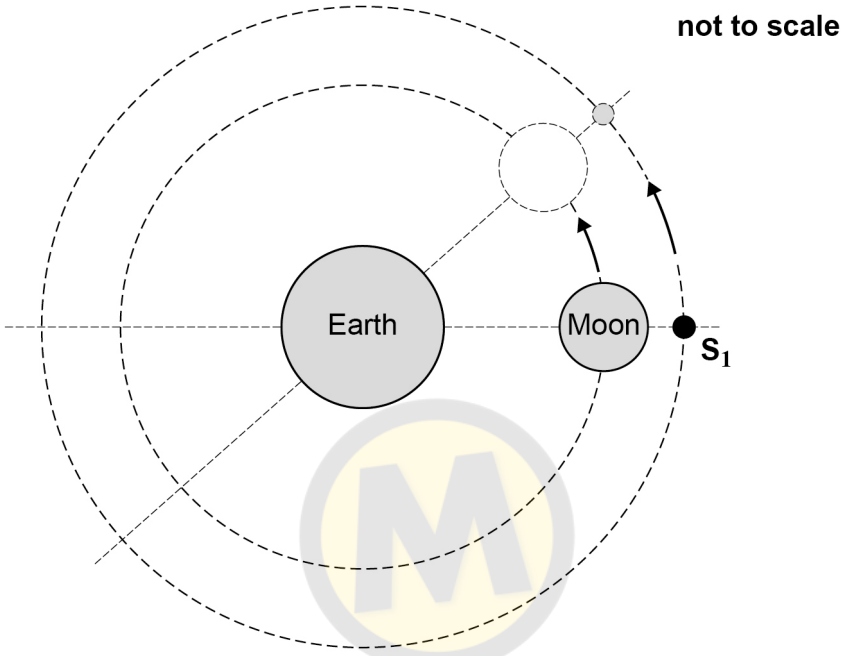
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0 5 A satellite S_1 is placed in a circular orbit around the Earth so that observations of the far side of the Moon can be made continuously.

S_1 has the same angular speed as the Moon so that the centres of the Earth, the Moon and S_1 are always in a straight line.

Figure 7 shows two positions of the Moon and S_1 as they orbit the Earth.

Figure 7



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0 5 . 1 The resultant force on $\mathbf{S_1}$ is due to the gravitational forces from the Earth and the Moon.
The magnitude of the Earth's gravitational field strength at the orbital radius of $\mathbf{S_1}$ is 1.98×10^{-3} N kg^{-1}.
The magnitude of the Moon's gravitational field strength at the orbital radius of $\mathbf{S_1}$ is g_M.

Show that g_M is approximately 1.2×10^{-3} N kg^{-1}.

period of the Moon's orbit = 27.3 days
orbital radius of $\mathbf{S_1}$ = 4.489×10^5 km

[3 marks]

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0 5 . 2 Calculate the distance from $\mathbf{S_1}$ to the centre of the Moon.

mass of the Moon = 7.35×10^{22} kg

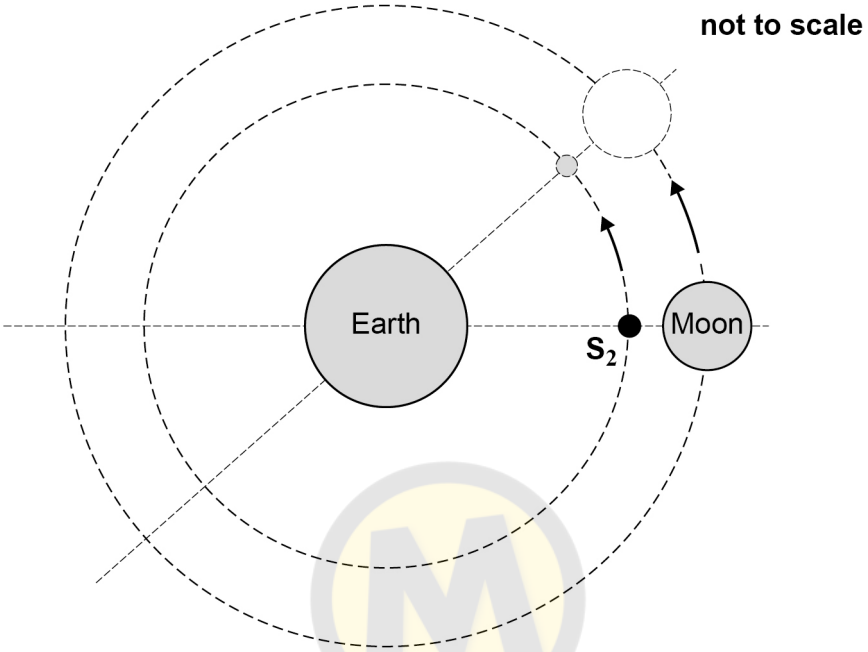
[2 marks]

distance = _____ m

Question 5 continues on the next page

0 5 . 3 Another satellite S_2 is placed in a circular orbit between the Earth and the Moon.
S_2 always views the near side of the Moon.
S_2 also has the same angular speed as the Moon so that the centres of the Earth, the Moon and S_2 are always in a straight line.
Figure 8 shows two positions of the Moon and S_2 as they orbit the Earth.

Figure 8



Explain how the resultant force on S_2 due to the gravitational fields of the Earth and the Moon causes S_2 to orbit with the same angular speed as the Moon.
No calculations are required.

[3 marks]

8

Question	Answers	Additional comments/Guidelines	Mark	AO
05.1	Max 2 ✓✓ • $\omega = \frac{2\pi}{T} = \frac{2\pi}{27.3\times24\times60\times60} = 2.664 \times 10^{-6}$ (rad s^{-1}) or $v = \frac{2\pi r}{T} = \frac{2\pi\times4.489\times10^{8}}{27.3\times24\times60\times60} = 1196$ (m s^{-1}) Substitution or value • Idea (resultant) gravitational field strength is equal to centripetal acceleration e.g. $g_R = a$ or $g_R = r\omega^2$ or $g_R = \frac{v^2}{r}$ (= 3.19 × 10^{-3} m s^{-2}) • Idea that $g_M = g_R - g_E$ $1.21 \times 10^{-3} \geq 3$ SF from correct working ✓ (N kg^{-1})	MP1 may be part of MP2 Ignore PoT, rounding errors and minor copy errors for MP1 and MP2 A substitution into $T^2 = \frac{4\pi^2 r^3}{GM}$ or equivalent is not accepted for the first bullet. In the second bullet point do not allow g_m MP3 must follow from correct working	3	AO2f

Question	Answers	Additional comments/Guidelines	Mark	AO
05.2	$r = \sqrt{\frac{GM}{g_M}} = \sqrt{\frac{6.67\times10^{-11}\times7.35\times10^{22}}{1.21\times10^{-3}}}$ ✓ 6.37×10^{7} (m) ✓	Allow ecf from **05.1** Allow 6.38×10^{7} or 6.39×10^{7} or 6.4×10^{7} (m)	2	AO2b

Question	Answers	Additional comments/Guidelines	Mark	AO
05.3	Idea that the force from the Moon is in the opposite direction to the force from the Earth ✓1 **An expression of why S_2 can't orbit Earth at this ω and r without the moon.** Idea that the Earth's gravitational force is too great or that it causes an angular velocity that is too great (without the presence of the Moon) OR Application of the idea that orbital period decreases with radius ✓2 **An expression of how the moon enables S_2 to orbit Earth with this ω and r.** Idea that the Moon and the Earth reduce the resultant or centripetal force (causes the angular speed of S_2 and the Moon to be the same) OR Effect of the moon's gravitational force or field is to reduce the (angular) speed or increase period ✓3	✓1 Allow gravitational field strength from the Moon is in the opposite direction to the gravitational field strength from the Earth. Condone balanced forces for ✓1 but not ✓3. ✓2 Alternatives (Without the Moon's gravitational influence) • the force or gravitational field (strength) from the Earth is too large • the force or gravitational field is larger as it is closer to the Earth (than **S_1** or the moon) • the required centripetal force is smaller as it closer to the Earth (than **S_1** or the moon, at the same angular velocity) • satellite's angular velocity is greater than the Moon • satellite's period is less than the moons • satellite's angular velocity would be too large. ✓3 The resultant/total centripetal force/centripetal acceleration/gravitational field strength is less (because of the Moon's presence and so orbits at the same angular velocity as the Moon). Reject the idea that the centripetal force on the moon and S_2 are the same. Do not credit arguments based around being closer to moon	3	AO2e

Total			8