AS
FURTHER MATHEMATICS
7366/2D

Paper 2 Discrete

Mark scheme

June 2025

Version: 1.0 Final

256A7366/2D/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

No student should be disadvantaged on the basis of their gender identity and/or how they refer to the gender identity of others in their exam responses.

A consistent use of 'they/them' as a singular and pronouns beyond 'she/her' or 'he/him' will be credited in exam responses in line with existing mark scheme criteria.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:
- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)
ISW	Ignore Subsequent Workings

Examiners should consistently apply the following general marking principles:

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking instructions	AO	Marks	Typical solution
1	Ticks 2nd box.	2.2a	B1	The maximum flow through the network is equal to 36 $m^3\ s^{-1}$
	Question total		**1**	

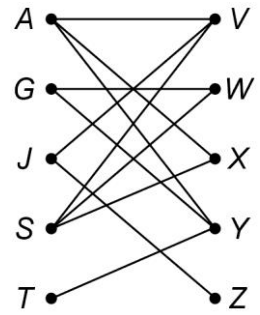
Q	Marking instructions	AO	Marks	Typical solution
2	Circles 1st answer.	1.2	B1	$n - 1$
	Question total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3(a)	Obtains the correct degree of each vertex.	1.1b	B1	**Vertex** / **Degree**: *A* 3; *B* 2; *C* 3; *D* 2
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3(b)(i)	Explains that the graph has vertices with odd degree and concludes Statement 1 is incorrect.	2.4	E1	Incorrect as the graph has vertices with odd degree.
	Subtotal		**1**	

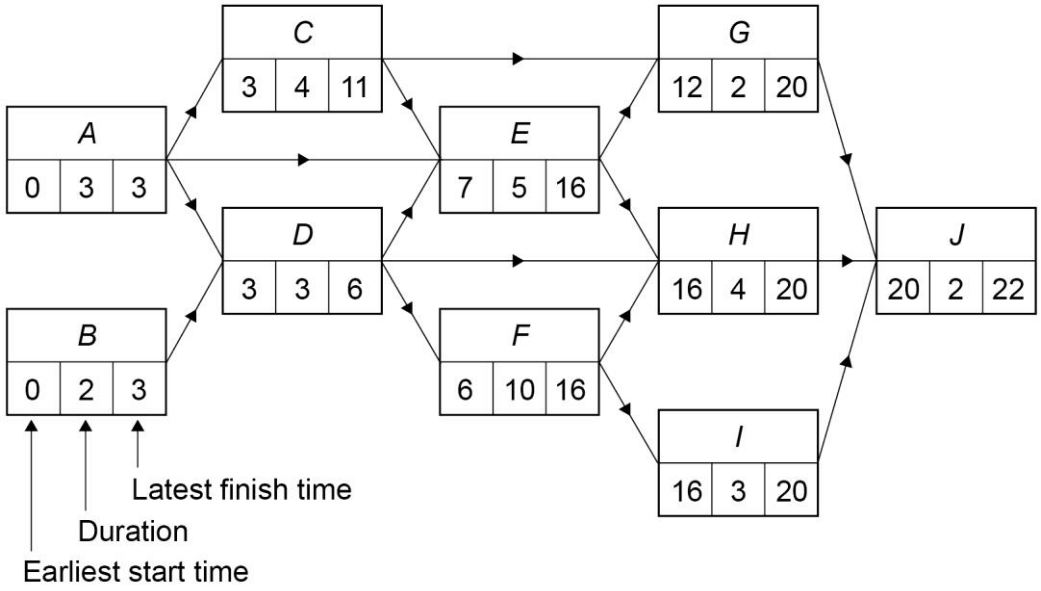
Q	Marking instructions	AO	Marks	Typical solution
3(b)(ii)	States a correct path starting and ending at vertices *A* and *C*	1.1b	B1	*ABCDAC*
	Subtotal		**1**	

	Question total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
4(a)	Draws a bipartite graph with two sets of five vertices labelled and at least 5 edges correctly shown.	1.1a	M1	
	Draws a fully correct and labelled bipartite graph.	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
4(b)	Obtains at least one correct assignment.	1.1a	M1	*A–V G–W J–Z S–X T–Y* *A–X G–W J–Z S–V T–Y*
	Obtains both correct assignments and no others.	1.1b	A1	
	Subtotal		**2**	

	Question total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
5(a)	Obtains the correct start times for *C*, *D*, *E* and *F*	1.1a	M1	See below
	Finds correctly the earliest start time for each activity on the network.	1.1b	A1	
	Finds correctly the latest finish time for each activity on the network.	1.1b	B1	
				
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
5(b)	States the float of activity *E* as 4 PI by reference to '5 or more days' or by reference to '4 or fewer days' or by reference to the minimum completion time (possibly) increasing by (up to) 3 days	1.1a	M1	The float of activity *E* is 4 Therefore, if the duration of activity *E* increases by 5 or more days, the minimum completion time of the project will increase.
	Infers that if the increase is 5 or more, the minimum completion time will increase or Infers that if the increase is 4 or less, the minimum completion time will not increase.	2.2b	R1	
	Subtotal		**2**	

	Question total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
6(a)	Substitutes 0 for a or b into $a + b - 2ab$	1.1a	M1	$a * 0 = a + 0 - 2 \times a \times 0$ $= a$
	Completes a reasoned argument to show that $a * 0 = a$ and $0 * b = b$ and concludes that 0 is the identity.	2.1	R1	$0 * b = 0 + b - 2 \times 0 \times b$ $= b$ Therefore 0 is the identity.
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
6(b)	Forms an equation of the form $a + b - 2ab = 0$	3.1a	M1	$a + b - 2ab = 0$ $b - 2ab = -a$
	Obtains $\frac{a}{2a-1}$ for the inverse of a **ACF**	1.1b	A1	$b(1 - 2a) = -a$ $b = \frac{-a}{1-2a}$ Therefore, $a^{-1} = \frac{a}{2a-1}$
	Subtotal		**2**	

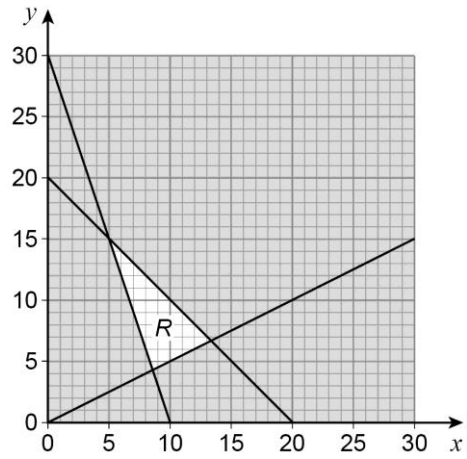
	Question total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
7(a)	Uses the nearest neighbour algorithm from A by finding the first two arcs of the correct Hamiltonian cycle.	1.1a	M1	$A - B - D - E - C - F - A$ 16 + 14 + 15 + 21 + 27 + 18 = 111 Upper bound = 111 miles
	Obtains 111 Condone missing units	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
7(b)	Translates the problem to a mathematical process by finding exactly four arcs with at least two in the minimum spanning tree.	3.1b	M1	BD = 14 DE = 15 BC = 17 DF = 21 Total of MST = 67 miles Two shortest arcs from A are 16 and 18 Lower bound = 67 + 16 + 18 = 101 miles
	Obtains 67 for the total weight of the minimum spanning tree Condone missing units	1.1b	A1	
	Obtains 101 FT the total weight of their spanning tree Condone missing units	1.1b	A1F	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
7(c)	Compares their lower bound with 100 miles and comments on the validity of Maria's claim. FT their lower bound from part **(b)**	2.3	E1F	As 101 > 100, Maria's claim is incorrect.
	Subtotal		**1**	

	Question total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
8(a)	Draws $x + y = 20$ correctly.	1.1b	B1	
	Draws $3x + y = 30$ correctly.	1.1b	B1	
	Draws $2y = x$ correctly.	1.1b	B1	
	Deduces fully correct feasible region and identifies it	2.2a	R1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
8(b)	Finds the coordinates of at least one vertex of the feasible region. PI by correct solution for x and y	3.1a	M1	Vertices of the feasible region are (5, 15), $\left(\frac{40}{3},\frac{20}{3}\right)$ and $\left(\frac{60}{7},\frac{30}{7}\right)$
	Deduces $\left(\frac{40}{3},\frac{20}{3}\right)$ as the coordinates of the optimal vertex. Accept (AWRT 13.3, AWRT 6.7)	2.2a	A1	At (5, 15), $P = 45$ At $\left(\frac{40}{3},\frac{20}{3}\right)$, $P = \frac{160}{3}$ Hence $P = \frac{160}{3}$ is the maximum value.
	Obtains $P = \frac{160}{3}$ AWRT 53.3	1.1b	A1	
	Subtotal		**3**	

	Question total		**7**	

Q	Marking instructions	AO	Marks	Typical solution
9(a)	Finds four or more correct row minima/column maxima.	3.1b	M1	row minima = $-2, -2, 2$ column maxima = 3, 4, 5 max(row minima) = 2 min(column maxima) = 3 max(row minima) = $2 \neq 3$ = min(column maxima) Hence no stable solution.
	Obtains max(row minima) = 2 and min(column maxima) = 3	1.1b	A1	
	Clearly shows that the correct value max(row minima) and correct value min(column maxima) are not equal and concludes that a stable solution does not exist.	3.2a	R1	
	Subtotal		**3**	

<table>
<tr><th>Q</th><th>Marking instructions</th><th>AO</th><th>Marks</th><th>Typical solution</th></tr>
<tr><td rowspan="6">9(b)</td><td>Deduces that strategy C₂ is dominated by strategy C₃
PI by a correct 2 × 3 pay-off matrix or by a correct 2 × 2 pay-off matrix</td><td>2.2a</td><td>E1</td><td rowspan="6">As $-2 < 2$, $k + 1 < 4$ and $k + 2 < 5$
strategy C₃ dominates strategy C₂

As $-2 < -1$ and $4 < 5$
strategy D₂ dominates strategy D₃

Pay off matrix becomes
<table><tr><th></th><th>D₁</th><th>D₂</th></tr><tr><td>C₁</td><td>3</td><td>–2</td></tr><tr><td>C₃</td><td>2</td><td>4</td></tr></table>
Let Caroline choose
strategy C₁ with probability p and
strategy C₃ with probability $1 - p$

If David plays strategy D₁
expected gain for Caroline is
$3p + 2(1 - p) = p + 2$

If David plays strategy D₂
expected gain for Caroline is
$-2p + 4(1 - p) = -6p + 4$

$-6p + 4 = p + 2$

$p = \frac{2}{7}$

Caroline should play
strategy C₁ $\frac{2}{7}$ of the time and
strategy C₃ $\frac{5}{7}$ of the time</td></tr>
<tr><td>Deduces that strategy D₃ is dominated by strategy D₂
PI by a correct 3 × 2 pay-off matrix or by a correct 2 × 2 pay-off matrix</td><td>2.2a</td><td>E1</td></tr>
<tr><td>Introduces a probability variable for Caroline and finds at least one expected gain for Caroline in terms of the probability variable.</td><td>3.3</td><td>M1</td></tr>
<tr><td>Uses the model and finds both correct expected gains in terms of the probability variable.</td><td>3.4</td><td>A1</td></tr>
<tr><td>Equates their expressions for the expected gains and solves to find their correct value for p</td><td>1.1a</td><td>M1</td></tr>
<tr><td>Obtains $p = \frac{2}{7}$ and explains that Caroline should play
strategy C₁ $\frac{2}{7}$ of the time and
strategy C₃ $\frac{5}{7}$ of the time.</td><td>3.5a</td><td>R1</td></tr>
<tr><td></td><td>Subtotal</td><td></td><td>6</td><td></td></tr>
</table>

	Question total		**9**	

	Question Paper total		**40**	