Maths with Matt

Physics and maths resources

Edexcel A Level Further Mathematics

9FM0/01 - Core Pure Mathematics 1

Tuesday 20 May 2025

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2025 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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Question 1

Question 1

For $\mathbf{M} = \begin{pmatrix} 3 & 6 & 0 \\ a & 3 & 1 \\ 2 & a & a \end{pmatrix}$: (a) find the singular values of a. (b) Given $\det \mathbf{M} = -108$ and $a < 0$, find $\mathbf{M}^{-1}$.

Model solution

(a) A matrix is singular exactly when its determinant is zero.

$$\det \mathbf{M} = 3(3a - a) - 6(a^2 - 2) = 6a - 6a^2 + 12.$$

$$6a - 6a^2 + 12 = 0 \iff a^2 - a - 2 = 0 \iff (a-2)(a+1) = 0.$$

$$a = 2 \text{ or } a = -1.$$

(3) (b) First use the determinant condition to identify the numerical matrix.

$$6a - 6a^2 + 12 = -108 \iff a^2 - a - 20 = 0, \quad a = -4 \quad (a < 0).$$

$$\mathbf{M}^{-1} = \frac{1}{-108}\begin{pmatrix} -8 & 24 & 6 \\ -14 & -12 & -3 \\ 10 & 24 & 33 \end{pmatrix} = \begin{pmatrix} \frac{2}{27} & -\frac{2}{9} & -\frac{1}{18} \\ \frac{7}{54} & \frac{1}{9} & \frac{1}{36} \\ -\frac{5}{54} & -\frac{2}{9} & -\frac{11}{36} \end{pmatrix}.$$

$$\mathbf{M}^{-1} = \begin{pmatrix} \frac{2}{27} & -\frac{2}{9} & -\frac{1}{18} \\ \frac{7}{54} & \frac{1}{9} & \frac{1}{36} \\ -\frac{5}{54} & -\frac{2}{9} & -\frac{11}{36} \end{pmatrix}.$$

(3)

Question 2

Question 2

Determine the exact values of x for which $\sinh 2x = 3\sinh x$.

Model solution

The double angle is the clue to use $\sinh 2x = 2\sinh x\cosh x$, *allowing the equation to factor.*

$$2\sinh x\cosh x = 3\sinh x \iff \sinh x(2\cosh x - 3) = 0.$$

Thus $x = 0$, *or* $\cosh x = \frac{3}{2}$. *The inverse hyperbolic cosine gives*

$$x = \pm\ln\left(\frac{3}{2} + \sqrt{\frac{9}{4} - 1}\right) = \pm\ln\left(\frac{3+\sqrt{5}}{2}\right).$$

$$x = 0,\ \pm\ln\left(\frac{3+\sqrt{5}}{2}\right).$$

(5)

Question 3

Question 3

For $z = a + bi$, with $a, b \in \mathbb{R}$, z/z^* is purely imaginary. (a) Show that $a^2 = b^2$. Given also $zz^* = 50$, (b) find all possible z.

Model solution

(a) Rationalise the quotient; a purely imaginary number has zero real part.

$$\frac{z}{z^*} = \frac{a+bi}{a-bi}\frac{a+bi}{a+bi} = \frac{a^2-b^2+2abi}{a^2+b^2}.$$

$$\Re\left(\frac{z}{z^*}\right) = 0 \implies a^2 - b^2 = 0 \implies a^2 = b^2.$$

(2) (b) Multiplication by a conjugate gives the squared modulus.

$$zz^* = (a+bi)(a-bi) = a^2 + b^2 = 50.$$

Together with $a^2 = b^2$, $a^2 = b^2 = 25$, *so* $a = \pm 5$ *and* $b = \pm 5$.

$$z = 5 + 5i, 5 - 5i, -5 + 5i, -5 - 5i$$

(3)

Question 4

Question 4

(a) Solve $y'' - 4y' + 4y = 2e^{3x}$. (b) Given $y(0) = 5$ and $y'(0) = 12$, find the particular solution.

Model solution

(a) Use the auxiliary equation for the homogeneous part, then a trial exponential for the exponential forcing term.

$$m^2 - 4m + 4 = (m-2)^2 = 0 \implies y_{CF} = (A + Bx)e^{2x}.$$

Put $y_{PI} = \lambda e^{3x}$. *Substitution gives* $(9 - 12 + 4)\lambda e^{3x} = 2e^{3x}$, *so* $\lambda = 2$.

$$y = (A + Bx)e^{2x} + 2e^{3x}.$$

(5) (b) Apply the two initial conditions to determine A *and* B.

$$y(0) = A + 2 = 5 \implies A = 3, \qquad y' = (2A + B + 2Bx)e^{2x} + 6e^{3x}.$$

$$12 = y'(0) = 6 + B + 6 \implies B = 0.$$

$$y = 3e^{2x} + 2e^{3x}.$$

(3)

Question 5

Question 5

Use differences to prove, for $n > 2$, that $\sum_{r=2}^{n} \frac{4}{r^2-1} = \frac{(pn+q)(n-1)}{n(n+1)}$, finding p, q.

Model solution

Factor the denominator into linear factors so that partial fractions create a telescoping sum.

$$\frac{4}{r^2-1} = \frac{4}{(r-1)(r+1)} = \frac{2}{r-1} - \frac{2}{r+1}.$$

$$\sum_{r=2}^{n} \frac{4}{r^2-1} = 2\left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n-1} - \frac{1}{3} - \cdots - \frac{1}{n} - \frac{1}{n+1}\right).$$

$$= 3 - \frac{2}{n} - \frac{2}{n+1} = \frac{3n(n+1) - 2(n+1) - 2n}{n(n+1)} = \frac{(3n+2)(n-1)}{n(n+1)}.$$

$$p = 3, q = 2.$$

(5)

Question 6

Question 6

$f(z) = z^3 + az^2 + bz + c$, with real coefficients, has roots forming a triangle. Given one root is $2+4i$ and the triangle has area 12, determine the two possible functions $f(z)$.

Model solution

Real coefficients make the conjugate $2-4i$ *another root. These points are vertically* 8 *units apart, so use the triangle area to find the real third root* h.

$$12 = \frac{1}{2}(8)|h-2| \implies |h-2| = 3 \implies h = 5 \text{ or } -1.$$

Form the monic cubic from the three roots.

$$f(z) = (z-(2+4i))(z-(2-4i))(z-h) = (z^2-4z+20)(z-h).$$

$$h = 5: \ f(z) = z^3 - 9z^2 + 40z - 100; \qquad h = -1: \ f(z) = z^3 - 3z^2 + 16z + 20.$$

$$f(z) = z^3 - 9z^2 + 40z - 100 \text{or} f(z) = z^3 - 3z^2 + 16z + 20.$$

(5)

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Question 7

Question 7

(a) Express $\dfrac{2x^3+10x^2+9x+22}{(x+2)(x^2+3)}$ in partial fractions. (b) Hence show that its integral from 0 to 1 is $2+\ln\left(\dfrac{27}{4}\right)-\dfrac{\pi}{6\sqrt{3}}$.

Model solution

(a) Equal numerator and denominator degrees require a constant term. The irreducible quadratic needs a linear numerator.

$$\frac{2x^3+10x^2+9x+22}{(x+2)(x^2+3)} = A+\frac{B}{x+2}+\frac{Cx+D}{x^2+3}.$$

$$2x^3+10x^2+9x+22 = A(x+2)(x^2+3)+B(x^2+3)+(Cx+D)(x+2).$$

Comparing coefficients gives $A=2$, $B=4$, $C=2$, $D=-1$.

$$2+\frac{4}{x+2}+\frac{2x-1}{x^2+3}.$$

(4) (b) Integrate the partial fractions. The $2x$ *term uses the logarithmic derivative of* x^2+3.

$$\int\left(2+\frac{4}{x+2}+\frac{2x-1}{x^2+3}\right)dx = 2x+4\ln(x+2)+\ln(x^2+3)-\frac{1}{\sqrt{3}}\arctan\frac{x}{\sqrt{3}}.$$

$$\begin{aligned} I &= \left[2x+4\ln(x+2)+\ln(x^2+3)-\frac{1}{\sqrt{3}}\arctan\frac{x}{\sqrt{3}}\right]_0^1 \\ &= 2+4\ln 3+\ln 4-\frac{\pi}{6\sqrt{3}}-4\ln 2-\ln 3 \\ &= 2+\ln\left(\frac{27}{4}\right)-\frac{\pi}{6\sqrt{3}}. \end{aligned}$$

$$2+\ln\left(\frac{27}{4}\right)-\frac{\pi}{6\sqrt{3}}.$$

(4)

Question 8

Question 8

A rambler's vertical displacement x metres, t hours after 10 am, satisfies $\sin t \dfrac{dx}{dt} - x\cos t = A\sin 2t \sin t$, $t \geqslant 0$. At $t = 1$, $x = 213$ and $dx/dt = 273.2$. (a) Find A to 3 s.f. (b) Find $x = f(t)$. (c) Find the return time to the bottom. (d) The top is 300.68 m high: find t there. (e) State a limitation.

Model solution

(a) Substitute the two measurements into the differential equation.

$$\sin 1(273.2) - 213\cos 1 = A\sin 2\sin 1 \implies A = 150.043\ldots$$

$$A = 150 (3\text{s.f.}).$$

(1) (b) After division by $\sin t$, $-\cot t$ is the coefficient of x, so use integrating factor $\csc t$.

$$\frac{dx}{dt} - x\cot t = A\sin 2t, \qquad \frac{d}{dt}(x\csc t) = A\sin 2t\csc t = 2A\cos t.$$

$$x\csc t = 2A\sin t + C \implies x = 300\sin^2 t + C\sin t.$$

$$213 = 300\sin^2 1 + C\sin 1 \implies C = 0.687\ldots$$

$$x = 300\sin^2 t + 0.687\sin t$$

(5) (c) Set the displacement to zero. The root $t = 0$ is the start, so take the next positive root.

$$\sin t(300\sin t + 0.687) = 0 \implies t = \pi.$$

$$t = \pi \text{h}, \text{about} 13:08.$$

(3) (d) Equate the model displacement to the hill height and solve the quadratic in $\sin t$.

$$300\sin^2 t + 0.687\sin t = 300.68 \implies t \approx 1.57.$$

$$t = 1.57\text{h}.$$

(2) (e) The peak is reached at about half the total time in this model.

The model makes ascent and descent take similar times, whereas in reality walking uphill would normally take longer.

(1)

Question 9

Question 9

(i) The curves $y = \frac{3}{4}\sinh x$ and $y = \tanh x + \frac{1}{5}$ meet once at P. (a) Show its x coordinate satisfies $15e^{4x} - 48e^{3x} + 32e^{x} - 15 = 0$. (b) Show $e^{x} = 3$ is a solution. (c) Give P. (ii) Show that $\int_{-4}^{0} \frac{e^{1/x}}{x^2}\,dx = e^{-1/4}$.

Model solution

(i)(a) Replace the hyperbolic functions by exponential definitions, then clear denominators.

$$\frac{3}{4}\left(\frac{e^{x} - e^{-x}}{2}\right) = \frac{e^{2x} - 1}{e^{2x} + 1} + \frac{1}{5}.$$

Writing $u = e^{x}$ *and multiplying the resulting equation by* $40u(u^2 + 1)$ *gives*

$$15(u^2 - 1)(u^2 + 1) = 16u(3u^2 - 2).$$

Thus, on expanding and restoring $u = e^{x}$,

$$15e^{4x} - 48e^{3x} + 32e^{x} - 15 = 0,$$

as required. *(3) (i)(b) Direct substitution is enough to verify the proposed solution.*

$$15(3)^4 - 48(3)^3 + 32(3) - 15 = 1215 - 1296 + 96 - 15 = 0.$$

$$e^{x} = 3 \text{ is a solution.}$$

(1) (i)(c) Since the intersection is stated to be unique, $e^{x} = 3$ *gives* $x = \ln 3$. *Substitute in either curve.*

$$y = \tanh(\ln 3) + \frac{1}{5} = \frac{9 - 1}{9 + 1} + \frac{1}{5} = 1.$$

$$P = (\ln 3, 1)$$

(1) (ii) The upper limit is improper because of $x = 0$. *Use* $u = 1/x$, *whose derivative produces the* $1/x^2$ *factor, and retain the limiting process.*

$$u = \frac{1}{x}, \qquad du = -\frac{1}{x^2}\,dx, \qquad \int \frac{e^{1/x}}{x^2}\,dx = -e^{1/x}.$$

$$\int_{-4}^{0} \frac{e^{1/x}}{x^2}\,dx = \lim_{t \to 0^-} \left[-e^{1/x}\right]_{-4}^{t} = \lim_{t \to 0^-} \left(-e^{1/t} + e^{-1/4}\right).$$

As $t \to 0^-$, $1/t \to -\infty$ *and* $e^{1/t} \to 0$. *Hence*

$$\int_{-4}^{0} \frac{e^{1/x}}{x^2}\,dx = e^{-1/4}.$$

(4)

Question 10

Question 10

(a) Given $z^n + z^{-n} = 2\cos n\theta$ and $z^n - z^{-n} = 2i\sin n\theta$, show $8\sin^4\theta = \cos 4\theta - 4\cos 2\theta + 3$. (b) The region $0 \leqslant y \leqslant 8\pi/5$, $0 \leqslant x \leqslant \sin^2(\frac{1}{2}y)$ is revolved about the y-axis. Find its volume to 2 s.f. (c) Wood density is 0.85 g cm^{-3} and the ornament mass is 6 g. Comment on the model.

Model solution

(a) Taking $n = 1$ in the second identity expresses $\sin\theta$ in terms of z; raise this to the fourth power and expand symmetrically.

$$8\sin^4\theta = 8\left(\frac{z - z^{-1}}{2i}\right)^4 = \frac{1}{2}(z - z^{-1})^4 = \frac{1}{2}\left(z^4 - 4z^2 + 6 - 4z^{-2} + z^{-4}\right).$$

$$= \frac{1}{2}\{(z^4 + z^{-4}) - 4(z^2 + z^{-2}) + 6\} = \frac{1}{2}(2\cos 4\theta - 8\cos 2\theta + 6).$$

$$8\sin^4\theta = \cos 4\theta - 4\cos 2\theta + 3$$

(5) (b) Rotation about the y-axis produces circular discs of radius $x = \sin^2(y/2)$, so use $V = \pi\int x^2\,dy$. The result in (a), with $\theta = y/2$, supplies an algebraic integrand.

$$V = \pi\int_0^{8\pi/5} \sin^4\left(\frac{y}{2}\right)dy = \frac{\pi}{8}\int_0^{8\pi/5}(3 - 4\cos y + \cos 2y)\,dy.$$

$$V = \frac{\pi}{8}\left[3y - 4\sin y + \frac{1}{2}\sin 2y\right]_0^{8\pi/5} = 7.300\ldots$$

$$V = 7.3 \text{ cm}^3 (2\text{s.f.}).$$

(5) (c) Convert the model volume into a predicted mass and compare it with the given mass.

$$m = 0.85(7.300\ldots) = 6.205\ldots \text{ g}.$$

This is close to 6 g (within 10%), so

the model is suitable/good.

(2)