Maths with Matt

Physics and maths resources

Edexcel A Level Further Mathematics

9FM0/02 - Core Pure Mathematics 2

Thursday 22 May 2025

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2025 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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Question 1

Question 1

Given $z = 2 - 2\sqrt{3}\mathrm{i}$ and $w = -1 + \sqrt{3}\mathrm{i}$, show that (a) $\left|\frac{z}{w}\right| = \frac{|z|}{|w|}$ and (b) $\arg(zw) = \arg z + \arg w$.

Model solution

(a) Calculate the two moduli directly. This verifies the quotient rule without relying on a calculator.

$$|z| = \sqrt{2^2 + (-2\sqrt{3})^2} = 4, \qquad |w| = \sqrt{(-1)^2 + (\sqrt{3})^2} = 2.$$

$$\frac{z}{w} = \frac{2 - 2\sqrt{3}\mathrm{i}}{-1 + \sqrt{3}\mathrm{i}} \frac{-1 - \sqrt{3}\mathrm{i}}{-1 - \sqrt{3}\mathrm{i}} = -2.$$

$$\left|\frac{z}{w}\right| = |-2| = 2 = \frac{4}{2} = \frac{|z|}{|w|}.$$

$$\boxed{\left|\frac{z}{w}\right| = \frac{|z|}{|w|} = 2.}$$

(3)

(b) The arguments can be checked directly here; this avoids assuming the result which is to be shown.

$$\arg z = -\frac{\pi}{3}, \qquad \arg w = \frac{2\pi}{3}.$$

$$zw = (2 - 2\sqrt{3}\mathrm{i})(-1 + \sqrt{3}\mathrm{i}) = 4 + 4\sqrt{3}\mathrm{i}, \qquad \arg(zw) = \frac{\pi}{3}.$$

Also $\arg z + \arg w = -\pi/3 + 2\pi/3 = \pi/3$*, the argument found for* zw*. Hence*

$$\boxed{\arg(zw) = \arg z + \arg w.}$$

(3)

Question 5

Question 5

The planes $2x - y + z = 3$, $x + py - 3z = q$ and $3x + y - 2z = 4$ form a sheaf. Find (i) p and (ii) q.

Model solution

A sheaf consists of planes through one common line. Their normal vectors must therefore be linearly dependent.

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & p & -3 \\ 3 & 1 & -2 \end{vmatrix} = 0 \quad \Longrightarrow \quad 14 - 7p = 0,$$

$$p = 2.$$

(2)

With $p = 2$, add the first two plane equations. Their left side is the third plane's left side, so their constants must add in the same way.

$$(2x - y + z) + (x + 2y - 3z) = 3x + y - 2z = 3 + q = 4.$$

$$q = 1.$$

(2)

Question 6

Question 6

The roots of $2x^4+Ax^3-Ax^2-5x+6=0$ are $\alpha, \beta, \gamma, \delta$. (a) Find $\frac{3}{\alpha}+\frac{3}{\beta}+\frac{3}{\gamma}+\frac{3}{\delta}$. Given $\alpha^2+\beta^2+\gamma^2+\delta^2 = -\frac{3}{4}$, (b) find the possible values of A.

Model solution

(a) Vieta's relations give the product of the roots and the sum of triple products, which are exactly what is needed for a sum of reciprocals. Dividing by 2,

$$x^4 + \frac{A}{2}x^3 - \frac{A}{2}x^2 - \frac{5}{2}x + 3 = 0.$$

Hence $\alpha\beta\gamma\delta = 3$ *and*

$$\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta = \frac{5}{2}.$$

$$\frac{3}{\alpha} + \frac{3}{\beta} + \frac{3}{\gamma} + \frac{3}{\delta} = 3\frac{\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta}{\alpha\beta\gamma\delta} = 3\left(\frac{5/2}{3}\right).$$

$$\frac{5}{2}.$$

(3)

(b) Use $(\sum \alpha)^2 = \sum \alpha^2 + 2\sum \alpha\beta$. *Vieta gives* $\sum \alpha = -A/2$ *and* $\sum \alpha\beta = -A/2$.

$$\left(-\frac{A}{2}\right)^2 = -\frac{3}{4} + 2\left(-\frac{A}{2}\right) \quad \Longrightarrow \quad A^2 + 4A + 3 = 0.$$

$$(A+1)(A+3) = 0.$$

$$A = -1 \text{ or } A = -3.$$

(5)

Question 7

Question 7

The polar curve C is $r = a(1 + \sin\theta)$, $-\pi \leqslant \theta \leqslant \pi$. Tangents at A and B are perpendicular to the initial line. (a) Find their polar coordinates. The distance AB is 10 m. (b) Show $a = 20\sqrt{3}/9$. (c) Find the enclosed surface area.

Model solution

(a) A tangent perpendicular to the initial line is vertical, so differentiate $x = r\cos\theta$ *and set* $dx/d\theta = 0$.

$$x = a(1 + \sin\theta)\cos\theta,$$

$$\frac{dx}{d\theta} = a\left(\cos^2\theta - \sin\theta - \sin^2\theta\right) = a(1 - 2\sin^2\theta - \sin\theta).$$

$$2\sin^2\theta + \sin\theta - 1 = 0 \quad \Longrightarrow \quad \sin\theta = \frac{1}{2} \quad (\sin\theta = -1 \textit{ gives the cusp}).$$

Thus $r = a(1 + 1/2) = 3a/2$ *and* $\theta = \pi/6, 5\pi/6$.

$$A = (3a/2, \pi/6) \textit{ and } B = (3a/2, 5\pi/6).$$

(6)

(b) The two points have equal and opposite x*-coordinates, so their horizontal separation is* $2r\cos(\pi/6)$.

$$10 = 2\left(\frac{3a}{2}\right)\frac{\sqrt{3}}{2} = \frac{3\sqrt{3}a}{2} \quad \Longrightarrow \quad a = \frac{20\sqrt{3}}{9}.$$

$$a = \frac{20\sqrt{3}}{9}.$$

(2)

(c) For a region described once by a polar curve, use $\frac{1}{2}\int r^2\,d\theta$ *over a full turn. Expand before integrating.*

$$\textit{Area} = \frac{1}{2}\int_{-\pi}^{\pi} a^2(1 + \sin\theta)^2\,d\theta = \frac{a^2}{2}\int_{-\pi}^{\pi}\left(\frac{3}{2} + 2\sin\theta - \frac{1}{2}\cos 2\theta\right)d\theta = \frac{3\pi a^2}{2}.$$

$$\textit{Area} = \frac{3\pi}{2}\left(\frac{20\sqrt{3}}{9}\right)^2.$$

$$\frac{200\pi}{9}\ m^2.$$

(4)

Question 2

Question 2

An arrow is fired from $A = (3, -5, 2)$ towards the plane $2x + 4y - z = 3$, and hits it at $B = (6, -2, 1)$. Determine (a) the shortest distance from A to the plane, (b) a vector equation for its flight path, (c) the acute angle made with the plane, (d) the travelled distance, and (e) whether this is likely to be the actual distance.

Model solution

(a) The perpendicular distance from a point to a plane uses the plane's normal vector $(2, 4, -1)$.

$$d = \frac{|2(3) + 4(-5) - 2 - 3|}{\sqrt{2^2 + 4^2 + (-1)^2}} = \frac{19}{\sqrt{21}}.$$

$$\frac{19}{\sqrt{21}} \text{ units.}$$

(2)

(b) A line through A *and* B *has direction* $\overrightarrow{AB} = B - A$.

$$\overrightarrow{AB} = (3, 3, -1), \qquad \mathbf{r} = \begin{pmatrix} 3 \\ -5 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 3 \\ -1 \end{pmatrix}.$$

$$\mathbf{r} = (3, -5, 2) + \lambda(3, 3, -1).$$

(2)

(c) The angle between a line and a plane is the complement of the angle between the line direction and the plane normal. Equivalently use $\sin\phi = |\mathbf{d} \cdot \mathbf{n}|/(|\mathbf{d}||\mathbf{n}|)$.

$$\sin\phi = \frac{|(3, 3, -1) \cdot (2, 4, -1)|}{\sqrt{19}\sqrt{21}} = \frac{19}{\sqrt{399}}, \qquad \phi = 72.0\ldots^\circ.$$

$$72^\circ \text{ (nearest degree).}$$

(2)

(d) The model says the path is the straight segment AB*, so its length is the magnitude of the direction vector.*

$$|AB| = \sqrt{3^2 + 3^2 + (-1)^2} = \sqrt{19}.$$

$$\sqrt{19} \text{ units (about } 4.36 \text{ units).}$$

(1)

(e) In reality gravity makes an arrow follow a curved trajectory, rather than the straight line assumed by the model.

The actual distance is unlikely to match $\sqrt{19}$, because the flight path will not be straight.

(1)

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Question 3

Question 3

$A = \begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix}$. (a) Prove by induction that $A^n = \begin{pmatrix} 1 & 5(2^n-1) \\ 0 & 2^n \end{pmatrix}$ for $n \in \mathbb{N}$. $B = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$. (b) Describe B. If P followed by Q (matrix A^n) is R (matrix C), (c) find C, and (d) given that $R(27, 1) = (a, a)$, find the matrix for Q.

Model solution

(a) Induction is appropriate because the proposed formula concerns every positive integer power. For $n = 1$,

$$\begin{pmatrix} 1 & 5(2^1-1) \\ 0 & 2^1 \end{pmatrix} = \begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix} = A.$$

Assume the result holds for $n = k$: $A^k = \begin{pmatrix} 1 & 5(2^k-1) \\ 0 & 2^k \end{pmatrix}$. *Then*

$$A^{k+1} = A^k A = \begin{pmatrix} 1 & 5(2^k-1) \\ 0 & 2^k \end{pmatrix} \begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 5+10(2^k-1) \\ 0 & 2^{k+1} \end{pmatrix} = \begin{pmatrix} 1 & 5(2^{k+1}-1) \\ 0 & 2^{k+1} \end{pmatrix}.$$

So truth for k *implies truth for* $k+1$*; together with the base case, the formula holds for all* $n \in \mathbb{N}$.

$$A^n = \begin{pmatrix} 1 & 5(2^n-1) \\ 0 & 2^n \end{pmatrix}.$$

(6)

(b) The x*-coordinate changes sign while the* y*-coordinate is unchanged.*

Reflection in the y*-axis (the line* $x = 0$*).*

(2)

*(c) "*P *followed by* Q*" means first multiply by* B*, then by* A^n*, so* $C = A^n B$.

$$C = \begin{pmatrix} 1 & 5(2^n-1) \\ 0 & 2^n \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 5(2^n-1) \\ 0 & 2^n \end{pmatrix}.$$

$$C = \begin{pmatrix} -1 & 5(2^n-1) \\ 0 & 2^n \end{pmatrix}.$$

(1)

(d) Equal output coordinates give an equation for n.

$$\begin{pmatrix} -1 & 5(2^n-1) \\ 0 & 2^n \end{pmatrix} \begin{pmatrix} 27 \\ 1 \end{pmatrix} = \begin{pmatrix} -27+5(2^n-1) \\ 2^n \end{pmatrix} = \begin{pmatrix} a \\ a \end{pmatrix}.$$

$$-27 + 5(2^n - 1) = 2^n \implies 4 \cdot 2^n = 32 \implies 2^n = 8 \implies n = 3.$$

$$Q = A^3 = \begin{pmatrix} 1 & 35 \\ 0 & 8 \end{pmatrix}.$$

(3)

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Question 4

Question 4

(i) $z_1 = a + bi$, $z_2 = c + di$, with $b > d$, $z_1 + z_2$ real, $|z_1| = \sqrt{13}$, $|z_2| = 5$, and $\mathrm{Re}(z_2 - z_1) = 2$. Show $a = 2$ and find b, c, d. (ii) For $|z - 12| = 7$ and $|w - 5i| = 4$, sketch the loci and find the range of $|z - w|$.

Model solution

(i) The condition that a complex number is real makes its imaginary part zero. The real-part condition provides a second linear relation.

$$b + d = 0, \qquad c - a = 2.$$

The moduli give

$$a^2 + b^2 = 13, \qquad c^2 + d^2 = 25.$$

Thus $d = -b$ *and* $c = a + 2$. *Substitute these into the modulus equations.*

$$(a + 2)^2 + b^2 = 25, \qquad a^2 + b^2 = 13.$$

$$4a + 4 = 12 \quad \Longrightarrow \quad a = 2.$$

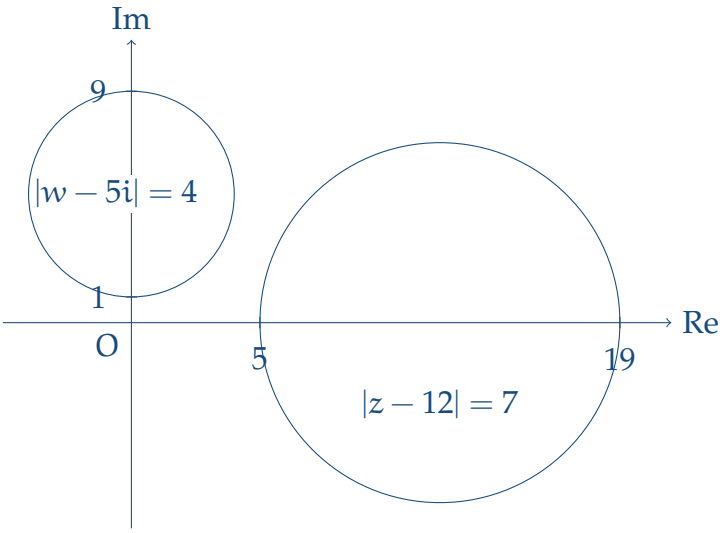
Then $c = 4$, $b^2 = 13 - 4 = 9$, *and* $d = -b$. *Since* $b > d$, $b = 3$ *and* $d = -3$.

$$(a, b, c, d) = (2, 3, 4, -3).$$

$$a = 2,\ b = 3,\ c = 4,\ d = -3.$$

(5)

(ii)(a) A modulus equation $|z - z_0| = r$ *is a circle with centre* z_0 *and radius* r. *The required Argand diagram is therefore:*



(2)

(ii)(b) The centres are 13 *units apart. The smallest and greatest possible separation of points on circles of radii* 7 *and* 4 *are therefore* $13-(7+4)$ *and* $13+(7+4)$.

$$2 \leqslant |z-w| \leqslant 24.$$

$$2 \leqslant |z-w| \leqslant 24.$$

(3)

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Question 8

Question 8

Given $y = \cos x \sinh x$, (a) show that $d^4y/dx^4 = ky$, finding k, and (b) find the first three non-zero terms of its Maclaurin series.

Model solution

(a) Repeated product differentiation is appropriate because the question asks for a derivative relation. The cycle of derivatives of sine/cosine and sinh/cosh will simplify after four derivatives.

$$y' = -\sin x \sinh x + \cos x \cosh x,$$

$$y'' = -2 \sin x \cosh x,$$

$$y''' = -2 \cos x \cosh x - 2 \sin x \sinh x,$$

$$y'''' = -4 \cos x \sinh x.$$

$$\frac{d^4y}{dx^4} = -4y, \text{ so } k = -4.$$

(5)

(b) The derivative relation makes the Maclaurin coefficients quick to obtain at $x = 0$.

$$y(0) = 0, \quad y'(0) = 1, \quad y''(0) = 0, \quad y'''(0) = -2,$$

and differentiating $y'''' = -4y$ *gives* $y^{(5)}(0) = -4y'(0) = -4$.

$$y = y(0) + y'(0)x + \frac{y''(0)}{2!}x^2 + \frac{y'''(0)}{3!}x^3 + \frac{y^{(5)}(0)}{5!}x^5 + \cdots.$$

$$y = x - \frac{x^3}{3} - \frac{x^5}{30} + \cdots.$$

(3)

Question 9

Question 9

Given $y = \arcsin(3x)$, $-1/3 \leqslant x \leqslant 1/3$, (a) find dy/dx. The curve C is $y = \cos(\arcsin(3x))$. (b) find its derivative in simplest form and (c) find the x-coordinate where its gradient is 4.

Model solution

(a) Differentiate inverse sine using the chain rule; the inner function is $3x$.

$$\frac{dy}{dx} = \frac{3}{\sqrt{1-(3x)^2}}.$$

$$\boxed{\frac{dy}{dx} = \frac{3}{\sqrt{1-9x^2}}.}$$

(2)

(b) Apply the chain rule to the outer cosine, then use $\sin(\arcsin(3x)) = 3x$ *to remove the inverse trigonometric function.*

$$\frac{dy}{dx} = -\sin(\arcsin(3x))\frac{3}{\sqrt{1-9x^2}} = -\frac{9x}{\sqrt{1-9x^2}}.$$

$$\boxed{\frac{dy}{dx} = -\frac{9x}{\sqrt{1-9x^2}}.}$$

(2)

(c) Set the derivative equal to the specified gradient. The positive gradient requires $x < 0$, *which will choose the correct root after squaring.*

$$-\frac{9x}{\sqrt{1-9x^2}} = 4 \quad \Longrightarrow \quad 81x^2 = 16(1-9x^2) \quad \Longrightarrow \quad 225x^2 = 16.$$

$$x = \pm\frac{4}{15}, \qquad x < 0.$$

$$\boxed{x = -\frac{4}{15}.}$$

(3)