Maths with Matt

Physics and maths resources

Edexcel A Level Further Mathematics

9FM0/3A - Further Pure Mathematics 1

Wednesday 18 June 2025

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2025 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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Question 1

Question 1

(a) Given $y = e^{\csc^2 x}$, complete the table at $x = 2$:

x	1.5	1.75	2	2.25	2.5
y	2.73	2.81		5.22	16.31

(b) Use Simpson's rule with this completed table to estimate $\int_{1.5}^{2.5} e^{\csc^2 x}\,dx$. (c) Hence estimate $\int_{1.5}^{2.5} e^{\cot^2 x}\,dx$.

Model solution

(a) Substitute the missing x-value directly into the given function, retaining enough figures before rounding.

$$y = e^{\csc^2 2} = 3.35 \quad \textit{(to 2 d.p.)}.$$

$$y = 3.35$$

(1)

(b) There are four equal strips, so Simpson's rule is appropriate and has step length $h = (2.5 - 1.5)/4 = 0.25$. The end values have coefficient 1, the two interior odd values coefficient 4, and the middle value coefficient 2.

$$\int_{1.5}^{2.5} e^{\csc^2 x}\,dx \approx \frac{0.25}{3}\{2.73 + 16.31 + 4(2.81 + 5.22) + 2(3.35)\} = 4.8.$$

$$\int_{1.5}^{2.5} e^{\csc^2 x}\,dx \approx 4.8.$$

(3)

(c) The clue is that $\cot^2 x = \csc^2 x - 1$. This converts the new integrand into a constant multiple of the integrand estimated in part (b).

$$\int_{1.5}^{2.5} e^{\cot^2 x}\,dx = \frac{1}{e}\int_{1.5}^{2.5} e^{\csc^2 x}\,dx \approx \frac{4.8}{e} = 1.8 \quad \textit{(1 d.p.)}.$$

$$\int_{1.5}^{2.5} e^{\cot^2 x}\,dx \approx 1.8.$$

(2)

Question 2

Question 2

Given $\frac{d^2y}{dx^2}+3\frac{dy}{dx}-2xy=4$, (a) show that $\frac{d^4y}{dx^4}=a\frac{dy}{dx}+bx\frac{d^2y}{dx^2}+c\frac{d^3y}{dx^3}$, determining the integers a, b, c. Hence, if $y=1$ and $dy/dx=1$ at $x=2$, (b) find the Taylor series about $x=2$ through $(x-2)^4$.

Model solution

(a) Differentiate the differential equation twice. This is needed because a fourth derivative is required, and the product $2xy$ requires the product rule.

$$y''=4-3y'+2xy,$$

$$y'''=-3y''+2y+2xy',$$

$$y''''=-3y'''+2y'+2y'+2xy''=4y'+2xy''-3y'''.$$

$$\frac{d^4y}{dx^4}=4\frac{dy}{dx}+2x\frac{d^2y}{dx^2}-3\frac{d^3y}{dx^3}.$$

(4)

(b) First find the derivatives at the centre $x=2$, then insert them into Taylor's formula.

$$y''(2)=4-3(1)+2(2)(1)=5,$$

$$y'''(2)=-3(5)+2(1)+2(2)(1)=-9,$$

$$y''''(2)=4(1)+2(2)(5)-3(-9)=51.$$

$$\begin{aligned}y&=y(2)+(x-2)y'(2)+\frac{(x-2)^2}{2!}y''(2)+\frac{(x-2)^3}{3!}y'''(2)+\frac{(x-2)^4}{4!}y''''(2)\\&=1+(x-2)+\frac{5}{2}(x-2)^2-\frac{3}{2}(x-2)^3+\frac{17}{8}(x-2)^4.\end{aligned}$$

$$y=1+(x-2)+\frac{5}{2}(x-2)^2-\frac{3}{2}(x-2)^3+\frac{17}{8}(x-2)^4.$$

(4)

Question 3

Question 3

Use algebra to determine the values of x for which $\dfrac{x^2-4}{|x-5|} > 4x+8$.

Model solution

The absolute value changes form at $x = 5$, which is excluded from the domain. Split into the two sign cases so that multiplication is by a known positive quantity.

$$x > 5: \quad \frac{x^2-4}{x-5} > 4x+8 \iff x^2-4 > (4x+8)(x-5) \iff 3x^2-12x-36 < 0.$$

$$(x-6)(x+2) < 0 \implies 5 < x < 6.$$

$$x < 5: \quad \frac{x^2-4}{5-x} > 4x+8 \iff x^2-4 > (4x+8)(5-x) \iff 5x^2-12x-44 > 0.$$

$$(x+2)(5x-22) > 0 \implies x < -2 \text{ or } \frac{22}{5} < x < 5.$$

The strict inequality excludes every equality boundary, and $x = 5$ is not defined.

$$x < -2, \quad \frac{22}{5} < x < 5, \quad \text{or} \quad 5 < x < 6.$$

(6)

Question 4

Question 4

Let $y = (15 + e^{2x})^{1/2}$. (a) Show the stated expression for d^2y/dx^2. (b) Find the Maclaurin expansion through x^2. (c) Find the first two non-zero terms of $\sin 3x$. (d) Hence show that $\lim_{x\to 0} \dfrac{(15+e^{2x})^{1/2}-4}{\sin 3x} = \dfrac{1}{12}$.

Model solution

(a) Differentiate by the chain rule, then differentiate again using the product rule. Keeping a common denominator makes the required simplification visible.

$$\frac{dy}{dx} = \frac{e^{2x}}{(15+e^{2x})^{1/2}},$$

$$\begin{aligned}\frac{d^2y}{dx^2} &= \frac{2e^{2x}}{(15+e^{2x})^{1/2}} - \frac{e^{4x}}{(15+e^{2x})^{3/2}} \\ &= \frac{2e^{2x}(15+e^{2x}) - e^{4x}}{(15+e^{2x})^{3/2}} = \frac{e^{2x}(30+e^{2x})}{(15+e^{2x})^{3/2}}.\end{aligned}$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{e^{2x}(30+e^{2x})}{(15+e^{2x})^{3/2}}.}$$

(3)

(b) A Maclaurin series uses values at $x = 0$.

$$y(0) = 4, \qquad y'(0) = \frac{1}{4}, \qquad y''(0) = \frac{31}{64}.$$

$$y = 4 + \frac{1}{4}x + \frac{31/64}{2}x^2 + \cdots$$

$$\boxed{y = 4 + \frac{x}{4} + \frac{31x^2}{128} + \cdots.}$$

(2)

(c) Substitute $3x$ into $\sin u = u - u^3/6 + \cdots$.

$$\boxed{\sin 3x = 3x - \frac{9}{2}x^3 + \cdots}$$

(1)

(d) Use the two expansions. The constant terms cancel, so both numerator and denominator have leading term proportional to x.

$$\frac{(15+e^{2x})^{1/2}-4}{\sin 3x} = \frac{\frac{1}{4}x + \frac{31}{128}x^2 + \cdots}{3x - \frac{9}{2}x^3 + \cdots} = \frac{\frac{1}{4} + \frac{31}{128}x + \cdots}{3 - \frac{9}{2}x^2 + \cdots}.$$

$$\lim_{x\to 0}\frac{(15+e^{2x})^{1/2}-4}{\sin 3x}=\frac{1/4}{3}=\frac{1}{12}.$$

(2)

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Question 5

Question 5

With $t = \tan(x/2)$, (a) show the transformed integral for $\int \frac{1}{1+\csc x}\,dx$. (b) Hence show that it equals $Ax + \dfrac{B}{1+\tan(x/2)} + k$, determining A and B.

Model solution

(a) The tangent half-angle substitution converts both $\sin x$ and dx into rational functions of t, which is why it is chosen here.

$$\sin x = \frac{2t}{1+t^2}, \qquad \csc x = \frac{1+t^2}{2t}, \qquad dx = \frac{2}{1+t^2}\,dt.$$

$$\begin{aligned}\int \frac{1}{1+\csc x}\,dx &= \int \frac{1}{1+(1+t^2)/(2t)}\,\frac{2}{1+t^2}\,dt \\ &= \int \frac{4t}{(1+t^2)(1+t)^2}\,dt.\end{aligned}$$

$$\int \frac{1}{1+\csc x}\,dx = \int \frac{4t}{(1+t^2)(1+t)^2}\,dt.$$

(3)

(b) The rational integrand has a quadratic factor and a repeated linear factor, so use partial fractions.

$$\frac{4t}{(1+t^2)(1+t)^2} = \frac{2}{1+t^2} - \frac{2}{(1+t)^2}.$$

$$\int \frac{4t}{(1+t^2)(1+t)^2}\,dt = 2\arctan t + \frac{2}{1+t} + k.$$

$$2\arctan\left(\tan\frac{x}{2}\right) = x,$$

$$\int \frac{1}{1+\csc x}\,dx = x + \frac{2}{1+\tan(x/2)} + k, \quad A = 1,\ B = 2.$$

(5)

Question 6

Question 6

(i) Use L'Hospital's rule to show that $\lim_{x\to 0}\dfrac{1-\cos 7x}{x\sin 9x}=\dfrac{49}{18}$. (ii) For $y=x^2e^{3x}$, (a) use Leibnitz's theorem to find d^ky/dx^k in the given form; (b) solve $d^9y/dx^9=0$.

Model solution

(i) Substitution gives $0/0$, so L'Hospital's rule applies. The first differentiation still gives $0/0$, hence differentiate numerator and denominator a second time.

$$\begin{aligned}\lim_{x\to 0}\frac{1-\cos 7x}{x\sin 9x}&=\lim_{x\to 0}\frac{7\sin 7x}{\sin 9x+9x\cos 9x}\\&=\lim_{x\to 0}\frac{49\cos 7x}{18\cos 9x-81x\sin 9x}=\frac{49}{18}.\end{aligned}$$

$$\frac{49}{18}.$$

(4)

(ii)(a) Leibnitz's theorem is efficient because derivatives of x^2 vanish after the second derivative. Let $u=x^2$, $v=e^{3x}$.

$$u'=2x,\quad u''=2,\quad u^{(r)}=0\ (r\geqslant 3),\qquad v^{(r)}=3^re^{3x}.$$

$$\begin{aligned}\frac{d^ky}{dx^k}&=uv^{(k)}+\binom{k}{1}u'v^{(k-1)}+\binom{k}{2}u''v^{(k-2)}\\&=3^{k-2}e^{3x}\left(9x^2+6kx+k(k-1)\right).\end{aligned}$$

$$\frac{d^ky}{dx^k}=3^{k-2}e^{3x}\{9x^2+6kx+k(k-1)\},\ \text{so}\ A=9,B=6,C=1.$$

(5)

(ii)(b) The exponential factor and power of 3 cannot be zero, so only the quadratic bracket needs solving.

$$9x^2+54x+72=0\iff x^2+6x+8=0\iff (x+2)(x+4)=0.$$

$$x=-4\ \text{or}\ x=-2.$$

(2)

Question 7

Question 7

The concentration P is modelled by $\frac{1}{P}\frac{dP}{dt} = 4tP^2 - 1$. (a) With $x = P^{-2}$, show that $dx/dt - 2x = -8t$. Given $P = 0.5$ at $t = 0$, (b) solve to show the stated model for P. (c) Comment using the observed value $P = 0.034$ at $t = 3$.

Model solution

(a) The reciprocal-square substitution is suggested by the cubic P^3: its derivative produces P^{-3}, cancelling that power.

$$\frac{dx}{dt} = -2P^{-3}\frac{dP}{dt} = -2P^{-2}(4tP^2 - 1) = -8t + 2P^{-2}.$$

$$\boxed{\frac{dx}{dt} - 2x = -8t.}$$

(3) (b) This is a linear differential equation. Use the integrating factor e^{-2t}.

$$\frac{d}{dt}\left(xe^{-2t}\right) = -8te^{-2t}.$$

$$xe^{-2t} = \int -8te^{-2t}\,dt = (4t + 2)e^{-2t} + k,$$

$$x = 4t + 2 + ke^{2t}.$$

At $t = 0$, $x = P^{-2} = 4$, giving $k = 2$. Now return to P.

$$P^{-2} = 4t + 2 + 2e^{2t},$$

$$\boxed{P^2 = \frac{1}{4t + 2 + 2e^{2t}}.}$$

(6)

(c) Test the model at the observed time, then compare like quantities in context.

$$P(3) = \left(14 + 2e^6\right)^{-1/2} = 0.034903\ldots \text{ mg m}^{-3}.$$

$$\boxed{\begin{array}{c}\text{The model is reliable at } t = \\ 3, \text{ since } 0.0349 \text{ mg m}^{-3} \text{ is close to the observed } 0.034 \text{ mg m}^{-3}.\end{array}}$$

(2)

Question 8

Question 8

The ellipse is $x^2/64 + y^2/36 = 1$ and $l : y = mx + c$. (a) Show the stated quadratic equation for intersection x-coordinates. (b) If l is tangent and passes through $(10, 20)$, find its possible equations.

Model solution

(a) At an intersection both equations hold, so substitute the line equation into the ellipse and clear denominators.

$$\frac{x^2}{64} + \frac{(mx+c)^2}{36} = 1 \iff 36x^2 + 64m^2x^2 + 128cmx + 64c^2 - 2304 = 0.$$

$$(16m^2+9)x^2 + 32cmx + 16c^2 - 576 = 0, \text{so } k = 576.$$

(2)

(b) A tangent meets the ellipse once, so the discriminant of this quadratic is zero. The point condition supplies a second equation for m and c.

$$(32cm)^2 - 4(16m^2+9)(16c^2-576) = 0 \iff 64m^2 + 36c^2 = 2304.$$

$$20 = 10m + c \implies c = 20 - 10m.$$

$$64m^2 + 36(20-10m)^2 = 2304 \iff 36m^2 - 400m + 364 = 0,$$

$$m = 1 \text{ or } \frac{91}{9}, \qquad c = 10 \text{ or } -\frac{730}{9}.$$

$$y = x + 10 \quad \text{or} \quad y = \frac{91}{9}x - \frac{730}{9}.$$

(5)

Question 9

Question 9

The parallelogram P has vertices $E(10,-1,-6)$, $F(7,-2,7)$, $G(10,2,9)$, $H(13,3,-4)$. (a) Find its exact area. (b) Find the line through E perpendicular to its plane in the form $(\mathbf{r}-\mathbf{a})\times\mathbf{b}=\mathbf{0}$. (c) The opposite face lies in a parallel plane through $(25,-4,13)$; find the volume.

Model solution

(a) Use two adjacent sides from E. The magnitude of their cross product is the parallelogram area.

$$\overrightarrow{EF}=(-3,-1,13), \qquad \overrightarrow{EH}=(3,4,2),$$

$$\overrightarrow{EF}\times\overrightarrow{EH}=(-54,45,-9),$$

$$\left|\overrightarrow{EF}\times\overrightarrow{EH}\right|=\sqrt{54^2+45^2+9^2}=9\sqrt{62}.$$

$$\text{Area}=9\sqrt{62}.$$

(3)

(b) A normal vector to the plane is the cross product from part (a). A line through E in this normal direction has the required vector equation.

$$\left(\mathbf{r}-\begin{pmatrix}10\\-1\\-6\end{pmatrix}\right)\times\begin{pmatrix}6\\-5\\1\end{pmatrix}=\mathbf{0}.$$

(2)

(c) The opposite face is parallel to P, hence has the same normal $(6,-5,1)$. Form its plane using the given point, then find the perpendicular separation of the planes.

$$6x-5y+z=d, \qquad d=6(25)-5(-4)+13=183.$$

$$\textit{Plane containing P}: \quad 6x-5y+z=59.$$

$$\textit{height}=\frac{|183-59|}{\sqrt{6^2+(-5)^2+1^2}}=\frac{124}{\sqrt{62}}.$$

$$V=(9\sqrt{62})\left(\frac{124}{\sqrt{62}}\right)=1116.$$

$$\text{Volume}=1116.$$

(5)