Maths with Matt

Physics and maths resources

Edexcel A Level Further Mathematics

9FM0/3C - Further Mechanics 1

Friday 21 June 2019

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2019 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

Question wording is reproduced from the user-provided 2019 paper for study/reference. Pearson retains copyright in the original examination material.

Contents

Question 1 2

Question 2 4

Question 3 6

Question 4 7

Question 5 9

Question 6 10

Question 7 12

www.mathswithmatt.co.uk

Question 1

Question 1

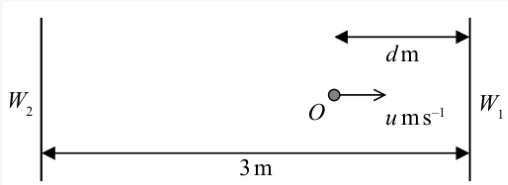


Figure 1

: plan of part of a smooth horizontal floor, where W_1 and W_2 are ical walls. The walls are 3 metres apart.

t a point O on the floor between the two walls, where the point O , from W_1

Two smooth parallel vertical walls are 3 m apart. A particle at O, distance d m from W_1, is projected perpendicularly towards W_1 at speed u m s^{-1}. The coefficient of restitution at each wall is $\frac{2}{3}$. It returns to O, after one impact with each wall, at time T.

(**a**) Show that $T = \dfrac{45 - 5d}{4u}$.

(**b**) With u fixed and d variable, find the least possible value of T, giving a reason.

Model solution

(a) Split the journey into its three constant-speed sections. At a smooth wall the speed parallel to the line of motion is multiplied by the coefficient of restitution.
Immediately after impact with W_1, the speed is

$$\frac{2}{3}u.$$

After the impact with W_2, it is

$$\frac{2}{3}\left(\frac{2}{3}u\right) = \frac{4}{9}u.$$

The distances travelled are d, then 3, then $3 - d$, respectively. Hence

$$T = \frac{d}{u} + \frac{3}{(2u/3)} + \frac{3-d}{(4u/9)} = \frac{d}{u} + \frac{9}{2u} + \frac{9(3-d)}{4u}.$$

Putting these terms over the common denominator $4u$,

$$T = \frac{4d + 18 + 27 - 9d}{4u} = \frac{45 - 5d}{4u}.$$

$$T = \frac{45 - 5d}{4u} \text{ s.}$$

(6)

(b) The expression for T decreases as d increases, since the coefficient of d is negative. Physically, this replaces part of the final, slowest section by travel at the original, greatest speed.

Since $0 < d \leqslant 3$*, the greatest possible value is* $d = 3$*. Therefore*

$$T_{min} = \frac{45 - 5(3)}{4u} = \frac{30}{4u} = \frac{15}{2u}.$$

$$T_{min} = \frac{15}{2u} \text{ s, attained when } d = 3.$$

(2)

www.mathswithmatt.co.uk

Question 2

Question 2

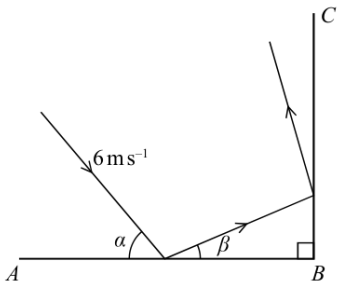


Figure 2

ts the plan view of part of a horizontal floor, where AB and BC are fixed
h AB perpendicular to BC.

A ball is projected towards wall AB at 6 m s^{-1}, with $\tan\alpha = \frac{4}{3}$. After hitting AB, its path makes angle β with AB, where $\tan\beta = \frac{1}{3}$. The coefficient of restitution at AB is e, and at perpendicular wall BC it is $\frac{1}{2}$. The floor and walls are smooth.

(**a**) Show that $e = \frac{1}{4}$.

(**b**) Find the speed immediately after the ball hits BC.

(**c**) Suggest two refinements to the model.

Model solution

(a) Resolve perpendicular and parallel to AB. *Smoothness means that the component parallel to the wall is unchanged; only the perpendicular component is changed by restitution. As* $\tan\alpha = 4/3$, *use the* $3:4:5$ *triangle:*

$$6\cos\alpha = 6\left(\frac{3}{5}\right) = \frac{18}{5}, \qquad 6\sin\alpha = 6\left(\frac{4}{5}\right) = \frac{24}{5}.$$

The component parallel to AB *after impact is still* $18/5$. *The perpendicular component after impact has magnitude* $e(24/5)$. *Thus*

$$\tan\beta = \frac{e(24/5)}{18/5} = \frac{4e}{3}.$$

Using $\tan\beta = 1/3$,

$$\frac{1}{3} = \frac{4e}{3} \quad \Longrightarrow \quad e = \frac{1}{4}.$$

$$e = \frac{1}{4}.$$

(5)

(b) Use the components found above. On impact with vertical wall BC, *the vertical component is parallel to the wall and remains unchanged; the horizontal component is reversed and halved.*

Just before hitting BC, *the components are* $18/5$ *towards* BC *and* $6/5$ *upwards. Hence just after impact they are*

$$-\frac{1}{2}\left(\frac{18}{5}\right) = -\frac{9}{5} \quad \text{and} \quad \frac{6}{5}.$$

Therefore the required speed is

$$\sqrt{\left(\frac{9}{5}\right)^2 + \left(\frac{6}{5}\right)^2} = \frac{\sqrt{117}}{5} = \frac{3\sqrt{13}}{5}.$$

$$\frac{3\sqrt{13}}{5} \text{ m s}^{-1}.$$

(4)

(c) *The model says that all contacts are smooth and the ball is a particle. Real motion can depart from both assumptions.*

For example: include friction between the ball and floor, and allow the ball to have finite size rather than modelling it as a particle.

(2)

www.mathswithmatt.co.uk

Question 3

Question 3

A particle of mass 0.5 kg has velocity $(4\mathbf{i} + 4\mathbf{j})$ m s^{-1}. It receives an impulse $\mathbf{I}$ of magnitude 2.5 Ns, and its direction is deflected through 45°. Given $\mathbf{I} = (\lambda\mathbf{i} + \mu\mathbf{j})$ Ns, find all possible pairs of λ and μ.

Model solution

The initial velocity is at 45° to each coordinate axis. A deflection of 45° therefore leaves the final motion parallel to either the $\mathbf{i}$*-axis or the* $\mathbf{j}$*-axis. Use impulse = change in momentum and then impose the given impulse magnitude.*
First suppose the final velocity is $a\mathbf{i}$*, where* $a > 0$*. Then*

$$\mathbf{I} = 0.5(a\mathbf{i}) - 0.5(4\mathbf{i} + 4\mathbf{j}) = \left(\frac{a}{2} - 2\right)\mathbf{i} - 2\mathbf{j}.$$

Since $|\mathbf{I}| = 2.5$*,*

$$\left(\frac{a}{2} - 2\right)^2 + (-2)^2 = (2.5)^2,$$

$$\frac{1}{4}a^2 - 2a + \frac{7}{4} = 0 \quad \Longrightarrow \quad a^2 - 8a + 7 = 0.$$

Thus $a = 1$ *or* $a = 7$*, giving*

$$\mathbf{I} = -\frac{3}{2}\mathbf{i} - 2\mathbf{j} \quad \textit{or} \quad \mathbf{I} = \frac{3}{2}\mathbf{i} - 2\mathbf{j}.$$

By symmetry, if the final velocity is parallel to $\mathbf{j}$*, the other two impulses are*

$$\mathbf{I} = -2\mathbf{i} - \frac{3}{2}\mathbf{j} \quad \textit{or} \quad \mathbf{I} = -2\mathbf{i} + \frac{3}{2}\mathbf{j}.$$

$$(\lambda, \mu) = (\tfrac{3}{2}, -2), (-\tfrac{3}{2}, -2), (-2, \tfrac{3}{2}), (-2, -\tfrac{3}{2}).$$

(9)

Question 4

Question 4

A 600 kg car pulls a 150 kg trailer. The trailer's resistance is 200 N and the car's resistance at speed v is $(200 + \lambda v)$ N. On level ground the engine works at 15 kW and the speed is constantly 25 m s^{-1}.
(**a**) Show that $\lambda = 8$.
The car then travels uphill with $\sin\theta = 1/15$, still with engine power 15 kW. The trailer resistance remains 200 N and car resistance is $(200 + 8v)$ N. When $v = 10$ m s^{-1}, the towbar breaks.
(**b**) Find the car's acceleration immediately afterwards.
(**c**) The trailer comes to rest d m uphill from the breaking point. Use work-energy to find d.

Model solution

(a) At constant speed the resultant force is zero. First obtain the driving force from power $P = Fv$, then balance it against both resistances.

$$F = \frac{15000}{25} = 600 \text{ N}.$$

Therefore

$$600 - \{200 + 200 + 25\lambda\} = 0,$$

$$600 = 400 + 25\lambda \quad \Longrightarrow \quad \lambda = 8.$$

$$\lambda = 8.$$

(4)

(b) Immediately after the break, apply Newton's second law to the car alone, taking uphill as positive. Its engine force is still obtained from $P = Fv$.

$$F_{\text{engine}} = \frac{15000}{10} = 1500 \text{ N}.$$

The forces downhill are the resistance $200 + 8(10) = 280$ N and the component of weight

$$600g\sin\theta = 600(9.8)\left(\frac{1}{15}\right) = 392 \text{ N}.$$

Hence

$$1500 - 280 - 392 = 600a, \qquad a = 1.38 \text{ m s}^{-2}.$$

$$1.38 \text{ m s}^{-2} \text{ uphill.}$$

(4)

(c) For the trailer after the break, the initial kinetic energy is spent doing work against its resistance and the downhill component of its weight. Its final kinetic energy is zero.

$$\frac{1}{2}(150)(10)^2 = \left(200 + 150(9.8)\frac{1}{15}\right) d.$$

Thus

$$7500 = 298\mathrm{d}, \qquad \mathrm{d} = 25.167\ldots$$

$$\mathrm{d} = 25.2 \textit{ m (3 s.f.).}$$

(4)

www.mathswithmatt.co.uk

Question 5

Question 5

Particles P and Q, of masses $3m$ and $2m$, respectively, collide directly on a smooth horizontal plane. Before collision they move towards one another with speeds u and $2u$. After collision they move in opposite directions; the coefficient of restitution is e.
(**a**) Find the range of possible values of e, justifying the answer.
Given that Q loses 75% of its kinetic energy in the collision, (**b**) find e.

Model solution

(a) Take the initial direction of P *as positive. Let* P*'s final speed to the left be* v*, and* Q*'s final speed to the right be* w*. Conservation of momentum and restitution give two equations for these speeds.*
Conservation of linear momentum gives

$$3mu - 4mu = 2mw - 3mv, \quad \textit{so} \quad -u = -3v + 2w.$$

The relative speed of separation is $v + w$*, while the relative speed of approach was* $3u$*, so*

$$v + w = 3ue.$$

Solving the two equations,

$$5w = u(9e - 1), \qquad 5v = u(1 + 6e).$$

For the stated directions, $v > 0$ *and* $w > 0$*. Since* $u > 0$*, the second expression is positive for every physical* $e \geqslant 0$*, whereas*

$$9e - 1 > 0 \quad \Longrightarrow \quad e > \frac{1}{9}.$$

Also a coefficient of restitution cannot exceed 1*. Therefore*

$$\frac{1}{9} < e \leqslant 1.$$

(8)

(b) A 75% loss means that Q's final kinetic energy is 25% of its initial kinetic energy. Since its mass is unchanged, its final speed is half of its original speed.
Thus $w = \frac{1}{2}(2u) = u$*. Using* $5w = u(9e - 1)$*,*

$$5u = u(9e - 1) \quad \Longrightarrow \quad 9e = 6.$$

$$e = \frac{2}{3}.$$

(3)

Question 6

Question 6

Perpendicular unit vectors $\mathbf{i}, \mathbf{j}$ lie in a horizontal plane. Smooth spheres A and B, of masses 0.2 kg and 0.4 kg, collide obliquely. Before collision,

$$\mathbf{u}_A = 3\mathbf{i} + 2\mathbf{j}, \qquad \mathbf{u}_B = -4\mathbf{i} - \mathbf{j} \quad \text{m s}^{-1}.$$

The line of centres is parallel to $\mathbf{i}$, and $e = 3/7$.
(**a**) Find the velocity of A immediately after collision.
(**b**) Find the magnitude of the impulse received by A.
(**c**) Find, to the nearest degree, the angle through which A's direction is deflected.

Model solution

(a) For smooth spheres, impulse acts along the line of centres. Thus the $\mathbf{j}$*-components, perpendicular to that line, are unchanged. Apply momentum and restitution only to the* $\mathbf{i}$*-components.*
So A*'s final* $\mathbf{j}$*-component is* 2*. Let final* $\mathbf{i}$*-components of* A, B *be* v, w*. Momentum parallel to the line of centres gives*

$$0.2(3) + 0.4(-4) = 0.2v + 0.4w, \quad \textit{so} \quad 2v + 4w = -10.$$

Restitution gives

$$w - v = \frac{3}{7}\{3 - (-4)\} = 3.$$

Hence $w = v + 3$*, and substitution gives*

$$2v + 4(v + 3) = -10, \qquad 6v = -22, \qquad v = -\frac{11}{3}.$$

$$\mathbf{v}_A = -\frac{11}{3}\mathbf{i} + 2\mathbf{j} \ \textit{m s}^{-1}.$$

(7)

(b) The impulse on A *is its change in momentum. Only its* $\mathbf{i}$*-component changes.*

$$|\mathbf{I}_A| = 0.2\left|-\frac{11}{3} - 3\right| = 0.2\left(\frac{20}{3}\right) = \frac{4}{3}.$$

$$\frac{4}{3} \ \textit{Ns.}$$

(2)

(c) The angle between the initial and final velocity vectors is found most directly from their scalar product.

$$\cos\theta = \frac{(3\mathbf{i} + 2\mathbf{j}) \cdot (-\frac{11}{3}\mathbf{i} + 2\mathbf{j})}{|3\mathbf{i} + 2\mathbf{j}|\,|-\frac{11}{3}\mathbf{i} + 2\mathbf{j}|} = \frac{-7}{\sqrt{13}\,(\sqrt{157}/3)}.$$

Therefore

$$\theta = 118.0\ldots^\circ .$$

The direction is deflected through 118° *(nearest degree).*

(3)

www.mathswithmatt.co.uk

Question 7

Question 7

A particle of mass m hangs from a light elastic spring of natural length (a) and modulus kmg, fixed at its upper end O. It is released from rest at A, where $OA = 3a$, and first comes to rest at B, where $OB = \frac{1}{2}a$.
(**a**) Show that $k = \frac{4}{3}$.
(**b**) Find the acceleration immediately after release at A.
(**c**) Find the maximum speed as it moves from A to B.

Model solution

(a) Both endpoints are instantaneous rest positions, so conservation of mechanical energy is appropriate. Compare elastic potential energy and gravitational potential energy.
At A, the extension is $2a$, so the elastic energy is

$$\frac{(kmg)(2a)^2}{2a} = 2kmga.$$

At B, the spring is compressed by $a/2$, so its elastic energy is

$$\frac{(kmg)(a/2)^2}{2a} = \frac{1}{8}kmga.$$

From A to B, the particle rises $\frac{5}{2}a$. Therefore

$$2kmga = \frac{1}{8}kmga + mg\left(\frac{5}{2}a\right).$$

Cancelling mga gives $2k - k/8 = 5/2$, hence

$$k = \frac{4}{3}.$$

(3)

(b) At A, use Hooke's law for the spring tension and then Newton's second law. The extension is $2a$, and the spring force acts upwards.

$$T = \frac{\lambda x}{a} = \frac{(4mg/3)(2a)}{a} = \frac{8}{3}mg.$$

Thus the resultant upward force is

$$\frac{8}{3}mg - mg = \frac{5}{3}mg,$$

so

$$\frac{5}{3}g \text{ m s}^{-2} \text{ upwards.}$$

(3)

(c) The speed is greatest where the resultant force is zero: the equilibrium position. Find that position, then use energy from A to it.

If the equilibrium extension is e, then

$$\frac{(4mg/3)e}{a} = mg \quad \Longrightarrow \quad e = \frac{3}{4}a.$$

The equilibrium position is $a + 3a/4 = 7a/4$ below O, so it is $5a/4$ above A. Conservation of energy from A to this point gives

$$\frac{8}{3}mga = \frac{3}{8}mga + mg\left(\frac{5}{4}a\right) + \frac{1}{2}mv^2.$$

Hence

$$\frac{1}{2}mv^2 = \frac{25}{24}mga, \qquad v^2 = \frac{25}{12}ga.$$

$$v_{\max} = \frac{5}{2}\sqrt{\frac{ga}{3}} \ m\ s^{-1}.$$

(6)

www.mathswithmatt.co.uk