Maths with Matt

Physics and maths resources

Edexcel A Level Further Mathematics

9FM0/3C - Further Mechanics 1

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Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2023 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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Question 1

Question 1

Impulse and deflection of 2 kg particle: $\mathbf{u} = 4\mathbf{i} + 3\mathbf{j}$, $\mathbf{J} = -6\mathbf{i} + 42\mathbf{j}$.

Model solution

Use $\mathbf{J} = m(\mathbf{v} - \mathbf{u})$.

$$\mathbf{v} = \mathbf{i} + 24\mathbf{j}, \quad |\mathbf{v}| = 25.$$

$$25\text{m s}^{-1}.$$

(4)Use the scalar product for the angle between the velocity vectors.

$$\cos\alpha = \frac{(4,3)\cdot(1,24)}{5\cdot 25} = \frac{76}{125}, \quad \alpha = 37.5\ldots^\circ.$$

$$37.5^\circ.$$

(2)

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Question 2

Question 2

Car: resistance 400 N, power 16 kW; then it tows a 600 kg trailer, resistance 300 N.

Model solution

At constant speed use $P = Fv$ *and zero resultant force.*

$$16000/v = 400, \quad v = 40.$$

$$40\text{m s}^{-1}.$$

(3)At $20/3\text{m s}^{-1}$, $F = 16000/(20/3) = 2400$ *N. The rope imposes a common acceleration.*

$$2400 - 400 - 300 = 1600a, \ a = 17/16; \quad T - 300 = 600a.$$

$$T = 937.5\text{N}.$$

(5)

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Question 3

Question 3

Direct collision: P has mass $2m$, velocity $3u$; Q mass m, velocity $-2u$; restitution $e > 4/5$.

Model solution

Take P's initial direction positive; use momentum and restitution.

$$4u = 2v_P + v_Q, \quad v_Q - v_P = 5eu.$$

$$v_P = (4 - 5e)u/3, \quad v_Q = (4 + 10e)u/3.$$

Q's speed is $(4 + 10e)u/3$.

(6)After the wall, Q's return speed must exceed P's reversed speed.

$$f(4 + 10e)u/3 > (5e - 4)u/3.$$

$$(5e - 4)/(4 + 10e) < f \leqslant 1.$$

(4)

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Question 4

Question 4

A light elastic string has natural length $2a$ and modulus $4mg$. It is fixed at O and supports a particle P of mass m. Initially P is in equilibrium at E. It is then pulled to A, where $OA = 4a$, and released; resistance has magnitude $\frac{1}{4}mg$. Finally it is held at O and released, with the same resistance.

Model solution

At equilibrium the string tension balances the weight. If $x = OE - 2a$, Hooke's law gives $T = \lambda x/l$.

$$mg = \frac{4mg}{2a}x, \qquad x = \frac{a}{2}.$$

$$OE = 2a + \tfrac{a}{2} = \tfrac{5a}{2}.$$

(4) After moving upwards a distance a, the extension decreases from $2a$ to a. Apply work--energy: elastic potential energy released supplies the gain in gravitational potential energy, the work against resistance, and kinetic energy.

$$\frac{1}{2}\frac{4mg}{2a}(2a)^2 - \frac{1}{2}\frac{4mg}{2a}a^2 = mga + \frac{1}{4}mga + \frac{1}{2}mv^2.$$

$$4mga - mga = mga + \frac{1}{4}mga + \frac{1}{2}mv^2, \qquad \frac{1}{2}mv^2 = \frac{7}{4}mga.$$

$$v = \sqrt{\tfrac{7ag}{2}}.$$

(7) At maximum speed the acceleration is zero. Let $OB = x$; the string extension is $x - 2a$, and the forces opposing the downward motion are tension and resistance.

$$mg - \frac{1}{4}mg - \frac{4mg}{2a}(x - 2a) = 0.$$

$$\frac{2mg}{a}(x - 2a) = \frac{3}{4}mg, \qquad x = 2a + \frac{3a}{8}.$$

$$OB = \tfrac{19a}{8}.$$

(4)

Question 5

Question 5

A smooth sphere S, of mass m, moves with speed U on a smooth horizontal plane and collides obliquely with an equal-radius sphere of mass M, initially at rest. The initial direction of S makes angle α with the line of centres; after impact its direction makes angle β. The coefficient of restitution is e.

Model solution

The component perpendicular to the line of centres is unchanged, so it is $U\sin\alpha$ after impact. Let v_1 be S's component along the line of centres and v_2 be the speed of the other sphere. Use momentum and restitution along that line.

$$mU\cos\alpha = mv_1 + Mv_2, \qquad eU\cos\alpha = -v_1 + v_2.$$

$$v_1 = \frac{m - eM}{m + M}U\cos\alpha.$$

$$\tan\beta = \frac{U\sin\alpha}{v_1} = \frac{(m+M)\tan\alpha}{m - eM}.$$

$$\tan\beta = \frac{(m+M)\tan\alpha}{m - eM}.$$

(8) If $m = eM$, the expression for the component of S's velocity along the line of centres gives $v_1 = 0$. Thus S moves perpendicular to the line of centres; the other sphere moves along it.

The directions of motion immediately after collision are perpendicular.

(2)

Question 6

Question 6

A particle P of mass m falls vertically with speed u onto a fixed smooth plane inclined at angle α, where $0 < \alpha \leqslant 45°$. It rebounds horizontally with speed v. The coefficient of restitution is $e > 0$.

Model solution

There is no impulse along the smooth plane, so resolve parallel to it. Then resolve the impulse perpendicular to the plane.

$$mu\sin\alpha = mv\cos\alpha, \qquad I = m\{v\sin\alpha - (-u\cos\alpha)\}.$$

$$v = u\tan\alpha, \qquad I = m(u\tan\alpha\sin\alpha + u\cos\alpha) = mu\sec\alpha.$$

The magnitude of the impulse exerted on the plane by P *is* $mu\sec\alpha$.

(5) The normal component of separation is $v\sin\alpha$*, and the normal component of approach is* $u\cos\alpha$*. Use restitution with the unchanged parallel component.*

$$v\sin\alpha = eu\cos\alpha, \qquad v\cos\alpha = u\sin\alpha.$$

$$v^2 = u^2\sin^2\alpha + e^2u^2\cos^2\alpha.$$

$$v^2 = u^2(\sin^2\alpha + e^2\cos^2\alpha).$$

(3) Subtract the kinetic energy after impact from that before impact and use $\sin^2\alpha + \cos^2\alpha = 1$.

$$\textit{KE lost} = \frac{1}{2}mu^2 - \frac{1}{2}mu^2(\sin^2\alpha + e^2\cos^2\alpha) = \frac{1}{2}mu^2(1-e^2)\cos^2\alpha.$$

$$\frac{1}{2}mu^2(1-e^2)\cos^2\alpha.$$

(2) Dividing the two component equations gives $\tan^2\alpha = e$*, so* $\cos^2\alpha = 1/(1+e)$.

$$\textit{KE lost} = \frac{1}{2}mu^2\frac{1-e^2}{1+e} = \frac{1}{2}mu^2(1-e).$$

$$\frac{1}{2}mu^2(1-e).$$

(2)

Question 7

Question 7

A smooth snooker ball is projected from corner A of a horizontal rectangular table. It travels $A \to P$ on side DC, then $P \to Q$ on side CB, and then returns to A. The angles are $\angle APD = \alpha$, $\angle QPC = \beta$, $\angle AQB = \gamma$, and the coefficients of restitution at DC and CB are e_1 and e_2, respectively. Its speeds on AP, PQ, QA are U, V, W.

Model solution

At DC, *the component parallel to the side is unchanged, while the normal component is multiplied by* e_1.

$$V\sin\beta = e_1 U\sin\alpha, \qquad V\cos\beta = U\cos\alpha.$$

$$\tan\beta = e_1\tan\alpha.$$

$$\tan\beta = e_1\tan\alpha.$$

(4) At CB, *resolving parallel and perpendicular to that side gives* $\tan\gamma = e_2\cot\beta$. *Substitute the result from part (a).*

$$\tan\gamma = e_2\cot\beta = \frac{e_2}{e_1\tan\alpha}.$$

$$e_1\tan\alpha = e_2\cot\gamma.$$

(3) The other two angles of triangle APQ *have sum* $270^\circ - \alpha - \gamma$. *For the ball to return to* A, *this must be less than* 180°.

$$270^\circ - \alpha - \gamma < 180^\circ \quad \Rightarrow \quad \alpha + \gamma > 90^\circ.$$

$$\tan\alpha > \cot\gamma \quad \Rightarrow \quad e_1\tan\alpha > e_1\cot\gamma.$$

$$e_2\cot\gamma > e_1\cot\gamma \quad \Rightarrow \quad e_2 > e_1,$$

since $\cot\gamma > 0$.

It is possible only if $e_2 > e_1$.

(6) If $e_1 = e_2$, *part (b) gives* $\tan\alpha = \cot\gamma = \tan(90^\circ - \gamma)$, *so* $\alpha + \gamma = 90^\circ$.

After hitting CB *at* Q, *the ball travels parallel to* AP *(its initial direction).*

(1)