Maths with Matt

Physics and maths resources

Edexcel A Level Further Mathematics

9FM0/3C - Further Mechanics 1

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Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2025 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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Question 1

Question 1

Two particles of masses $4m$ and $3m$ collide directly on a smooth horizontal table. Before impact their speeds are u and ku respectively, in opposite directions; afterwards their directions are reversed and their speeds are $2v$ and $3v$. The impulse on the $4m$ particle has magnitude $20mv$.

Model solution

(a) Take the initial direction of particle A as positive. Impulse equals change in momentum, so it is the direct way to use the given impulse. A reverses direction, hence its velocity changes from u to $-2v$.

$$20mv = |4m(-2v) - 4m(u)| = 4m(2v + u).$$

$$5v = 2v + u \quad \Rightarrow \quad u = 3v.$$

$$u = 3v.$$

(3)

(b) Particle B receives an equal and opposite impulse. Its initial velocity is $-ku$ and its final velocity is $3v$, so its change in momentum has magnitude $3m(3v + ku)$.

$$20mv = 3m(3v + ku).$$

Substituting $u = 3v$ and cancelling mv gives

$$20 = 9 + 9k \quad \Rightarrow \quad k = \frac{11}{9}.$$

$$k = \frac{11}{9}.$$

(3)

Question 2

Question 2

A 900 kg van has resistance $50v$ N at speed v m s^{-1}. Its engine power is 12 kW and, at speed V, its acceleration is 0.2 m s^{-2}. Later it climbs a slope with $\sin\alpha = 1/12$, resistance $50v$ N, and power 15 kW at constant speed U.

Model solution

(a) The engine force is not constant, because the engine supplies constant power. First use Newton's second law to express that force, then use $P = Fv$. Take forwards as positive.

$$F - 50V = 900(0.2) = 180.$$

Since $P = FV$,

$$F = \frac{12000}{V}, \qquad \frac{12000}{V} - 50V = 180.$$

Multiplying by V gives

$$50V^2 + 180V - 12000 = 0.$$

The positive root is $V = 13.796\ldots$; the negative root is not a speed.

$$V = 14 \text{ m s}^{-1} \text{ (to 2 s.f.).}$$

(4)

(b) At constant speed the acceleration is zero. Along the slope, the opposing forces are the resistance and the component $mg\sin\alpha$ of the weight.

$$F - 50U - 900g\left(\frac{1}{12}\right) = 0.$$

Using $g = 9.8$ m s^{-2} and $F = 15000/U$,

$$\frac{15000}{U} - 50U - 735 = 0,$$

$$50U^2 + 735U - 15000 = 0.$$

The physically valid root is $U = 11.49\ldots$.

$$U = 11 \text{ m s}^{-1} \text{ (to 2 s.f.).}$$

(4)

Question 3

Question 3

A particle of mass 0.4 kg changes velocity from $7\mathbf{i}$ m s^{-1} to $\lambda(2\mathbf{i} + \mathbf{j})$ m s^{-1} when it receives an impulse of magnitude $\sqrt{1.6}$ N s. Find the two possible values of λ.

Model solution

Impulse is a vector change in momentum. The stated magnitude therefore gives an equation by squaring and adding its two components.

$$\mathbf{I} = 0.4\,[\lambda(2\mathbf{i} + \mathbf{j}) - 7\mathbf{i}] = (0.8\lambda - 2.8)\mathbf{i} + 0.4\lambda\mathbf{j}.$$

Hence

$$(\sqrt{1.6})^2 = (0.8\lambda - 2.8)^2 + (0.4\lambda)^2.$$

$$1.6 = (0.8\lambda - 2.8)^2 + 0.16\lambda^2 \quad \Rightarrow \quad 5\lambda^2 - 28\lambda + 39 = 0.$$

$$\lambda = \frac{28 \pm \sqrt{28^2 - 4(5)(39)}}{10} = \frac{28 \pm 2}{10}.$$

$$\lambda = 3 \text{ or } \lambda = \frac{13}{5}.$$

(6)

Question 4

Question 4

A 0.5 kg package is projected up a rough ramp at 8 m s^{-1} and comes to rest 18 m up the slope. The ramp has $\sin\theta = 1/14$. The work done against friction is W and the coefficient of friction is μ.

Model solution

(a) The package starts and ends at known speeds and moves a known distance, so work--energy accounts directly for both the rise in gravitational potential energy and the energy dissipated by friction.

$$\frac{1}{2}(0.5)(8)^2 = 0.5g(18\sin\theta) + W.$$

Thus, using $g = 9.8$ *and* $\sin\theta = 1/14$*,*

$$16 = 0.5(9.8)\left(\frac{18}{14}\right) + W = 6.3 + W,$$

$$W = 9.7.$$

$W = 9.7$ ***J, as required.***

(3)

(b) The normal reaction is $R = mg\cos\theta$*, so the friction magnitude is* μR*. Its work over* 18 *m is the value found in part (a).*

$$9.7 = (\mu \times 0.5g\cos\theta)(18).$$

Since $\cos\theta = \sqrt{1-(1/14)^2} = \dfrac{\sqrt{195}}{14}$*,*

$$\mu = \frac{9.7}{9(9.8)\sqrt{195}/14} = 0.110\ldots.$$

$$\mu = 0.11.$$

(4)

Question 5

Question 5

Balls A, B, C, of masses $2m, 3m, 4m$, lie in that order on a smooth horizontal line. A, initially projected towards B at $5u$, collides directly with B, initially at rest; B then has speed $3u$. Afterwards C is projected towards B at speed u. The coefficient of restitution for the B–C collision is f. A second A–B collision occurs.

Model solution

(a) Use conservation of linear momentum for the A--B collision. Take A's original direction as positive and let A's final velocity be V.

$$2m(5u) = 2mV + 3m(3u).$$

$$10u = 2V + 9u \quad \Rightarrow \quad V = \frac{u}{2}.$$

The speed of A is $\frac{u}{2}$.

(3)

(b) For a direct collision, coefficient of restitution is speed of separation divided by speed of approach. After impact B moves at $3u$ and A at $u/2$, so their separation speed is $3u - u/2$.

$$e = \frac{3u - u/2}{5u - 0} = \frac{u}{2u} = \frac{1}{2}.$$

$$e = \frac{1}{2}.$$

(3)

(c) Keep the same positive direction. Just before B and C collide, B has velocity $3u$ and C has velocity $-u$. Let their velocities afterwards be x and y respectively. Momentum and restitution determine x, then the stated second collision gives the required inequality.

$$3m(3u) + 4m(-u) = 3mx + 4my,$$

$$5u = 3x + 4y. \tag{1}$$

The speed of separation is $y - x$ and the speed of approach is $3u - (-u) = 4u$, so

$$y - x = 4fu. \tag{2}$$

From $y = x + 4fu$, substitute into (1):

$$5u = 3x + 4(x + 4fu) = 7x + 16fu,$$

$$x = \frac{(5-16f)u}{7}.$$

After the first collision A travels right at $u/2$. B is initially to the right of A, so a second collision occurs precisely when B's velocity after its collision with C is less than A's velocity; this includes B turning left.

$$\frac{(5-16f)u}{7} < \frac{u}{2} \quad \Rightarrow \quad 5-16f < \frac{7}{2} \quad \Rightarrow \quad f > \frac{3}{32}.$$

As a coefficient of restitution, $f \leqslant 1$.

$$\frac{3}{32} < f \leqslant 1.$$

(7)

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Question 6

Question 6

On a smooth plane with $\sin\theta = 1/3$, A, B and C lie on a line of greatest slope, with A above B and $AC = 3a$. A parcel of mass m is attached at B to a string fixed at A; its natural length is $2a$ and its modulus is $8mg/3$. It is released from rest at C.

Model solution

(a) At equilibrium the tension balances the component of weight down the smooth slope. Combine this with Hooke's law to find the extension.

$$T = mg\sin\theta = \frac{mg}{3}, \qquad T = \frac{\lambda e}{l} = \frac{(8mg/3)e}{2a}.$$

$$\frac{mg}{3} = \frac{4mge}{3a} \quad \Rightarrow \quad e = \frac{a}{4}, \qquad AB = 2a + e.$$

$$AB = \frac{9a}{4}, \text{ as required.}$$

(4)

(b) Use $EPE = \lambda e^2/(2l)$. *The extensions at C and B are* a *and* $a/4$ *respectively.*

$$\Delta E = \frac{8mg/3}{2(2a)}\left[a^2 - \left(\frac{a}{4}\right)^2\right] = \frac{5mga}{8}.$$

$$\text{Elastic potential energy lost} = \frac{5mga}{8}.$$

(3)

(c) From C to B, the parcel rises $CB = 3a - 9a/4 = 3a/4$ *up the plane. Conservation of mechanical energy converts the EPE loss into the increase in GPE and kinetic energy.*

$$\frac{5mga}{8} = mg\left(\frac{3a}{4}\right)\left(\frac{1}{3}\right) + \frac{1}{2}mv^2 = \frac{mga}{4} + \frac{1}{2}mv^2.$$

$$v^2 = \frac{3ga}{4}.$$

$$v = \sqrt{\frac{3ga}{4}} \text{ m s}^{-1}.$$

(3)

Question 7

Question 7

Uniform spheres P (0.3 kg) and Q (0.4 kg) collide obliquely on a smooth plane. Before collision their speeds are 5 and 3 m s^{-1} at angles 30° and 60° to the line of centres; after collision P has speed v at 60°, and Q has speed w at angle θ.

Model solution

(a) For smooth spheres the impulse is along the line of centres, so P's perpendicular velocity component is unchanged.

$$5\sin 30^\circ = v\sin 60^\circ \quad \Rightarrow \quad v = \frac{5\sqrt{3}}{3}.$$

$$v = \frac{5\sqrt{3}}{3} \text{ m s}^{-1}, \text{ as required.}$$

(3)

(b)(i), (ii) Q's perpendicular component is also unchanged. Conserve momentum parallel to the line of centres, since the collision impulse has no perpendicular component.

$$w\sin\theta = 3\sin 60^\circ = \frac{3\sqrt{3}}{2}. \tag{1}$$

$$0.3(5\cos 30^\circ) - 0.4(3\cos 60^\circ) = 0.4w\cos\theta - 0.3v\cos 60^\circ.$$

Using $v = 5\sqrt{3}/3$ *gives*

$$w\cos\theta = \frac{5\sqrt{3}-3}{2}. \tag{2}$$

Thus

$$w^2 = \left(\frac{3\sqrt{3}}{2}\right)^2 + \left(\frac{5\sqrt{3}-3}{2}\right)^2 = \left(\frac{3\sqrt{3}}{2}\right)^2 + \left(\frac{5\sqrt{3}-3}{2}\right)^2, \qquad \tan\theta = \frac{3\sqrt{3}}{5\sqrt{3}-3}.$$

$$w = 3.8 \text{ m s}^{-1} \text{ and } \theta = 43^\circ \text{ (both to 2 s.f.).}$$

(7)

(c) Newton's law of restitution uses only the components along the line of centres. The approach speed is $5\cos 30^\circ + 3\cos 60^\circ$ *and the separation speed is* $v\cos 60^\circ + w\cos\theta$.

$$e = \frac{(5\sqrt{3}/3)\cos 60^\circ + (5\sqrt{3}-3)/2}{5\cos 30^\circ + 3\cos 60^\circ} = 0.73300\ldots.$$

$$e = 0.73 \text{ (to 2 s.f.).}$$

(3)

Question 8

Question 8

A 0.25 kg ball strikes a smooth wall RS parallel to $\mathbf{i}$. Its velocity afterwards is $(3\mathbf{i} + 2\mathbf{j})$ m s^{-1} and $e = 1/3$. It then strikes a smooth wall ST parallel to $3\mathbf{i} + 4\mathbf{j}$, again with $e = 1/3$.

Model solution

(a) A smooth wall exerts impulse perpendicular to itself, so the component parallel to RS is unchanged. The outgoing perpendicular component is $2\mathbf{j}$; restitution makes the incoming perpendicular component three times as large and oppositely directed.

$$\mathbf{v} = 3\mathbf{i} - 6\mathbf{j}.$$

The full velocities give the kinetic-energy loss:

$$\Delta\text{KE} = \frac{1}{2}(0.25)(3^2 + 6^2) - \frac{1}{2}(0.25)(3^2 + 2^2) = \frac{1}{8}(45 - 13) = 4.$$

Kinetic energy lost $= 4$ J, as required.

(6)

(b) Resolve $3\mathbf{i} + 2\mathbf{j}$ in unit directions parallel and perpendicular to ST: $\mathbf{t} = (3\mathbf{i} + 4\mathbf{j})/5$ and $\mathbf{n} = (-4\mathbf{i} + 3\mathbf{j})/5$. The parallel component is unchanged, while the perpendicular component reverses and is multiplied by $1/3$.

$$(3\mathbf{i} + 2\mathbf{j}) \cdot \mathbf{t} = \frac{17}{5}, \qquad (3\mathbf{i} + 2\mathbf{j}) \cdot \mathbf{n} = -\frac{6}{5}.$$

Thus the outgoing parallel and perpendicular components are respectively $\frac{17}{5}\mathbf{t}$ and $\frac{2}{5}\mathbf{n}$, giving

$$\mathbf{w} = \frac{17}{5}\left(\frac{3\mathbf{i} + 4\mathbf{j}}{5}\right) + \frac{2}{5}\left(\frac{-4\mathbf{i} + 3\mathbf{j}}{5}\right) = \frac{43}{25}\mathbf{i} + \frac{74}{25}\mathbf{j}.$$

$$\mathbf{w} = \frac{43}{25}\mathbf{i} + \frac{74}{25}\mathbf{j} \text{ m s}^{-1}.$$

(6)