Maths with Matt

Physics and maths resources

Edexcel A Level Further Mathematics

9FM0/3C - Further Mechanics 1

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Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2021 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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Question 1

Question 1

A van of mass 900 kg has resistance $(500 + 7v)$ N at speed v m s^{-1}. Its engine works at a constant rate of 18 kW. (a) Find its constant speed V on a horizontal road. (b) It then travels up a road with $\sin\theta = 1/21$. Find its acceleration when $v = 15$ m s^{-1}.

Model solution

(a) At constant speed the resultant force is zero, so the driving force equals the resistance. Since the engine power is given, use $P = FV$ *to connect the driving force to the speed.*

$$F = 500 + 7V, \qquad 18000 = (500 + 7V)V.$$

$$7V^2 + 500V - 18000 = 0.$$

The positive root is $V = 26.309\ldots$*; the negative root has no physical meaning.*

$$V = 26.3 \text{ m s}^{-1} \text{ (3 s.f.).}$$

(5)

(b) Take up the slope as positive. The driving force is P/v*, while both resistance and the component* $mg\sin\theta$ *act down the slope.*

$$\frac{18000}{15} - (500 + 7 \times 15) - 900g\left(\frac{1}{21}\right) = 900a.$$

$$a = 0.194\ldots \text{ m s}^{-2}.$$

$$a = 0.19 \text{ m s}^{-2}\text{, so the van is accelerating up the slope.}$$

(4)

Question 2

Question 2

Particles A and B, of masses $5m$ and $3m$, collide directly on a smooth horizontal surface while moving in opposite directions. Their coefficient of restitution is $e > 0$. After collision their speeds are v and $2v$, and they move in the same direction. (a) Find the possible values of e. (b) Given that A's final kinetic energy is 16% of its initial kinetic energy, find (i) e and (ii) the magnitude of the impulse on A, in terms of m and v.

Model solution

(a) Choose the common direction after collision as positive. Let the initial velocities of A and B be x and $-y$, where $x, y > 0$. Conservation of linear momentum applies because there is no external horizontal impulse.

$$5mx - 3my = 5mv + 3m(2v), \qquad 5x - 3y = 11v. \tag{1}$$

The impact law uses relative speed of separation $=$ e times relative speed of approach.

$$2v - v = e\big(x - (-y)\big), \qquad v = e(x + y). \tag{2}$$

Solving (1)--(2),

$$x = \frac{v(11e + 3)}{8e}, \qquad y = \frac{v(5 - 11e)}{8e}.$$

For the stated initial directions $x > 0$ and $y > 0$. Since $e > 0$, the second condition requires $5 - 11e > 0$.

$$0 < e < \frac{5}{11}.$$

(8)

(b)(i) The 16% kinetic-energy statement determines the initial speed x of A.

$$\frac{1}{2}(5m)v^2 = \frac{16}{100}\left[\frac{1}{2}(5m)x^2\right] \quad \Longrightarrow \quad x = \frac{5}{2}v.$$

Use momentum equation (1) to find the other initial speed:

$$11v = 5\left(\frac{5}{2}v\right) - 3y \quad \Longrightarrow \quad y = \frac{1}{2}v.$$

Then substitute both initial speeds in the impact equation (2):

$$v = e\left(\frac{5}{2}v + \frac{1}{2}v\right) \quad \Longrightarrow \quad e = \frac{1}{3}.$$

$$e = \frac{1}{3}.$$

(b)(ii) Impulse is change in momentum. Using positive as the final direction, A changes velocity from $\frac{5}{2}v$ to v.

$$I_A = 5m\left(v - \frac{5}{2}v\right) = -\frac{15}{2}mv.$$

The question asks for magnitude, so take the positive modulus.

$$|I_A| = \frac{15}{2}mv \text{ N s.}$$

(6)

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Question 3

Question 3

In a horizontal plane, smooth spheres P (0.3 kg) and Q (0.5 kg), of equal radius, collide obliquely. Before impact their velocities are $(u\mathbf{i}+2\mathbf{j})$ and $(-4\mathbf{i}+3\mathbf{j})$ m s^{-1} respectively, and the line of centres is parallel to $\mathbf{i}$. The coefficient of restitution is 3/5. P is deflected through 90° and Q through α°. (a) Find u. (b) Find α. (c) State how equal radii were used.

Model solution

(a) For smooth spheres the impulse is along the line of centres, here the $\mathbf{i}$ direction. Thus the $\mathbf{j}$ components are unchanged. Since P turns through 90°, its final velocity must be perpendicular to $(u, 2)$, with $\mathbf{j}$ component 2:

$$\mathbf{v}_P = \left(-\frac{4}{u}\mathbf{i} + 2\mathbf{j}\right) \text{ m s}^{-1}.$$

Let the final $\mathbf{i}$ component of Q be w. Apply momentum and restitution along the line of centres:

$$0.3u + 0.5(-4) = 0.3\left(-\frac{4}{u}\right) + 0.5w, \tag{1}$$

$$w - \left(-\frac{4}{u}\right) = \frac{3}{5}(u - (-4)). \tag{2}$$

Equivalently, after clearing factors,

$$3u - 20 = -\frac{12}{u} + 5w, \qquad w + \frac{4}{u} = \frac{3}{5}(u + 4).$$

Solving gives $u = 32/32 = 1$ (the positive value is required).

$$u = 1.$$

(8)

(b) Substituting $u = 1$ in either collision equation gives $w = -1$. The final velocity of Q is therefore $(-\mathbf{i} + 3\mathbf{j})$ m s^{-1}. Compare this direction with Q's initial direction $(-4\mathbf{i} + 3\mathbf{j})$.

$$\alpha = \tan^{-1}(3) - \tan^{-1}\left(\frac{3}{4}\right) = 34.695\ldots^\circ.$$

$$\alpha = 34.7^\circ \text{ (or } 35^\circ).$$

(5)

(c) Equal radii mean that, at contact, the line joining the centres is parallel to the horizontal surface; hence the impact impulse has no vertical component.

The equal radii justify treating the line of centres, and therefore the impulse, as horizontal.

(1)

Question 4

Question 4

A particle of mass 0.5 kg initially has velocity $8\mathbf{i}$ m s^{-1} and receives impulse $\lambda(-\mathbf{i}+\mathbf{j})$ N s, where $\lambda > 0$. Its final speed is $4\sqrt{10}$ m s^{-1}. Find (i) λ and (ii) the angle $\theta°$ through which its direction changes.

Model solution

(i) An impulse gives a change in momentum, so first find the final velocity vector.

$$0.5\mathbf{v} - 0.5(8\mathbf{i}) = \lambda(-\mathbf{i}+\mathbf{j}), \qquad \mathbf{v} = (8-2\lambda)\mathbf{i} + 2\lambda\mathbf{j}.$$

Use the given final speed to determine λ.

$$(8-2\lambda)^2 + (2\lambda)^2 = (4\sqrt{10})^2 = 160,$$

$$8\lambda^2 - 32\lambda - 96 = 0 \implies \lambda^2 - 4\lambda - 12 = 0 \implies (\lambda-6)(\lambda+2) = 0.$$

Since λ is positive, $\lambda = 6$.

$$\lambda = 6.$$

(ii) The final velocity is $-4\mathbf{i}+12\mathbf{j}$, which is in quadrant II. Its acute angle above the negative $\mathbf{i}$ direction is $\tan^{-1}(12/4) = \tan^{-1}(3)$.

$$\theta = 180° - \tan^{-1}(3) = 108.435\ldots°.$$

$$\theta = 108° \textit{ (3 s.f.).}$$

(8)

Question 5

Question 5

A small particle ball travels on a smooth horizontal floor, hitting perpendicular smooth vertical walls AB then BC. Immediately before AB it has speed v at angle θ to AB. The coefficients of restitution with AB and BC are $1/3$ and e respectively. It loses half its kinetic energy at each impact. (a) Find $\cos\theta$. (b) Find e.

Model solution

(a) Resolve into components parallel and perpendicular to AB. *A smooth wall leaves the parallel component unchanged and reverses and multiplies the perpendicular component by the coefficient of restitution. Thus, after the first impact, the components have magnitudes* $v\cos\theta$ *and* $\frac{1}{3}v\sin\theta$.

$$\frac{1}{2}m\left(v\cos\theta\right)^2+\frac{1}{2}m\left(\frac{1}{3}v\sin\theta\right)^2=\frac{1}{2}\left(\frac{1}{2}mv^2\right).$$

Cancelling $\frac{1}{2}mv^2$ *gives*

$$\cos^2\theta+\frac{1}{9}\sin^2\theta=\frac{1}{2}\quad\Longrightarrow\quad\frac{8}{9}\cos^2\theta=\frac{7}{18}.$$

The angle shown is acute, so take the positive root.

$$\cos\theta=\frac{\sqrt{7}}{4}.$$

(5)

(b) At BC, *the component parallel to* BC *is unchanged and the perpendicular component is multiplied by* e. *The speed components before that impact are* $\frac{1}{3}v\sin\theta$ *and* $v\cos\theta$, *so the final kinetic energy is*

$$\frac{1}{2}m\left[\left(\frac{1}{3}v\sin\theta\right)^2+\left(ev\cos\theta\right)^2\right].$$

It is half the kinetic energy remaining after the first impact:

$$\left(\frac{1}{3}\sin\theta\right)^2+e^2\cos^2\theta=\frac{1}{2}\left[\left(\frac{1}{3}\sin\theta\right)^2+\cos^2\theta\right].$$

With $\cos^2\theta=7/16$ *and* $\sin^2\theta=9/16$,

$$\frac{1}{16}+\frac{7}{16}e^2=\frac{1}{2}\left(\frac{1}{16}+\frac{7}{16}\right)=\frac{1}{4},\qquad e^2=\frac{9}{49}.$$

$$e=\frac{3}{7}.$$

(5)

Question 6

Question 6

A light elastic spring has natural length $3l$ and modulus $3mg$. It joins fixed point X on a rough plane to package P of mass m. The plane has $\tan\alpha = 3/4$. Initially P is at Y, a distance $XY = l$ up the line of greatest slope, and is released from rest and moves up the plane. The coefficient of friction is $1/3$. (a) Show that its initial acceleration is $17g/15$. (b) Find its speed when the spring first reaches length $3l$.

Model solution

(a) Initially the spring is compressed by $3l - l = 2l$, so it pushes P up the plane. Hooke's law for a spring with modulus λ is $T = \lambda x/l_0$.

$$T = \frac{3mg}{3l}(2l) = 2mg.$$

Take up the plane as positive. From $\tan\alpha = 3/4$, $\sin\alpha = 3/5$ and $\cos\alpha = 4/5$. Friction opposes the upward motion.

$$2mg - mg\sin\alpha - \frac{1}{3}mg\cos\alpha = ma,$$

$$2mg - \frac{3}{5}mg - \frac{1}{3}\left(\frac{4}{5}mg\right) = ma.$$

$$a = \frac{17}{15}g, \text{ as required.}$$

(5)

(b) Use energy over the $2l$ upward displacement to the natural length. The spring's initial elastic potential energy becomes kinetic energy, gravitational potential energy, and work done against friction.

$$\text{initial EPE} = \frac{(3mg)(2l)^2}{2(3l)} = 2mgl,$$

$$\Delta GPE = mg(2l)\sin\alpha = \frac{6}{5}mgl,$$

$$W_{\text{friction}} = \frac{1}{3}mg\cos\alpha(2l) = \frac{8}{15}mgl.$$

There is no elastic potential energy when the spring reaches its natural length, hence

$$\frac{1}{2}mv^2 + \frac{6}{5}mgl + \frac{8}{15}mgl = 2mgl.$$

$$v^2 = \frac{8}{15}gl.$$

$$v = \sqrt{\frac{8gl}{15}}.$$

(6)

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Question 7

Question 7

On a smooth horizontal floor, a 0.25 kg ball hits fixed smooth wall AB, whose direction is $\mathbf{i}+\mathbf{j}$. Its velocity before impact is $8\mathbf{i}+2\mathbf{j}$ m s^{-1}; the coefficient of restitution is $1/3$. (a) Show that its velocity after impact is $4\mathbf{i}+6\mathbf{j}$ m s^{-1}. (b) Find the magnitude of its impulse.

Model solution

(a) Resolve the initial velocity parallel and perpendicular to the wall. A unit vector parallel to the wall is $(\mathbf{i}+\mathbf{j})/\sqrt{2}$. The parallel scalar component is

$$(8\mathbf{i}+2\mathbf{j})\cdot\frac{\mathbf{i}+\mathbf{j}}{\sqrt{2}}=5\sqrt{2}.$$

This component is unchanged at a smooth wall. The perpendicular component is reversed and multiplied by $1/3$. The initial perpendicular vector is

$$(8\mathbf{i}+2\mathbf{j})-5(\mathbf{i}+\mathbf{j})=3\mathbf{i}-3\mathbf{j}.$$

Thus the final velocity is the unchanged parallel vector plus $-\frac{1}{3}$ times this perpendicular vector.

$$\mathbf{v}=5\mathbf{i}+5\mathbf{j}-\frac{1}{3}(3\mathbf{i}-3\mathbf{j})=4\mathbf{i}+6\mathbf{j}.$$

$$\mathbf{v}=4\mathbf{i}+6\mathbf{j}\ \textit{m s}^{-1}\textit{, as required.}$$

(6)

(b) Impulse equals change in momentum.

$$\mathbf{I}=0.25\big[(4\mathbf{i}+6\mathbf{j})-(8\mathbf{i}+2\mathbf{j})\big]=-\mathbf{i}+\mathbf{j}\ \textit{N s}.$$

$$|\mathbf{I}|=\sqrt{(-1)^2+1^2}.$$

$$|\mathbf{I}|=\sqrt{2}\ \textit{N s}.$$

(3)