Maths with Matt

Physics and maths resources

Edexcel A Level Further Mathematics

9FM0/3D - Decision Mathematics 1

Monday 24 June 2019

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2019 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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Question 1

Question 1

For the list $2.1, 1.7, 3.0, 1.9, 3.2, 1.2, 3.3, 1.4, 1.5, 0.2$, use first-fit packing in bins of size 5, quick-sort into descending order, and estimate the running time when $n = 11250$, given 2.32 seconds when $n = 450$.

Model solution

(a) First-fit considers the list in its given order and places each value in the first bin which has enough remaining capacity.

1	$2.1, 1.7, 1.2$	5.0
2	$3.0, 1.9$	4.9
3	$3.2, 1.4, 0.2$	4.8
4	$3.3, 1.5$	4.8

Four bins, with contents as shown.

(2) (b) For descending quick sort, values larger than the pivot go left. Choosing middle-right pivots gives these passes.

$$\begin{aligned}
&2.1, 1.7, 3.0, 1.9, 3.2, |1.2|, 3.3, 1.4, 1.5, 0.2 \\
\to\ &2.1, 1.7, 3.0, 1.9, 3.2, 3.3, 1.4, 1.5, 1.2, 0.2 && (|3.2|, |0.2|) \\
\to\ &3.3, 3.2, 2.1, 1.7, 3.0, 1.9, 1.4, 1.5, 1.2, 0.2 && (|3.3|, |1.9|) \\
\to\ &3.3, 3.2, 2.1, 3.0, 1.9, 1.7, 1.4, 1.5, 1.2, 0.2 && (|3.0|, |1.4|) \\
\to\ &3.3, 3.2, 3.0, 2.1, 1.9, 1.7, 1.5, 1.4, 1.2, 0.2 && (|2.1|, |1.5|).
\end{aligned}$$

$3.3, 3.2, 3.0, 2.1, 1.9, 1.7, 1.5, 1.4, 1.2, 0.2.$

(4) (c) An $n \log n$ running time is proportional to $n \log n$, so take a ratio.

$$t = 2.32 \frac{11250 \log 11250}{450 \log 450} = 88.6\ldots$$

88.6 *s.*

(2)

Question 2

Question 2

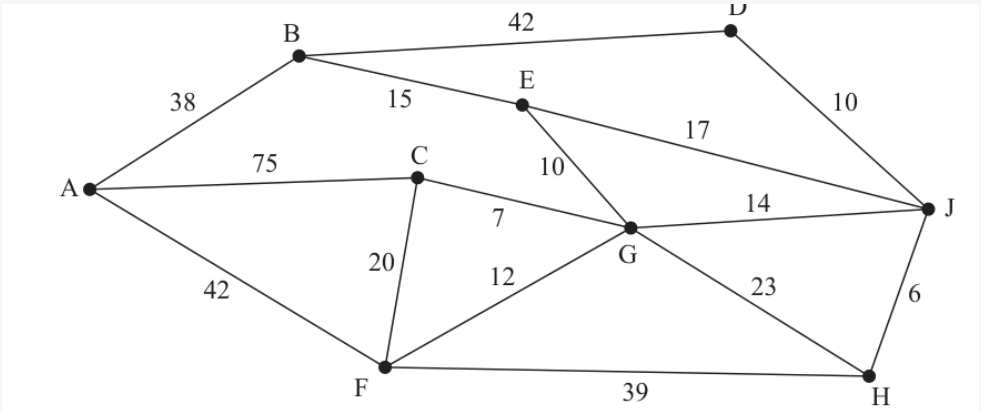


Figure 1

[*The total weight of the network is* 370]

gure 1 represents a network of corridors in a building. The number on each arc presents the length, in metres, of the corresponding corridor.

Corridor lengths are in metres. Find the shortest A-to-D path and the minimum routes which traverse all corridors, first from B to G, then after removing every arc incident with B.

Model solution

(a) Dijkstra's algorithm is appropriate because all weights are positive. The permanent labels and improvements are:

A	0	$B = 38, F = 42, C = 75$
B	38	$E = 53, D = 80$
F	42	$C = 62, G = 54, H = 81$
E	53	$J = 70$
G	54	$C = 61, J = 68, H = 77$
C	61	
J	68	$D = 78, H = 74$
H	74	
D	78	

Backtracking predecessors gives $D \leftarrow J \leftarrow G \leftarrow F \leftarrow A$.

$$A - F - G - J - D, \text{length} 78 \ m.$$

(6) (b) The original odd vertices are A, C, E, H. *For an open route from* B *to* G, *pair those vertices using minimum shortest paths.*

$A(FG)C + E(J)H$	$61 + 23 = 84$
$A(B)E + C(GJ)H$	$53 + 27 = 80$
$A(FGJ)H + C(G)E$	$74 + 17 = 91$

$$\text{Repeat} AB, BE, CG, GJ, JH.$$

(4) (c) Add the minimum duplication to the total original network weight.

$$370 + 80 = 450.$$

450 *m.*

(1) (d) After B *is removed,* D, C, G, H *are odd. The shortest connection among them is* $CG = 7$*, so duplicate this arc and leave* D, H *as endpoints.*

$$370 - 38 - 42 - 15 + 7 = 282.$$

StartD, finishH, orviceversa; minimumlength282 *m.*

(3)

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Question 3

Question 3

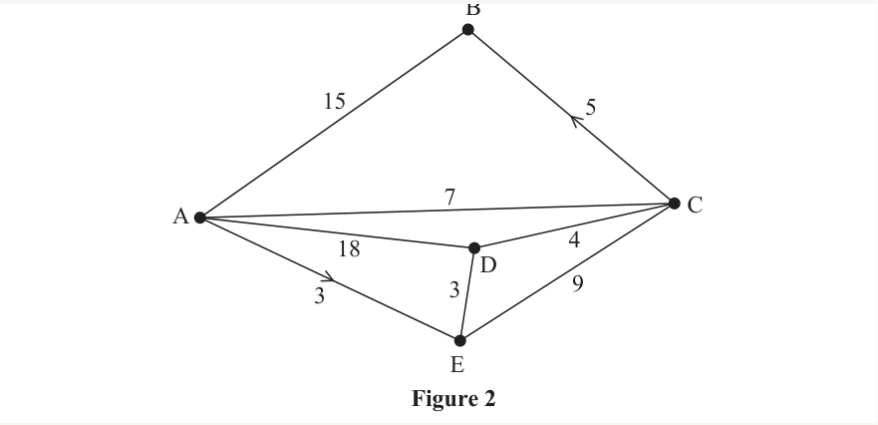


Figure 2

Use Floyd's algorithm; then recover the E-to-A route and apply nearest neighbour from D.

Model solution

(a) Initially use direct-road distances, ∞ if no direct road exists, and put the destination in every route-table cell.

$$D^{(0)} = \begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & - & 15 & 7 & 18 & 3 \\ B & 15 & - & \infty & \infty & \infty \\ C & 7 & 5 & - & 4 & 9 \\ D & 18 & \infty & 4 & - & 3 \\ E & \infty & \infty & 9 & 3 & - \end{array} \qquad R^{(0)} = \begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & A & B & C & D & E \\ B & A & B & C & D & E \\ C & A & B & C & D & E \\ D & A & B & C & D & E \\ E & A & B & C & D & E \end{array}.$$

(2) (b) At each iteration compare a current distance with the distance through the iteration vertex; change the route entry to the first leg when shorter.

k		A	B	C	D	E
A	A	-	15	7	18	3
	B	15	-	22	33	18
	C	7	5	-	4	9
	D	18	33	4	-	3
	E	∞	∞	9	3	-
B	A	-	15	7	18	3
	B	15	-	22	33	18
	C	7	5	-	4	9
	D	18	33	4	-	3
	E	∞	∞	9	3	-
C	A	-	12	7	11	3
	B	15	-	22	26	18
	C	7	5	-	4	9
	D	11	9	4	-	3
	E	16	14	9	3	-

$$R^{(A)} = R^{(B)} = \begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & A & B & C & D & E \\ B & A & B & A & A & A \\ C & A & B & C & D & E \\ D & A & A & C & D & E \\ E & A & B & C & D & E \end{array}, \quad R^{(C)} = \begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & A & C & C & C & E \\ B & A & B & A & C & A \\ C & A & B & C & D & E \\ D & C & C & C & D & E \\ E & C & C & C & D & E \end{array}.$$

***(5)** (c) Final-table entry* EA *is* D*, then* DA *is* C*, then* CA *is* A*.*

$$E - D - C - A.$$

***(3)** (d) Nearest neighbour gives* $D - E - C - B - A - D$*, with table distance*

$$3 + 7 + 5 + 15 + 6 = 36.$$

Expanding actual roads gives $D - E - D - C - B - A - E - D$*. The genuine cycle* $A - E - D - C - B - A$ *has length* $3 + 3 + 4 + 5 + 15 = 30 < 36$*.*

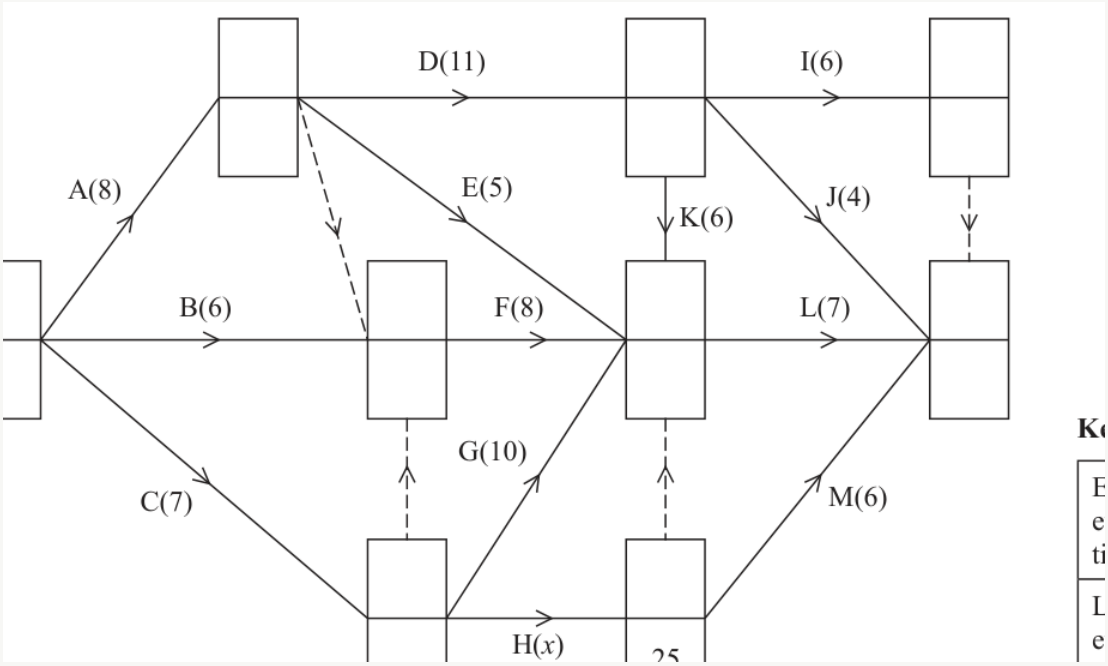
TheNNAcyclehaslength36milesandisnotoptimal.

(4)

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Question 4

Question 4



Activity H has total float 7. Complete the project analysis and a shortest-time worker schedule.

Model solution

(a) Total float is late event time at the finish minus early event time at the start minus duration. The start of H *has early time* 7*, and its finish has late time* 25*.*

$$25 - 7 - x = 7 \Rightarrow x = 11.$$

$$x = 11.$$

(2) (b) Forward-pass maxima and backward-pass minima give these event labels (*early, late*)*:*

$$(0,0), (8,8), (7,17), (19,19), (18,25), (25,25), (25,32), (32,32).$$

$$\text{Project duration 32 days.}$$

(3) (c) The total activity time is 95 *worker-days, so the worker lower bound is*

$$95/32 = 2.96875\ldots$$

$$3 \text{ workers.}$$

(1) (d) The bound cannot be attained under these precedence constraints. One valid four-worker schedule is

1	$A(0-8), D(8-19), I(19-25), L(25-32)$
2	$B(0-6), F(8-16), K(19-25)$
3	$C(0-7), G(7-17), J(19-23)$
4	$E(8-13), H(13-24), M(24-30)$

Fourworkerscompletetheprojectatday32.

(3)

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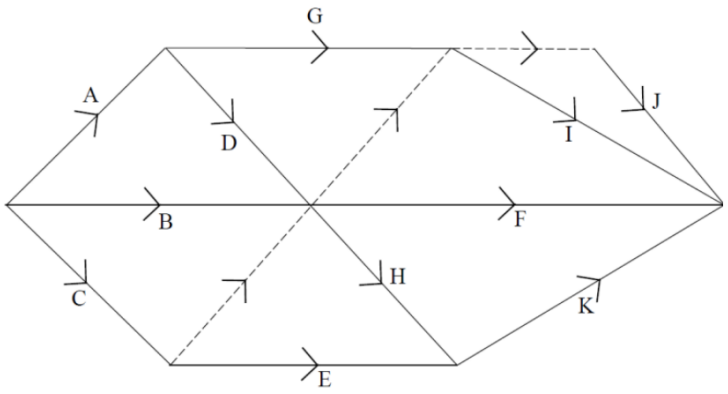
Question 5

Question 5

Draw a minimum-dummy activity-on-arc network for the stated precedence table, then find the critical path when all activities have equal duration.

Model solution

(a) An activity-on-arc diagram places activities on arrows. Dashed arrows are zero-duration dummies, used only where a predecessor relationship has to be enforced. This is a minimum-dummy network.



Critical path: ADHK

The two dummies into the B, C, D *join make* F, H *wait for all three; the remaining dummy makes* I, J *wait for* B, C, D, G. *(5) (b) Since every activity has the same duration, the critical path is the path containing the greatest number of activity arcs.*

ADHK.

(1)

Question 6

Question 6

Maximise $P = 2x + 2y - z$, subject to $3x + y + 2z \leqslant 30$, $x - y + z \geqslant 8$, $4y + 2z \geqslant 15$, and $x, y, z \geqslant 0$. Use the big-M method where necessary.

Model solution

(a) Ordinary Simplex needs $\leqslant$ constraints so that slack variables provide a basic feasible starting solution. This problem contains $\geqslant$ constraints.

Simplex cannot be used directly.

(1) (b) Add a slack to the first constraint; subtract a surplus and add an artificial variable to each $\geqslant$ constraint.

$$3x + y + 2z + s_1 = 30, \qquad x - y + z - s_2 + a_1 = 8, \qquad 4y + 2z - s_3 + a_2 = 15.$$

Penalise artificial variables in the maximisation objective. Substituting for a_1, a_2 gives

$$P = 2x + 2y - z - Ma_1 - Ma_2 = (2+M)x + (2+3M)y + (-1+3M)z - Ms_2 - Ms_3 - 23M.$$

b.v.	x	y	z	s_1	s_2	s_3	a_1	a_2	*Value*
s_1	3	1	2	1	0	0	0	0	30
a_1	1	-1	1	0	-1	0	1	0	8
a_2	0	4	2	0	0	-1	0	1	15
P	$-(2+M)$	$-(2+3M)$	$-(-1+3M)$	0	M	M	0	0	$-23M$

(7) (c) Non-basic variables are zero. Read the basic values from the supplied first-iteration tableau.

$$s_1 = 26.25, \ a_1 = 11.75, \ y = 3.75, \text{and} x = z = s_2 = s_3 = a_2 = 0.$$

(1) (d) Artificial variables must be zero in a feasible solution of the original problem.

$$\text{Thesolutionisinfeasiblebecause} a_1 = 11.75 \neq 0.$$

(1) (e) The most negative profit-row entry, $2 - 1.5M$, identifies the z-column. The ratio test uses positive entries only:

$$26.25/1.5 = 17.5, \qquad 11.75/1.5 = 7.833\ldots, \qquad 3.75/0.5 = 7.5.$$

$$\text{Pivoton} 0.5, \text{inthe} y - \text{rowandz} - \text{column}.$$

(2)

Question 7

Question 7

Let x be analogue watches and y digital watches. At least 60 watches are ordered, at most 80% are digital, $y + 3x \geqslant 140$, and $4y + x \geqslant 80$. An analogue watch costs five times a digital watch.

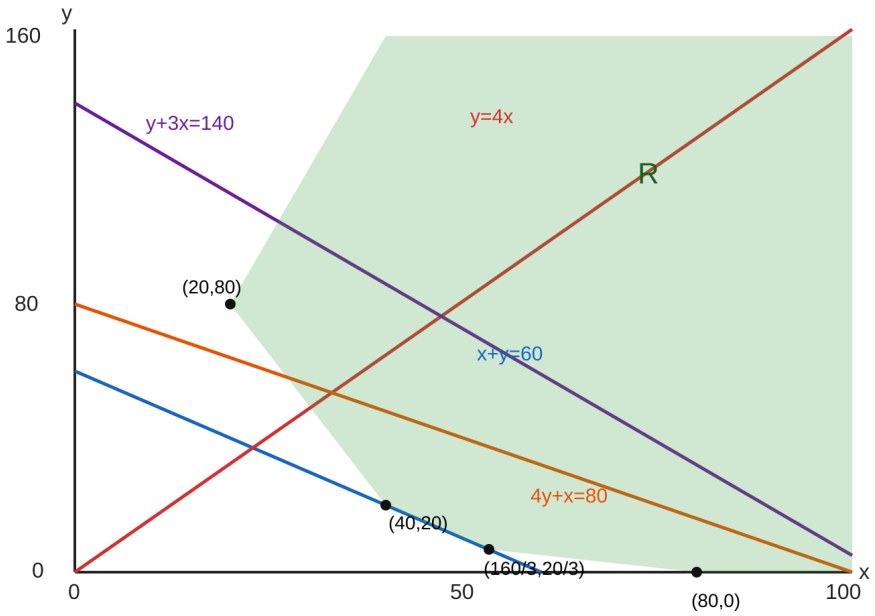
Model solution

(a) "At least" gives a lower bound on the total. Digital watches are no more than four fifths of the total.

$$x + y \geqslant 60, \qquad y \leqslant \frac{4}{5}(x + y), \quad \textit{equivalently } y \leqslant 4x.$$

(2) (b) Draw the boundary lines from these points and retain the stated sides.

$x + y = 60$	$(0, 60), (60, 0)$	*above*
$y = 4x$	$(0, 0), (25, 100)$	*below*
$y + 3x = 140$	$(0, 140), (140/3, 0)$	*above*
$4y + x = 80$	$(0, 20), (80, 0)$	*above*



Their intersection is feasible region R, *with vertices* $(20, 80), (40, 20), (80, 0), (160/3, 20/3)$.
(4) (c) If a digital watch costs d, *an analogue costs* $5d$, *so minimising cost is equivalent to minimising* $5x + y$. *A linear objective is optimised at a vertex.*

(x, y)	$(20, 80)$	$(40, 20)$	$(80, 0)$	$(160/3, 20/3)$
$5x + y$	180	220	400	$820/3$

Order20analoguewatchesand80digitalwatches.

(3) (d) Let the unit costs be a, d. *Use the chosen quantities, the stated total, and the five-to-one relationship.*

$$20a + 80d = 4455, \qquad a = 5d$$

$$20(5d) + 80d = 4455 \Rightarrow 180d = 4455 \Rightarrow d = 24.75, \qquad a = 123.75.$$

An analogue watch costs \$123.75 and a digital watch costs \$24.75.

(3)

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