Maths with Matt

Physics and maths resources

Edexcel A Level Further Mathematics

9FM0/3D - Decision Mathematics 1

Friday 24 June 2022

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2022 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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Question 1

Question 1

String lengths (m) are $4.3, 6.1, 5.1, 4.7, 2.5, 5.9, 3.4, 1.7, 2.1, 0.4, 1.3$. Each ball has 10 m.

Model solution

(a) Use the total length to give a lower bound: even with no waste, fewer balls cannot hold the total.

$$4.3 + 6.1 + 5.1 + 4.7 + 2.5 + 5.9 + 3.4 + 1.7 + 2.1 + 0.4 + 1.3 = 37.5, \qquad 37.5/10 = 3.75.$$

The lower bound is 4 balls.

(2) (b) First-fit places each length, in the given order, in the first ball with enough space.

1	4.3, 5.1, 0.4	9.8
2	6.1, 2.5, 1.3	9.9
3	4.7, 3.4, 1.7	9.8
4	5.9, 2.1	8.0

The first-fit packing uses four balls, as displayed.

(3)

Question 2

Question 2

Use the cycle-track network in Figure 1 for shortest-path and route-inspection questions.

Model solution

(a) Dijkstra's algorithm applies because every edge has positive length. The working values are recorded in the order in which they are obtained; the least current value is made permanent each time.

vertex	A	B	E	D	C	F	H	G	J	K
working values	0	8	12	14	18, 17, 15	29, 28, 25	31, 28	43, 39, 35, 34	51, 44	61, 59
permanent label	0	8	12	14	15	25	28	34	44	59
predecessor	–	A	B	A	E	C	E	H	G	G

Thus the permanent-label sequence is A, B, E, D, C, F, H, G, J, K. *Tracing predecessors from* K *gives* A – B – E – H – G – K.

(i) A – B – E – H – G – K. *(ii)* 59 *km.*

(6) (b) An open inspection route from D *to* E *must leave just those two vertices odd. Pair the remaining odd vertices by repeating* AB, BE, EH.

$$8 + 4 + 16 = 28, \qquad 299 + 28 = 327.$$

(i) Repeat AB, BE, EH. *(ii)* 327 *km.*

(2) (c) Starting at A *and ending at* K, *the relevant vertices to pair are* D, E, H, K. *Compare all three pairings using shortest paths.*

D(AB)E + H(G)K	26 + 31 = 57
D(FG)H + E(HG)K	30 + 47 = 77
D(FG)K + EH	49 + 16 = 65

The least additional distance is 57.

Minimum route length $= 299 + 57 = 356$ *km.*

(5)

Question 3

Question 3

Use Floyd's algorithm for the directed classroom network, then nearest neighbour from E.

Model solution

(a) A finite entry in row A of the initial matrix represents an arc leaving A.

$$A \to B : 12, \quad A \to C : 32, \quad A \to D : 24, \quad A \to E : 29, \quad A \to F : 11.$$

Add these five directed weighted arcs from A.

(2) (b) The next Floyd iterations allow routes via C, then via D; each entry is compared with the corresponding route through that vertex.

	A	B	C	D	E	F
A	–	12	29	20	29	11
B	12	–	17	8	29	23
C	29	17	–	4	12	40
D	24	21	4	–	16	13
E	41	29	12	16	–	12
F	11	23	40	13	12	–

	A	B	C	D	E	F
A	–	12	24	20	29	11
B	12	–	12	8	24	21
C	28	17	–	4	12	17
D	24	21	4	–	16	13
E	40	29	12	16	–	12
F	11	23	17	13	12	–

Left: third iteration; right: fourth iteration.

(4) (c) Nearest neighbour always chooses the shortest arc to an unvisited vertex.

$$E - C - D - F - A - B - E : 12 + 4 + 13 + 11 + 12 + 24 = 76,$$

$$E - F - A - B - D - C - E : 12 + 11 + 12 + 8 + 4 + 12 = 59.$$

The cycles have lengths 76 m and 59 m; E – F – A – B – D – C – E gives the better upper bound as it is shorter.

(4)

Question 4

Question 4

Interpret the initial big-M tableau and choose the first pivot.

Model solution

(a) Move the slack/artificial terms in each constraint row to obtain inequalities; the objective row supplies the objectives.

$$2x + 3y + 4z \leqslant 13, \quad x - 2y + 2z \geqslant 8, \quad 3x - 4z \geqslant 12, \quad x, y, z \geqslant 0.$$

Maximise $P = -2x + 3y + z$*, or minimise* $P = 2x - 3y - z$*.*

(4) (b) Since M *is large,* $2 - 4M$ *is the only negative objective-row entry, so* x *enters. The positive ratio test is*

$$13/2 = 6.5, \qquad 8/1 = 8, \qquad 12/3 = 4.$$

The pivot is 3*, in the* a_2*-row and* x*-column.*

(2)

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Question 5

Question 5

Analyse the activity network, its timing, resources and Gantt chart.

Model solution

(a) Immediately preceding activities are those that must be complete before the named activity can start.

activity	A	B	C	D	E	F	G	H	J	K
IPA	–	–	–	A	C	C	A, B, E	A, B, E	D, G	D, F, G, H

The IPA table is as displayed.

(2) (b) The event after D *has early time* 14*, set by the route through* G*. To ensure that* D *is not forced to be critical, it must arrive strictly before that event:* $4 + x < 14$*.*

$$x < 10.$$

The largest possible integer $m = 9$*.*

(1) (c) Work backward from the finish time 22*, taking the smaller late time at each merge.*

event	*start*	*after* A	*after* C	*middle*	*top right*	*bottom right*	*finish*
early	0	4	7	11	14	16	22
late	0	$15 - x$	7	11	15	16	22

An activity is critical precisely when early and late timings coincide.

Critical activities: C, E, H, K.

(3) (d) Total float is late finish minus early start minus duration.

$$15 - 11 - 3 = 1.$$

Total float of G *is* 1 *day.*

(1) (e) Read each activity's contribution from the resource histogram.

activity	A	B	C	D	E	F	G	H	J	K
workers	3	2	1	2	2	1	2	2	1	3

Worker requirements are as displayed.

(2) (f) A Gantt chart starts each bar at its early event time and shows float after the activity. The critical path gives $x = 6$.

A : 0~~4~~ (*float* 4)	B : 0~~5~~ (5)	C : 0~~7~~ (0)
D : 4~~1~~0 (6)	E : 7~~1~~1 (0)	F : 7~~1~~3 (3)
G : 11~~1~~4 (1)	H : 11~~1~~6 (0)	J : 14~~2~~1 (1)
K : 16~~2~~2 (0)		

Draw these bars on the time grid and shade the stated float; $x = 6$.

(5)

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Question 6

Question 6

Trace the Prim-comparison algorithm and use bubble sort.

Model solution

(a) For K_5, begin with $a = 1, b = 3, c = 3$, then repeat the update while $b > 0$.

n	a	b	c
5	1	3	3
	2	2	8
	3	1	13
	4	0	16

Output $= 16$.

(3) (b) Each bubble-sort pass moves the largest remaining entry rightwards.

1	17, 21, 14, 23, 13, 15, 19, 24, 20, 28
2	17, 14, 21, 13, 15, 19, 23, 20, 24, 28
3	14, 17, 13, 15, 19, 21, 20, 23, 24, 28
4	14, 13, 15, 17, 19, 20, 21, 23, 24, 28
5	13, 14, 15, 17, 19, 20, 21, 23, 24, 28
6	13, 14, 15, 17, 19, 20, 21, 23, 24, 28

There are $9 + 8 + 7 + 6 + 5 + 4 = 39$ *comparisons.*

(5) (c) There are ten values, so the worst-case count is $9 + 8 + \cdots + 1$.

$$\frac{9(10)}{2} = 45.$$

(1) (d) Use part (c) to determine the constant.

$$45 = \lambda(5)(4)(6)(3), \quad \lambda = \frac{1}{8}.$$

$$\tfrac{1}{8}(50)(49)(51)(48) = 749\,700 \text{ comparisons.}$$

(3)

Question 7

Question 7

Set up and complete the simplex method for $P = x + ky$, $k > 0$.

Model solution

(a) The feasible side of each boundary is the side containing R. Add a non-negative slack variable to each $\leqslant$ constraint.

$$x+y+s_1 = 8, \quad x+2y+s_2 = 12, \quad 7x+2y+s_3 = 46, \quad -2x+y+s_4 = 1, \quad P-x-ky = 0.$$

	x	y	s_1	s_2	s_3	s_4	*Value*
s_1	1	1	1	0	0	0	8
s_2	1	2	0	1	0	0	12
s_3	7	2	0	0	1	0	46
s_4	-2	1	0	0	0	1	1
P	-1	$-k$	0	0	0	0	0

Initial tableau as displayed.

(5) (b) In tableau T, basic variables take their row values and non-basic variables are zero.

$x = 2, y = 5, s_1 = 1, s_2 = 0, s_3 = 22, s_4 = 0$, *and* $P = 5k + 2$.

(1) (c) The s_2-column enters. Its pivot is the $-\frac{3}{5}$ entry in row 1. Using R_1 for that row, perform $R_1 \leftarrow 5R_1$, then $R_2 \leftarrow R_2 + 0.4R_1$, $R_3 \leftarrow R_3 - 2.4R_1$, $R_4 \leftarrow R_4 - 0.2R_1$, and $R_P \leftarrow R_P - (0.2k - 0.4)R_1$.

	x	y	s_1	s_2	s_3	s_4	*Value*
s_4	0	0	5	-3	0	1	5
x	1	0	2	-1	0	0	4
s_3	0	0	-12	5	1	0	10
y	0	1	-1	1	0	0	4
P	0	0	$2-k$	$k-1$	0	0	$4k+4$

The new basic solution is $(x, y) = (4, 4)$, so $P = 4k + 4$. For the second tableau not to be optimal, $-\frac{2}{5} + \frac{1}{5}k < 0$, giving $k < 2$. For this third tableau to be optimal, $2 - k \geqslant 0$ and $k - 1 \geqslant 0$, giving $k \geqslant 1$. Therefore

$$1 \leqslant k < 2 \quad \Longrightarrow \quad 8 \leqslant P < 12.$$

The third tableau is as displayed; $(x, y) = (4, 4)$ and $8 \leqslant P < 12$.

(9)