Maths with Matt

Physics and maths resources

# Edexcel A Level Further Mathematics

## 9FM0/3D - Decision Mathematics 1

Friday 23 June 2023

## *Questions and Fully Worked Model Solutions*

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2023 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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# Question 1

## Question 1

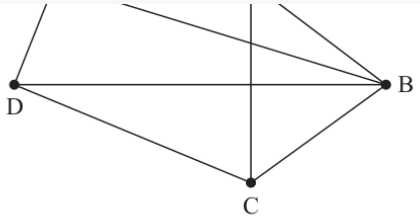



**Figure 1**

For the graph G in Figure 1:
(a) state whether G is Eulerian, semi-Eulerian or neither, giving a reason.
(b) write down a Hamiltonian cycle.
(c) state whether G is planar, justifying the answer.
(d) state how many arcs must be added to make G into $K_5$.

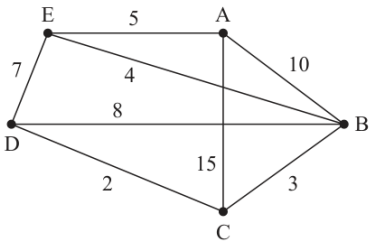



**Figure 2**

irect roads between five villages, A, B, C, D and E, are represented in Figure 2. The eight on each arc is the time, in minutes, required to travel along the corresponding ad. Floyd's algorithm is to be used to find the complete network of shortest times :tween the five villages.

) For the network represented in Figure 2, complete the initial time matrix in the answer book.

ıe time matrix after four iterations of Floyd's algorithm is shown in Table 1.

(e) Complete the initial time matrix for Figure 2. (f) The matrix after four iterations of Floyd's algorithm is given in the question. Perform the final iteration, showing the resulting matrix.

## *Model solution*

*(a) For an Euler trail or circuit, the useful clue is the number of odd-degree vertices. A graph has an Euler circuit only when all degrees are even, and an Euler trail only when exactly two are odd. Vertices A, C, D, E have odd degree (four odd vertices). Hence neither condition is met.*

*G is neither Eulerian nor semi-Eulerian.*

*(1)*

*(b) A Hamiltonian cycle visits every vertex exactly once before returning to its start.*

*A – B – C – D – E – A is a Hamiltonian cycle.*

*(1)*

*(c) Planarity concerns crossings in a drawing, rather than the particular shape of the printed diagram.*

*G is planar: move arc AC outside the graph, leaving no crossings.*

***(1)***

*(d) $K_5$ has an arc joining every pair of its five vertices. Comparing the existing arcs with this complete graph leaves two missing arcs.*

*2 arcs must be added.*

***(1)***

*(e) In the initial matrix, enter $0$ (or a dash) on the diagonal, the direct-road time when an arc exists, and $\infty$ when there is no direct road.*

| | A | B | C | D | E |
|---|---|---|---|---|---|
| A | – | 10 | 15 | $\infty$ | 5 |
| B | 10 | – | 3 | 8 | 4 |
| C | 15 | 3 | – | 2 | $\infty$ |
| D | $\infty$ | 8 | 2 | – | 7 |
| E | 5 | 4 | $\infty$ | 7 | – |

***(1)***

*(f) The final iteration uses E as the possible intermediate village. For each entry compare its current value with "via E". Only the improvements need to change.*

| | A | B | C | D | E |
|---|---|---|---|---|---|
| A | – | [9] | [12] | [12] | 5 |
| B | [9] | – | 3 | 5 | 4 |
| C | [12] | 3 | – | 2 | 7 |
| D | [12] | 5 | 2 | – | 7 |
| E | 5 | 4 | 7 | 7 | – |

*For example, A to B improves from $10$ to $5+4=9$, and A to C improves to $5+4+3=12$.*
***(2)***

# Question 2

## Question 2

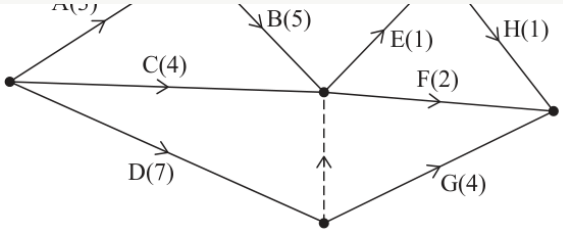



**Figure 3**

The network in Figure 3 shows the activities that need to be undertaken to complete a project. Each activity is represented by an arc and the duration, in hours, of the corresponding activity is shown in brackets.

(a) (i) Complete Diagram 1 in the answer book to show the early event times and the late event times

(a) Find the early and late event times and the minimum project completion time. (b) All activities start as early as possible. Draw the resource histogram from the stated worker requirements, and state when six workers are needed.

## *Model solution*

*(a) A forward pass gives earliest event times; a backward pass from the finish gives latest event times. At a merge take a maximum on the forward pass, and at a split take a minimum on the backward pass.*

| *event* | *start* | *after* A | *after* D | *central* | *after* E | *finish* |
|---|---|---|---|---|---|---|
| *early time* | 0 | 3 | 7 | 8 | 9 | 11 |
| *late time* | 0 | 4 | 7 | 9 | 10 | 11 |

*The finish event has early time* 11*, so this is the shortest possible completion time.*

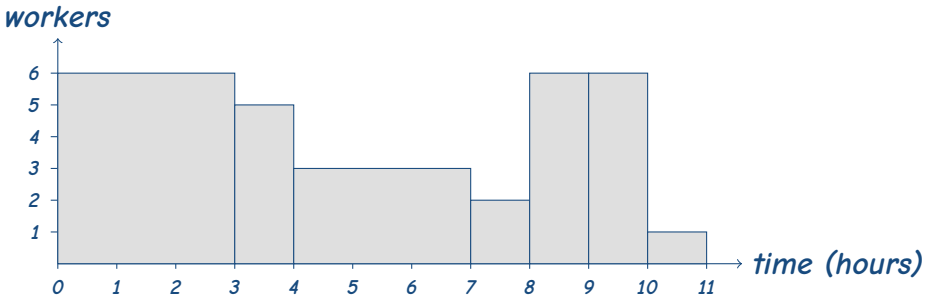
*Minimum completion time* $= 11$ *hours.*

*(3)*

*(b) Use each activity's earliest start and its duration to place its workers on a time line, then add the overlaps.*

| *time interval* | *activities in progress* | *workers* |
|---|---|---|
| 0--3 | A, C, D | $2+2+2=6$ |
| 3--4 | B, C, D | $1+2+2=5$ |
| 4--7 | B, D | $1+2=3$ |
| 7--8 | B, G | $1+1=2$ |
| 8--9 | E, F, G | $3+2+1=6$ |
| 9--10 | F, G, H | $2+1+3=6$ |
| 10--11 | G | 1 |

*The required resource histogram is:*




*Six workers are required during* 0--3 *and* 8--10 *hours.*

***(4)***

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# Question 3

## Question 3

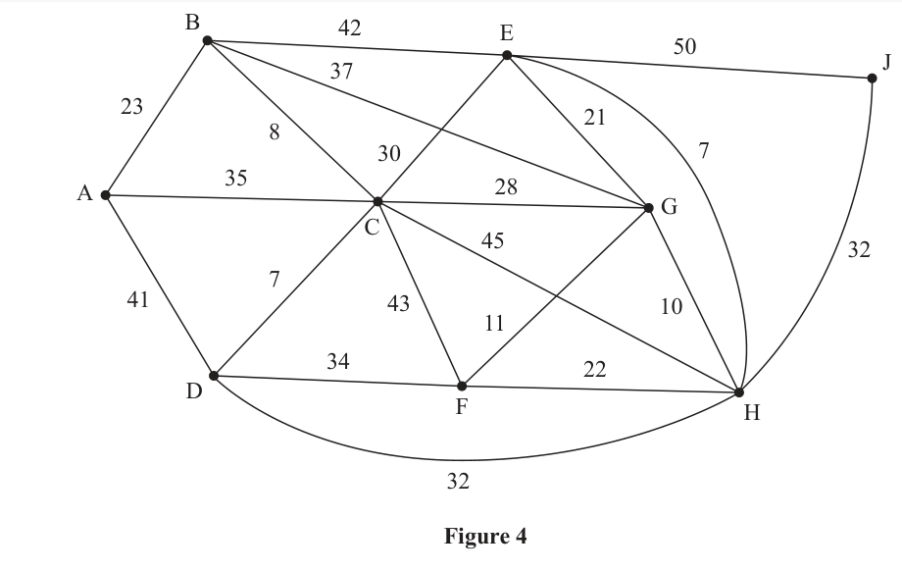



**Figure 4**

(a) Use Dijkstra's algorithm to find the shortest path from A to J and state its length. (b) Dijkstra's algorithm has order $n^2$. A 9-node network takes 0.0312 seconds; estimate the time, in minutes, for 9000 nodes.

## *Model solution*

*(a) All edge lengths are positive, so Dijkstra's algorithm is appropriate. Permanently label the unvisited node with the smallest tentative distance, then relax the arcs from it.*

| *permanent node* | *distance from* A | *new or improved tentative labels* |
|---|---|---|
| A | 0 | $B = 23,\ C = 35,\ D = 41$ |
| B | 23 | $C = 31,\ E = 65,\ G = 60$ |
| C | 31 | $D = 38,\ E = 61,\ F = 74,\ G = 59,\ H = 76$ |
| D | 38 | $F = 72,\ H = 70$ |
| G | 59 | $F = 70,\ H = 69$ |
| E | 61 | $H = 68,\ J = 111$ |
| H | 68 | $J = 100$ |
| F | 70 | *none* |
| J | 100 | *stop* |

*Tracing back the labels which created* $J = 100$ *gives*

$$A \longrightarrow B \longrightarrow C \longrightarrow E \longrightarrow H \longrightarrow J.$$

*Shortest path* ABCEHJ, *with length* 100.

*(6)*

*(b) Order* $n^2$ *means time is multiplied by the square of the ratio of node numbers.*

$$0.0312\left(\frac{9000}{9}\right)^2 = 31200 \text{ seconds} = \frac{31200}{60} = 520 \text{ minutes.}$$

*Approximately* 520 *minutes.*

*(2)*

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# Question 4

## Question 4

The distinct numbers $15, 22, 3, 9, 23, x, 5, 4, 18, 20, 13$ are packed into bins of size 40. Here $x$ is an integer below 40 and is the largest number. (a) Explain why three bins are impossible. (b) Given four bins are possible, find the range of $x$. (c) Apply first fit in the listed order. (d) Quick-sort the list into descending order, showing passes and pivots. (e) Given first-fit decreasing puts neither 15 nor 13 in bin 1, find $x$.

## *Model solution*

*(a) Compare the total size with the total capacity of three bins. The known numbers total* $132$*, so the total is* $132 + x > 120 = 3 \times 40$*.*

*Three bins cannot hold the numbers.*

*(1)*

*(b) Since* $x$ *is the largest listed number,* $x > 23$*. Four bins have capacity* $160$*, so* $132 + x \leqslant 160$*.*

$$x \leqslant 28.$$

$23 < x \leqslant 28$, *so* $x \in \{24, 25, 26, 27, 28\}$.

*(2)*

*(c) First fit checks bins in opening order and places each number in the first bin with enough remaining space.*

| *bin* | *contents* | *total* |
|---|---|---|
| 1 | $15, 22, 3$ | 40 |
| 2 | $9, 23, 5$ | 37 |
| 3 | $x, 4$ | $x + 4$ |
| 4 | $18, 20$ | 38 |
| 5 | $13$ | 13 |

*(3)*

*(d) Use the middle-left item as each pivot, recursively partitioning into values greater and less than the pivot because the required order is descending. The six passes are:*

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *start:* | 15 | 22 | 3 | 9 | 23 | $\boxed{x}$ | 5 | 4 | 18 | 20 | 13 |
| *pass 1:* | $\boxed{x}$ | 15 | 22 | 3 | 9 | 23 | 5 | 4 | 18 | 20 | 13 |
| *pass 2:* | $x$ | $\boxed{23}$ | 15 | 22 | 3 | 9 | 5 | 4 | 18 | 20 | 13 |
| *pass 3:* | $x$ | 23 | 15 | 22 | 9 | 18 | 20 | 13 | $\boxed{5}$ | 3 | 4 |
| *pass 4:* | $x$ | 23 | 22 | 20 | $\boxed{18}$ | 15 | 9 | 13 | 5 | 4 | 3 |
| *pass 5:* | $x$ | 23 | 22 | 20 | 18 | 15 | $\boxed{13}$ | 9 | 5 | 4 | 3 |
| *pass 6:* | $x$ | 23 | 22 | 20 | 18 | 15 | 13 | 9 | 5 | 4 | 3 |

$x, 23, 22, 20, 18, 15, 13, 9, 5, 4, 3.$

*(4)*

*(e) In first-fit decreasing, $x$ is placed in bin 1 first. The next bin begins with $23, 15$. If $15$ cannot fit in bin 1 then $x + 15 > 40$, so $x > 25$. Similarly, for $13$ not to fit in bin 1, $x + 13 > 40$, so $x > 27$. Together with $x \leqslant 28$ and integer $x$, this leaves only*

$$x = 28.$$

*(2)*

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# Question 5

## Question 5

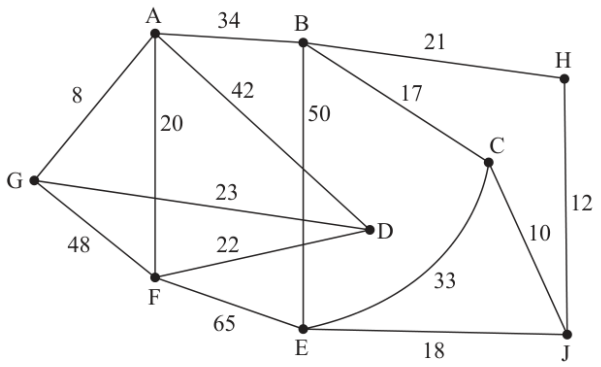



**Figure 5**

[*The total weight of the network is* 423]

Direct roads between nine towns, A, B, C, D, E, F, G, H and J, are represented in Figure 5. The number on each arc represents the length, in miles, of the corresponding road.

The table below shows the shortest distances, in miles, between the nine towns

The total network weight is 423 miles and the table of shortest distances is given in the question. (a) Find the route-inspection route which starts at F and finishes at J. (b) Starting at A, use Prim's algorithm on the shortest-distance table. (c) Starting at G, apply nearest neighbour for a travelling-salesperson upper bound. (d) Delete G and find a lower bound. (e) Interpret the route from (c) in terms of actual towns visited.

## *Model solution*

*(a) An open route from* F *to* J *must have exactly* F *and* J *odd. The original odd vertices are* C, D, F, G*; therefore pair* F *and* J *with two of these so that all other vertices become even. Compare all three pairings using the shortest-distance table.*

| *pairing* | *extra distance* |
|---|---|
| CD *and* FG | $82 + 28 = 110$ |
| CF *and* DG | $71 + 23 = 94$ |
| CG *and* DF | $59 + 22 = 81$ |

*The least extra distance is obtained by duplicating a shortest* C--G *path and the road* DF*. The shortest* C--G *path is* $C - B - A - G$*, so the roads traversed twice are* CB, BA, AG, DF*.*

$$423 + 81 = 504.$$

*Repeat* CB, BA, AG, DF*; route length* 504 *miles.*

*(5)*

*(b) Prim's algorithm grows one tree: at each step choose the shortest arc from the current tree to a new town.*

AG(8), AF(20), DF(22), AB(34), BC(17), CJ(10), HJ(12), EJ(18).

*The arc-selection order is* $\mathrm{AG}, \mathrm{AF}, \mathrm{DF}, \mathrm{AB}, \mathrm{BC}, \mathrm{CJ}, \mathrm{HJ}, \mathrm{EJ}.$

**(3)**

*(c) Nearest neighbour repeatedly travels to the nearest unvisited town, using the shortest-distance table, then returns to the start.*

$$\mathrm{G} - \mathrm{A} - \mathrm{F} - \mathrm{D} - \mathrm{B} - \mathrm{C} - \mathrm{J} - \mathrm{H} - \mathrm{E} - \mathrm{G}$$

$$8 + 20 + 22 + 65 + 17 + 10 + 12 + 30 + 87 = 271.$$

*An upper bound is* $271$ *miles, from* $\mathrm{GAFDBCJHEG}.$

**(2)**

*(d) For the deletion lower bound, find an MST after removing* $\mathrm{G}$*, then reconnect* $\mathrm{G}$ *with its two shortest arcs. The MST of the original table has weight* $141$ *and includes* $\mathrm{AG} = 8$*, so after deleting* $\mathrm{G}$ *the tree has weight* $141 - 8 = 133$*. The two shortest links from* $\mathrm{G}$ *are* $\mathrm{GA} = 8$ *and* $\mathrm{GD} = 23$*.*

$$133 + 8 + 23 = 164.$$

*Lower bound* $= 164$ *miles.*

**(2)**

*(e) Replace every table distance in the nearest-neighbour route by the corresponding actual shortest route. In particular,* $\mathrm{D}$ *to* $\mathrm{B}$ *is* $\mathrm{D} - \mathrm{A} - \mathrm{B}$*,* $\mathrm{H}$ *to* $\mathrm{E}$ *is* $\mathrm{H} - \mathrm{J} - \mathrm{E}$*, and* $\mathrm{E}$ *to* $\mathrm{G}$ *is* $\mathrm{E} - \mathrm{J} - \mathrm{C} - \mathrm{B} - \mathrm{A} - \mathrm{G}.$

$$\mathrm{G} - \mathrm{A} - \mathrm{F} - \mathrm{D} - \mathrm{A} - \mathrm{B} - \mathrm{C} - \mathrm{J} - \mathrm{H} - \mathrm{J} - \mathrm{E} - \mathrm{J} - \mathrm{C} - \mathrm{B} - \mathrm{A} - \mathrm{G}.$$

**(1)**

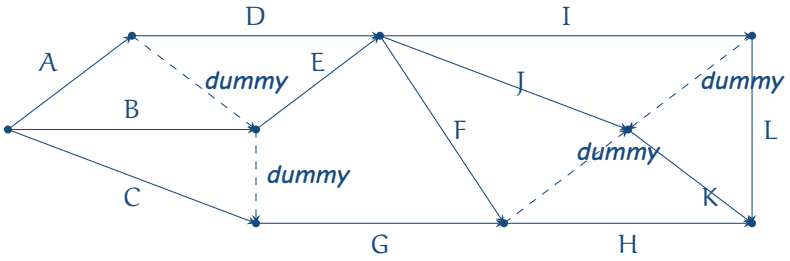
# Question 6

## Question 6

The precedence table for activities A to L is given in the question. (a) Draw the activity-on-arc network using the minimum number of dummies. (b) State the critical activities from the cascade chart. (c) Given that F has total float 2 hours, state its duration. (d) Find the stated floats of K and J in terms of $x$ and $y$.

## *Model solution*

*(a) Activities with the same immediate predecessors must start from the same event. Dummies are used only to transmit a prerequisite to a merge without introducing an extra prerequisite. The following activity-on-arc network uses four dummies.*




*Thus* E *follows* A, B*;* G *follows* A, B, C*;* F, I, J *follow* D, E*;* H *follows* F, G*;* K *follows* F, G, I, J*; and* L *follows* I. *(5)*

*(b) Critical activities are precisely those lying along the zero-float path in the cascade chart.*

B, E, I, L.

*(1)*

*(c) Activity* F *starts as early as time* 12 *and can finish as late as time* 18*. Its available window is therefore* 6 *hours. Total float is window minus duration.*

$$2 = 18 - 12 - \textit{duration of F}, \qquad \textit{duration of F} = 4.$$

4 *hours.*

*(1)*

*(d) Use the latest allowable finish of each activity and subtract its earliest start and duration. For* K*, the maximum available window is* 5 *hours, hence*

*maximum total float of* $K = 5 - y$ *hours.*

*For* J*, its available window before the completion of* K *is* $12 - y$ *hours, so*

*total float of* $J = 12 - x - y$ *hours.*

*(2)*

# Question 7

## Question 7

A publisher produces $x$ paperbacks, $y$ hardcovers and $z$ deluxe editions. Printing times are $4, 8, 15$ minutes; binding times are $1, 5, 12$ minutes. Printing is available for at most 150 hours, binding for at least 60 hours, at least 1600 books are required, and there must be at least three times as many paperbacks as hardcovers. Profits are £8, £20, £40. (a) Formulate the LP. (b) Set up the two-stage simplex tableau. (c) Carry out the stated iteration. (d)–(e) interpret the final tableau supplied in the question.

## *Model solution*

*(a) Convert hours to minutes first, so that the stated per-book times can be used directly. The word "at most" gives a $\leqslant$ constraint and "at least" gives a $\geqslant$ constraint.*

$$\textit{Maximise } P = 8x + 20y + 40z$$

*subject to*

$$\begin{aligned} 4x + 8y + 15z &\leqslant 9000, \\ x + 5y + 12z &\geqslant 3600, \\ x + y + z &\geqslant 1600, \\ x - 3y &\geqslant 0, \\ x, y, z &\geqslant 0. \end{aligned}$$

*(5)*

*(b) Add a slack variable to the $\leqslant$ constraint. For each $\geqslant$ constraint subtract a surplus variable; the first two of these also need artificial variables to provide an initial basis. Expressing the artificial variables in terms of the non-basic variables lets us form the Stage I objective.*

$$\begin{aligned} 4x + 8y + 15z + s_1 &= 9000, \\ x + 5y + 12z - s_2 + a_1 &= 3600, \\ x + y + z - s_3 + a_2 &= 1600, \\ -x + 3y + s_4 &= 0. \end{aligned}$$

$$I = -(a_1 + a_2), \qquad P - 8x - 20y - 40z = 0.$$

*Since*

$$a_1 = 3600 - x - 5y - 12z + s_2, \qquad a_2 = 1600 - x - y - z + s_3,$$

*the Stage I row is*

$$I - 2x - 6y - 13z + s_2 + s_3 = -5200.$$

*An initial tableau is*

| *b.v.* | $x$ | $y$ | $z$ | $s_1$ | $s_2$ | $s_3$ | $s_4$ | $a_1$ | $a_2$ | *Value* |
|---|---|---|---|---|---|---|---|---|---|---|
| $s_1$ | 4 | 8 | 15 | 1 | 0 | 0 | 0 | 0 | 0 | 9000 |
| $a_1$ | 1 | 5 | 12 | 0 | −1 | 0 | 0 | 1 | 0 | 3600 |
| $a_2$ | 1 | 1 | 1 | 0 | 0 | −1 | 0 | 0 | 1 | 1600 |
| $s_4$ | −1 | 3 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| $P$ | −8 | −20 | −40 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $I$ | −2 | −6 | −13 | 0 | 1 | 1 | 0 | 0 | 0 | −5200 |

*(6)*

*(c) In the supplied Stage II tableau, the most negative profit-row entry is $-56/11$ in the $s_3$ column, so $s_3$ enters. The ratio test selects the $s_1$ row: $(600)/(3) = 200$, so $s_1$ leaves. Divide its row by 3, then eliminate $s_3$ from every other row.*

| *b.v.* | $x$ | $y$ | $z$ | $s_1$ | $s_2$ | $s_3$ | $s_4$ | *Value* |
|---|---|---|---|---|---|---|---|---|
| $s_3$ | 0 | 0 | 0 | $\frac{1}{3}$ | $\frac{1}{3}$ | 1 | 0 | 200 |
| $z$ | 0 | $\frac{4}{11}$ | 1 | $-\frac{1}{33}$ | $-\frac{4}{33}$ | 0 | 0 | $\frac{1800}{11}$ |
| $x$ | 1 | $\frac{7}{11}$ | 0 | $\frac{4}{11}$ | $\frac{5}{11}$ | 0 | 0 | $\frac{18000}{11}$ |
| $s_4$ | 0 | $\frac{40}{11}$ | 0 | $\frac{4}{11}$ | $\frac{5}{11}$ | 0 | 1 | $\frac{18000}{11}$ |
| P | 0 | $-\frac{4}{11}$ | 0 | $\frac{56}{33}$ | $-\frac{40}{33}$ | 0 | 0 | $\frac{216000}{11}$ |

*The row operations are*

$$r_1 \leftarrow \frac{1}{3}r_1, \quad r_2 \leftarrow r_2 - \frac{1}{11}r_1, \quad r_3 \leftarrow r_3 + \frac{12}{11}r_1, \quad r_4 \leftarrow r_4 + \frac{12}{11}r_1, \quad r_P \leftarrow r_P + \frac{56}{11}r_1.$$

*(5)*

*(d) In the final tableau, non-basic variables are zero, so the Value column gives $x = 1125$, $y = 375$, $z = 100$ and $P = 20500$. To find binding time, use the original binding coefficients, not the simplex-row coefficients.*

$$1125 + 5(375) + 12(100) = 4200 \text{ minutes} = 70 \text{ hours}.$$

*(i) Maximum profit $= \$20\,500$. (ii) Binding is used for 70 hours.*

*(2)*

*(e) Linear programming maximises the profit calculated on books produced; it does not ensure that every book will be sold.*

*The publisher may not sell all of the books, so may not earn this profit.*

*(1)*