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Physics and maths resources

Edexcel A Level Further Mathematics

9FM0/3D - Decision Mathematics 1

Friday 20 June 2025

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2025 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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Question 1

Question 1

Pack $4.7, 2.9, 5.5, 1.4, 5.8, 2.8, 3.8, 6.5, 5.1, 6.3, 4.1$ into bins of capacity 13, using (a) first fit and (b) first-fit decreasing.

Model solution

(a) First fit considers the numbers in the order supplied. Each number goes into the first already-open bin with enough remaining space; a new bin is opened only when necessary.

bin	*contents*	*total*
1	4.7, 2.9, 1.4, 2.8	11.8
2	5.5, 5.8	11.3
3	3.8, 6.5	10.3
4	5.1, 6.3	11.4
5	4.1	4.1

First fit uses five bins, packed as shown.

(3) (b) First-fit decreasing uses exactly the same placement rule, but first sorts the items from largest to smallest. This gives the algorithm a better chance to fill a bin with smaller later items.

bin	*contents*	*total*
1	6.5, 6.3	12.8
2	5.8, 5.5, 1.4	12.7
3	5.1, 4.7, 2.9	12.7
4	4.1, 3.8, 2.8	10.7

First-fit decreasing uses four bins.

(2)

Question 2

Question 2

The list $271, 828, 182, 845, 904, 523, 536, 028, 747, 135$ is to be sorted in descending order. It is also used with the stated algorithm for approximating e.

Model solution

(a) In quick sort, choose a pivot and put all larger entries to its left and all smaller entries to its right. Repeating this on the unsorted sections produces descending order. Leading zeroes are retained in 028.

pivot(s) used	*list after the pass*
523	$828, 845, 904, 536, 747, \boxed{523}, 271, 182, 028, 135$
$904,\ 028$	$\boxed{904}, 828, 845, 536, 747, 523, 271, 182, 135, \boxed{028}$
$536,\ 182$	$904, 828, 845, 747, \boxed{536}, 523, 271, \boxed{182}, 135, 028$
845	$904, \boxed{845}, 828, 747, 536, 523, 271, 182, 135, 028$
747	$904, 845, \boxed{747}, 828, 536, 523, 271, 182, 135, 028$

$$904, 845, 828, 747, 536, 523, 271, 182, 135, 028.$$

(4) (b) Bubble sort compares adjacent entries. In the worst case, the first pass needs $n-1$ *comparisons, then* $n-2$*, down to* 1.

$$(n-1) + (n-2) + \cdots + 1 = \frac{n(n-1)}{2}.$$

$$\frac{n(n-1)}{2} \textit{ comparisons.}$$

(2) (c) The algorithm successively forms factorials in c *and adds their reciprocals to* d*; hence it is constructing* $1 + 1 + \frac{1}{2!} + \cdots + \frac{1}{6!}$.

a	b	c	d	*decimal form of* d
1	1	1	1	1
1	2	1	2	2
1	3	2	$\frac{5}{2}$	2.5
1	4	6	$\frac{8}{3}$	$2.666\ldots$
1	5	24	$\frac{65}{24}$	$2.708333\ldots$
1	6	120	$\frac{163}{60}$	$2.716666\ldots$
1	6	720	$\frac{1957}{720}$	$2.7180555\ldots$

$$\textit{The output is } \frac{1957}{720} = 2.7180555\ldots \textit{ (2.72 to 3 s.f.).}$$

(3) (d) Percentage error compares the absolute error with the accepted value of e.

$$\frac{|2.7180555\ldots - 2.7182818\ldots|}{2.7182818\ldots} \times 100 = 0.00832\ldots\%.$$

Percentage error $= 0.00832\%$ ***(3 s.f.).***

(1)

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Question 3

Question 3

For the directed road network in Figure 1, use Floyd's algorithm and then nearest neighbour from B.

Model solution

(a) The initial Floyd matrix records a direct travel time, ∞ when there is no direct arc, and 0 on the diagonal. The directed arcs must not be copied symmetrically.

	A	B	C	D	E	F	G
A	0	9	∞	25	35	∞	11
B	∞	0	40	8	∞	35	∞
C	∞	40	0	17	∞	∞	47
D	∞	8	17	0	11	31	∞
E	35	∞	∞	11	0	14	∞
F	∞	35	∞	31	14	0	12
G	11	∞	47	∞	24	12	0

Initial matrix as displayed.

(2) (b) The next two Floyd iterations allow village D, then village E, as an intermediate village. For each entry compare its current value with "via the new intermediate".

	A	B	C	D	E	F	G
A	0	9	34	17	28	44	11
B	∞	0	25	8	19	35	72
C	∞	25	0	17	28	48	47
D	∞	8	17	0	11	31	64
E	35	19	28	11	0	14	46
F	∞	35	48	31	14	0	12
G	11	20	45	28	24	12	0

(D-*iteration*)

	A	B	C	D	E	F	G
A	0	9	34	17	28	42	11
B	54	0	25	8	19	33	65
C	63	25	0	17	28	42	47
D	46	8	17	0	11	25	57
E	35	19	28	11	0	14	46
F	49	33	42	25	14	0	12
G	11	20	45	28	24	12	0

(E-*iteration*)

The two required matrices are shown.

(5) (c) Nearest neighbour always chooses the nearest unvisited village in the completed shortest-time network. Starting at B, this gives

$$B \to D \to E \to F \to G \to A \to C \to B.$$

$$8 + 11 + 14 + 12 + 11 + 34 + 25 = 115.$$

Some arcs in the complete network represent shortest routes through intermediate villages. Replacing each such arc by its actual route gives the physical journey.

(i) B – D – E – F – G – A – C – B; *(ii)* 115 *minutes; (iii) actual villages* B – D – E – F – G – A – B – D – C – D – B.

(3)

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Question 4

Question 4

For the corridor network of total weight 268 m, find routes from J to K via A, and postman routes as specified.

Model solution

(a) To force a route via A, run Dijkstra's algorithm from A. The final labels give the shortest distance from A to each office; tracing predecessors produces the two required legs.

$$\mathrm{J} \to \mathrm{E} \to \mathrm{B} \to \mathrm{C} \to \mathrm{A} \to \mathrm{C} \to \mathrm{G} \to \mathrm{F} \to \mathrm{K},$$

$$\mathrm{JA} = 10 + 24 + 6 + 5 = 45, \qquad \mathrm{AK} = 5 + 9 + 4 + 13 = 31, \qquad 45 + 31 = 76.$$

(i) JEBCACGFK*; (ii)* 76 *m.*

(6) (b) Since the route starts at J *and ends at* A*, these are the two vertices that may remain odd. The relevant vertices that must be paired are* B, C, D, K*, so test all three pairings.*

pairing of B, C, D, K	*added length (m)*
BC, D → G → F → K	6 + 18 = 24
B → F → G → D, C → G → F → K	15 + 26 = 41
B → F → K, C → G → D	23 + 10 = 33

The smallest addition is 24 *m, so repeat* BC, DG, FG, FK*.*

(i) Repeat BC, DG, FG, FK*. (ii) Minimum length* 268 + 24 = 292 *m.*

(5) (c) Blocking AB, CD, JK *removes lengths* 13, 11, 16*. The required new* J*-to-*A *route must duplicate a shortest* J*-to-*A *path of* 45 *m, so compare this new total with part (b).*

$$292 - [268 + 45 - (13 + 11 + 16)] = 19.$$

The new route is 19 *m longer.*

(2)

Question 5

Question 5

Maximise $P = 3x + 4y + 2z$, subject to $2x + 2y + 2z \leqslant 21$, $2x - y - z \leqslant 18$, $-3x + y - 2z \leqslant 1$, $x \geqslant 4, y \geqslant 1, z \leqslant -3$. Use $x = X + 4, y = Y + 1, z = -Z - 3, P = Q + 10$.

Model solution

(a) Ordinary Simplex starts from the origin, so its variables must have a non-negative feasible starting point. Here the lower bounds mean that the origin is not feasible.

There is no obvious basic feasible solution: the origin is not in the feasible region.

(1) (b) The substitutions shift the lower bounds to $X, Y, Z \geqslant 0$. Substitute into each constraint before simplifying, and use $P = Q + 10$ in the objective.

$$\begin{aligned} 2(X+4) + 2(Y+1) + 2(-Z-3) &\leqslant 21 &&\Rightarrow\quad 2X + 2Y - 2Z \leqslant 17, \\ 2(X+4) - (Y+1) - (-Z-3) &\leqslant 18 &&\Rightarrow\quad 2X - Y + Z \leqslant 8, \\ -3(X+4) + (Y+1) - 2(-Z-3) &\leqslant 1 &&\Rightarrow\quad -3X + Y + 2Z \leqslant 6, \\ Q + 10 = 3(X+4) + 4(Y+1) + 2(-Z-3) &&&\Rightarrow\quad Q = 3X + 4Y - \end{aligned}$$

Maximise $Q = 3X + 4Y - 2Z$, subject to the three displayed inequalities and $X, Y, Z \geqslant 0$.

(3) (c) Read the objective row of tableau T, then rearrange it with Q as the subject.

$$Q - \frac{5}{4}Z + \frac{15}{8}s_1 + \frac{1}{4}s_3 = \frac{267}{8}, \qquad Q = \frac{267}{8} + \frac{5}{4}Z - \frac{15}{8}s_1 - \frac{1}{4}s_3.$$

$$Q = \tfrac{267}{8} + \tfrac{5}{4}Z - \tfrac{15}{8}s_1 - \tfrac{1}{4}s_3.$$

(1) (d) A positive coefficient of the non-basic variable Z in this profit equation means increasing Z can increase Q. Therefore the current basic feasible solution cannot be optimal.

Z can be increased to increase the profit, so T is not optimal.

(1) (e) The positive Z-coefficient selects Z to enter. Only the s_2-row has a positive coefficient in the Z-column, so it is the pivot row and s_2 leaves. Divide that row by $9/4$, then clear the other Z-entries.

$$R_2 \leftarrow \frac{4}{9}R_2, \qquad R_1 \leftarrow R_1 + \frac{1}{3}R_2^{(\text{old})}, \qquad R_3 \leftarrow R_3 + \frac{1}{9}R_2^{(\text{old})}, \qquad R_4 \leftarrow R_4 + \frac{5}{9}R_2^{(\text{old})}.$$

b.v.	X	Y	Z	s_1	s_2	s_3	*Value*
X	1	0	0	$\frac{1}{6}$	$\frac{1}{3}$	0	$\frac{11}{2}$
Z	0	0	1	$\frac{1}{18}$	$\frac{4}{9}$	$\frac{1}{3}$	$\frac{13}{2}$
Y	0	1	0	$\frac{7}{18}$	$\frac{1}{9}$	$\frac{1}{3}$	$\frac{19}{2}$
Q	0	0	0	$\frac{35}{18}$	$\frac{5}{9}$	$\frac{2}{3}$	$\frac{83}{2}$

All objective-row coefficients are now non-negative, so this tableau is optimal. With non-basic slacks zero, $X = \frac{11}{2}, Y = \frac{19}{2}, Z = \frac{13}{2}, Q = \frac{83}{2}$. *Transform back to the original variables.*

$$x = X + 4 = \frac{19}{2}, \quad y = Y + 1 = \frac{21}{2}, \quad z = -Z - 3 = -\frac{19}{2}, \quad P = Q + 10 = \frac{103}{2}.$$

Maximum $P = 51.5$, ***at*** $(x, y, z) = (9.5, 10.5, -9.5)$.

(6)

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Question 6

Question 6

For the activity network in Figure 3, determine preceding activities, event times, a Gantt chart and workers per activity.

Model solution

(a) An immediately preceding activity is one that must finish before the listed activity can start. Read all arcs entering the event at which each activity begins.

A	–	B	–	C	–
D	A	E	C	F	C
G	A, B, E	H	D, G	I	A, B, E
J	H, I	K	A, B, E	L	A, B, E, F
M	A, B, E, F				

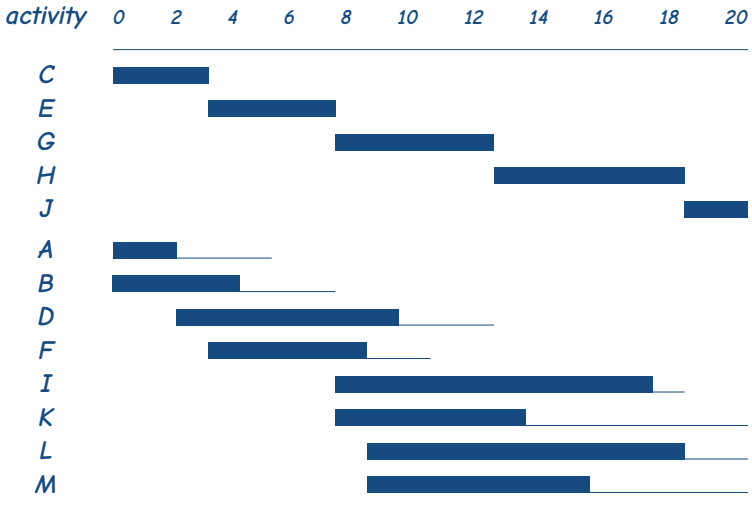
Immediately preceding activities as displayed.

(2) (b) Use a forward pass for early event times: at a merge take the largest incoming time. Then use a backward pass from the completion time 20: at a split take the smallest outgoing late time.

event (from left to right)	*start*	*after A*	*after C*	*central*	*upper middle*	*lower middle*	*upper right*
early	0	2	3	7	12	8	18
late	0	5	3	7	12	10	18

Early and late event times as displayed; minimum completion time = 20 hours.

(4) (c) A Gantt chart schedules each activity as early as possible, with its available float shown by the interval up to its latest finish. Critical activities have zero float.



activity scheduled from its earliest start; ____ available float.

The critical path is C – E – G – H – J, *with duration* 20 *hours.*

(4) *(d) Work backwards from the completion time and compare the hourly activity overlaps with the given histogram. The worker allocation must add to the histogram total in every hour.*

activity	A	B	C	D	E	F	G	H	I	J	K	L	M
workers	1	2	1	2	3	1	2	1	2	2	1	1	1

Workers per activity as displayed.

(3)

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Question 7

Question 7

The feasible region is given by $x \geqslant 1, 3x+y \leqslant 8, 2x+3y \geqslant 6, 3x+4y \leqslant 20$. Maximise $P = 11x+ky$, where $k > 0$, by the big-M method.

Model solution

(a) Constraints of type $\leqslant$ need slack variables. Constraints of type $\geqslant$ need a surplus variable and an artificial variable, because a surplus variable alone cannot provide an initial basic variable.

$$3x + 4y + s_1 = 20,$$
$$x - s_2 + a_1 = 1,$$
$$2x + 3y - s_3 + a_2 = 6,$$
$$3x + y + s_4 = 8.$$

Penalise artificial variables in the objective, then eliminate the initial basic artificial variables a_1, a_2 from its row.

b.v.	x	y	s_1	s_2	s_3	s_4	a_1	a_2	*Value*
s_1	3	4	1	0	0	0	0	0	20
a_1	1	0	0	-1	0	0	1	0	1
a_2	2	3	0	0	-1	0	0	1	6
s_4	3	1	0	0	0	1	0	0	8
P	$-(11+3M)$	$-(k+3M)$	0	M	M	0	0	0	$-7M$

Equivalently, the last row is obtained from $P - 11x - ky + Ma_1 + Ma_2 = 0$ after using the two artificial-variable rows.

The displayed tableau is a valid initial big-M tableau with two slack, two surplus and two artificial variables.

(7) (b) In the supplied third tableau, s_3 and s_4 are non-basic. For optimality their objective-row coefficients must be non-negative:

$$\frac{11}{7} - \frac{3k}{7} \geqslant 0, \qquad \frac{33}{7} - \frac{2k}{7} \geqslant 0.$$

$$k \leqslant \frac{11}{3}, \qquad k \leqslant \frac{33}{2}.$$

Thus the condition that this is optimal restricts $0 < k \leqslant \frac{11}{3}$. The objective-row value is $P = \frac{198}{7} + \frac{2k}{7}$, which increases with k, so use the largest permitted value.

$$k = \frac{11}{3}, \qquad P = \frac{198}{7} + \frac{2}{7}\left(\frac{11}{3}\right) = \frac{88}{3}.$$

$0 < k \leqslant \frac{11}{3}$; the greatest possible optimal value is $\boxed{\frac{88}{3}}$, attained when $k = \frac{11}{3}$.

(5)