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# Edexcel A Level Further Mathematics

## 9FM0/4C - Further Mechanics 2

Monday 24 June 2024

## *Questions and Fully Worked Model Solutions*

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2024 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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# Question 1

## Question 1

A particle starts from rest and has $a = 96/(3t+5)^3$. (a) Show $v = p - q/(3t+5)^2$. (b) Find its limiting velocity. (c) Find $x$ at $t = 2$.

## *Model solution*

*(a) Use $a = \frac{dv}{dt}$ and the given initial condition to integrate acceleration.*

$$v = -\frac{16}{(3t+5)^2} + C, \quad 0 = -\frac{16}{25} + C, \quad v = \frac{16}{25} - \frac{16}{(3t+5)^2}.$$

$$p = 16/25, \ q = 16.$$

*(4) (b) The reciprocal term tends to zero as $t \to \infty$.*

$$\lim v = \frac{16}{25}.$$

$$16/25 \text{ m s}^{-1}.$$

*(1) (c) Integrate $v = \frac{dx}{dt}$ and use $x = 0$ when $t = 0$.*

$$x = \frac{16t}{25} + \frac{16}{3(3t+5)} + D, \quad D = -\frac{16}{15}, \quad x(2) = \frac{192}{275}.$$

$$192/275 \text{ m} = 0.698\ldots \text{ m}.$$

*(4)*

# Question 2

## Question 2

Seven identical rods of length $4a$ form the framework shown. (a) Show its centre of mass is distance $d = 8\sqrt{3}a/7$ from ED. (b) When pivoted at C, find the horizontal holding force F at A in terms of weight $W$.

**2.**

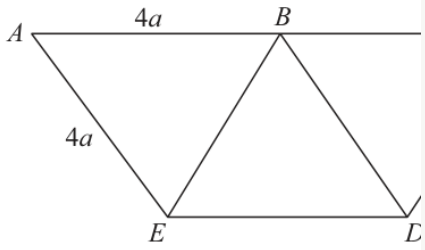



**Figure 1**

## Model solution

*(a) Identical rods have equal masses. Take moments of their masses about* ED.

$$(4)(4a)(2a\cos 30°) + (2)(4a)(4a\cos 30°) = (7)(4a)d.$$

$$32\sqrt{3}a^2 = 28ad \Rightarrow d = \frac{8\sqrt{3}}{7}a.$$

$$d = 8\sqrt{3}a/7.$$

*(4) (b) Take moments about* C *to remove the pivot reaction. The force arm is* $8a$ *and the weight arm is* $4a\cos 30° - d$.

$$8aF = \left(4a\cos 30° - \frac{8\sqrt{3}a}{7}\right)W.$$

$$F = \frac{3\sqrt{3}}{28}W.$$

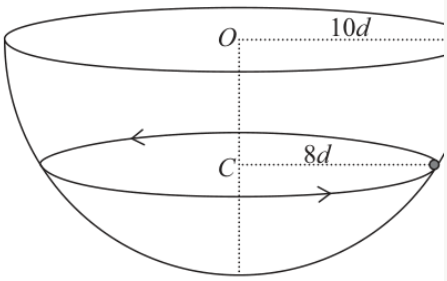
*(3)*

# Question 3

## Question 3

A particle moves on the smooth inside of a hemispherical bowl of radius 10d, in a horizontal circle of radius 8d. (a) Find its acceleration. (b) Find its period.

3.




## Model solution

*(a) The cross-section has vertical side* $6d$*, so* $\tan\theta = 6/8 = 3/4$*. Resolve the normal reaction and weight vertically and horizontally; the latter resultant is centripetal.*

$$R\sin\theta = mg, \qquad R\cos\theta = ma.$$

$$\frac{g}{a} = \tan\theta = \frac{3}{4} \Rightarrow a = \frac{4g}{3}.$$

$$4g/3 \text{ m s}^{-2}, \text{horizontally inwards.}$$

*(6) (b) Apply* $a = r\omega^2$*, then* $T = 2\pi/\omega$*.*

$$8d\omega^2 = \frac{4g}{3} \Rightarrow \omega^2 = \frac{g}{6d}, \qquad T = 2\pi\sqrt{\frac{6d}{g}}.$$

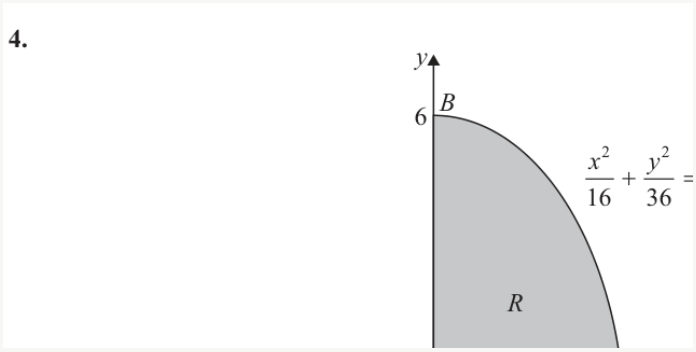
$$T = 2\pi\sqrt{6d/g} \text{ s.}$$

*(3)*

# Question 4

## Question 4

The lamina is the first-quadrant part of $x^2/16 + y^2/36 = 1$, with area $6\pi$. (a) Show $\bar{x} = 16/(3\pi)$. (b) Find $\bar{y}$. (c) Find suspension angle $\theta$.




## *Model solution*

*Use* $y = \sqrt{36 - \frac{9}{4}x^2}$, $0 \leqslant x \leqslant 4$. *(a) A vertical strip has* $y$*-axis moment* $xy\,dx$. *Substitute* $u = 36 - \frac{9}{4}x^2$.

$$6\pi\bar{x} = \int_0^4 x\sqrt{36 - \frac{9}{4}x^2}\,dx = \left[-\frac{2}{27}\left(36 - \frac{9}{4}x^2\right)^{3/2}\right]_0^4 = 32.$$

$$\bar{x} = 16/(3\pi).$$

*(5) (b) The centroid of a vertical strip is at* $y/2$*, so its* $x$*-axis moment is* $\frac{1}{2}y^2dx$.

$$6\pi\bar{y} = \frac{1}{2}\int_0^4 (36 - \frac{9}{4}x^2)\,dx = \left[18x - \frac{3}{8}x^3\right]_0^4 = 48.$$

$$\bar{y} = 8/\pi.$$

*(4) (c) In equilibrium the centre of mass lies vertically below* $A = (4, 0)$*, giving a right triangle.*

$$\tan\theta = \frac{\bar{y}}{4 - \bar{x}} = \frac{8/\pi}{4 - 16/(3\pi)}, \quad \theta = 47.882\ldots^\circ.$$

$$\theta = 47.9^\circ.$$

*(3)*

# Question 5

## Question 5

A particle has SHM with greatest acceleration 18 $\mathrm{m\,s^{-2}}$; at $x = 0.3$ m, its speed is 2.4 $\mathrm{m\,s^{-1}}$. (a) Show amplitude $a = 0.5$. (b) Find greatest speed. (c) Find the time S in one oscillation for which speed is at least 2 $\mathrm{m\,s^{-1}}$.

## *Model solution*

*(a) Use the SHM clues: maximum acceleration is* $a\omega^2$, *and* $v^2 = \omega^2(a^2 - x^2)$.

$$a\omega^2 = 18, \quad 2.4^2 = \omega^2(a^2 - 0.3^2).$$

$$\omega^2 = \frac{18}{a} \Rightarrow 5.76 = \frac{18}{a}(a^2 - 0.09) \Rightarrow 18a^2 - 5.76a - 1.62 = 0.$$

$$\Rightarrow a = 0.5 \quad \textit{(positive root)}.$$

$$a = 0.5 \text{ m}.$$

*(5) (b) Thus* $\omega = 6$, *and greatest speed is* $a\omega$.

$$v_{\max} = 0.5(6) = 3.$$

$$3 \text{ m s}^{-1}.$$

*(2) (c) Choose* $x = 0.5 \sin 6t$, *giving* $v = 3\cos 6t$. *In each symmetric interval the threshold is reached at* $3\cos 6t = 2$.

$$t_0 = \frac{\cos^{-1}(2/3)}{6} = 0.14017\ldots, \qquad S = 4t_0 = 0.5607\ldots.$$

$$S = 0.561 \text{ s}.$$

*(4)*

# Question 6

## Question 6

A cylinder of height 6 cm, radius 2 cm has density $\lambda(x+2)$. (a) Show its mass is $120\lambda\pi$. (b) Show its centre is 3.6 cm from O. A uniform hemisphere of radius 3 cm is attached; find toppling angle $\alpha$.

(b) Show that the centre of mass of the cylinder is 3.6 cm from $O$.

**(4)**

The point $O$ is the centre of one end of the cylinder. The point $A$ is the centre of the other end of the cylinder.

A uniform solid hemisphere of radius 3 cm has density $\lambda$ kg cm$^{-3}$. The hemisphere is attached to the cylinder with the centre of its circular face in contact with the point $A$ on the cylinder to form the model shown in Figure 5.

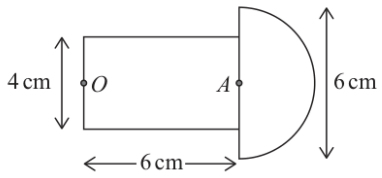



**Figure 5**

## Model solution

*(a) A disc of thickness* $\mathrm{d}x$ *has mass* $4\pi\lambda(x+2)\,\mathrm{d}x$.

$$M = \int_0^6 4\pi\lambda(x+2)\,\mathrm{d}x = 4\pi\lambda\left[\frac{x^2}{2}+2x\right]_0^6 = 120\lambda\pi.$$

$$120\lambda\pi \text{ kg}.$$

*(3) (b) Divide the moment of the discs about the* $yz$*-plane by their mass.*

$$M\bar{x} = \int_0^6 4\pi\lambda x(x+2)\,\mathrm{d}x = 4\pi\lambda\left[\frac{x^3}{3}+x^2\right]_0^6 = 432\lambda\pi, \quad \bar{x} = 3.6.$$

$$3.6\text{cm from O}.$$

*(4) (c) At limiting toppling take moments about the downhill rim. The hemisphere has mass* $18\lambda\pi$ *and centre* $9/8$ *cm beyond* A.

$$(120\lambda\pi)(3.6) + (18\lambda\pi)\left(6+\frac{9}{8}\right) = (138\lambda\pi)\mathrm{d}, \quad \mathrm{d} = \frac{747}{184}.$$

$$\tan\alpha = \frac{2}{\mathrm{d}}, \quad \alpha = 26.226\ldots^\circ.$$

$$\alpha = 26.2^\circ.$$

*(6)*

# Question 7

## Question 7

A stone is projected horizontally from the top of a smooth hemisphere of radius r, speed U. At B, OB makes angle θ to the upward vertical. (a) Find $v^2$. Given $U^2 = 2rg/3$, (b) show $W^2 = 8rg/9$. (c) Find impact speed. (d) Find impact angle α.

7.

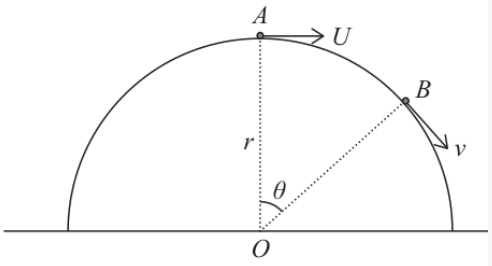



**Figure 6**

## *Model solution*

*(a) Energy is conserved; the vertical drop to* B *is* $r(1-\cos\theta)$.

$$\frac{1}{2}mU^2 + mgr(1-\cos\theta) = \frac{1}{2}mv^2.$$

$$v^2 = U^2 + 2gr(1-\cos\theta).$$

*(3) (b) At leaving,* $R = 0$. *The inward component of weight supplies centripetal force; combine this with the energy equation.*

$$mg\cos\theta = \frac{mW^2}{r}, \quad rg\cos\theta = \frac{2rg}{3} + 2rg(1-\cos\theta).$$

$$\cos\theta = \frac{8}{9}, \quad W^2 = rg\cos\theta = \frac{8rg}{9}.$$

$$W^2 = 8rg/9.$$

*(5) (c) The free projectile falls another* $r\cos\theta$.

$$V^2 = W^2 + 2gr\cos\theta = \frac{8rg}{3}.$$

$$V = \sqrt{8rg/3}.$$

*(3) (d) Horizontal velocity remains* $W\cos\theta$.

$$\cos\alpha = \frac{W\cos\theta}{V} = \frac{8\sqrt{3}}{27}.$$

$\alpha = 59°$ below the horizontal.

**(3)**

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