Maths with Matt

Physics and maths resources

Edexcel A Level Further Mathematics

9FM0/4C - Further Mechanics 2

Monday 23 June 2025

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2025 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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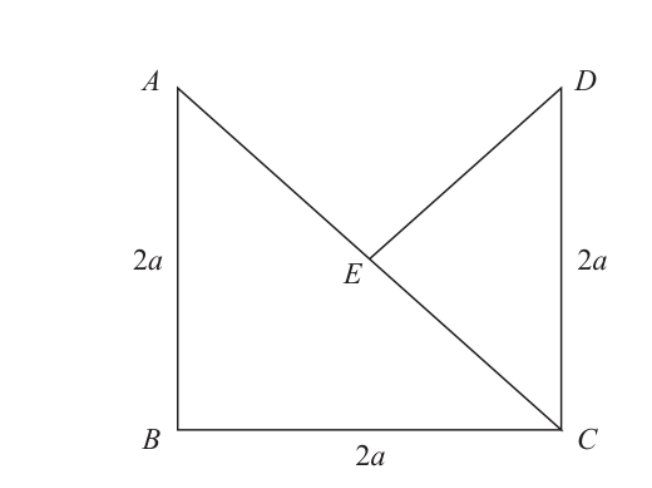
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Question 1

Question 1

A uniform rod is cut into six pieces to form the planar framework ABCDE shown below. $AB = BC = CD = 2a$, $AE = EC = ED = \sqrt{2}a$, E is the midpoint of AC, and $\angle ABC = \angle BCD = 90°$. The distance of its centre of mass from AB is d.



(a) Show that $d = \dfrac{12+7\sqrt{2}}{12+6\sqrt{2}}a$. **(4)**

The framework has mass M. A particle of mass kM is attached at B, and the centre of mass of the loaded framework is distance a from AB.

(b) Find k. **(3)**

Model solution

(a) Take moments about AB. Each rod has the same mass per unit length, so its mass is proportional to its length; the required distances are the perpendicular distances of the midpoints from AB.

piece	AB	BC	CD	AE	EC	ED
mass ratio	$2a$	$2a$	$2a$	$\sqrt{2}a$	$\sqrt{2}a$	$\sqrt{2}a$
distance from AB	0	a	$2a$	$\frac{a}{2}$	$\frac{3a}{2}$	$\frac{3a}{2}$

$$(2a+2a+2a+3\sqrt{2}a)d = 2a(a)+2a(2a)+\sqrt{2}a\left(\frac{a}{2}\right)+\sqrt{2}a\left(\frac{3a}{2}\right)+\sqrt{2}a\left(\frac{3a}{2}\right).$$

$$(6+3\sqrt{2})ad = \left(6+\frac{7\sqrt{2}}{2}\right)a^2, \qquad d = \frac{12+7\sqrt{2}}{12+6\sqrt{2}}a.$$

$$d = \frac{12+7\sqrt{2}}{12+6\sqrt{2}}a.$$

(4)

(b) The added particle is on AB, so it has zero moment about AB. Equating moments of

the whole loaded system gives an equation for its mass ratio.

$$Mg\left(\frac{12+7\sqrt{2}}{12+6\sqrt{2}}a\right)+kMg(0)=(1+k)Mg(a).$$

$$1+k=\frac{12+7\sqrt{2}}{12+6\sqrt{2}} \quad \Rightarrow \quad k=\frac{\sqrt{2}}{12+6\sqrt{2}}=0.0690\ldots$$

$$k=\frac{\sqrt{2}}{12+6\sqrt{2}} \textit{ (or } 0.0690\textit{).}$$

(3)

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Question 2

Question 2

The region under $y = 1/x$, between $x = 2$ and $x = 4$, is rotated about the x-axis. It models a uniform pedestal of height 2 m, upper radius 0.5 m and base radius 0.25 m, with volume $\pi/4$ m^3.

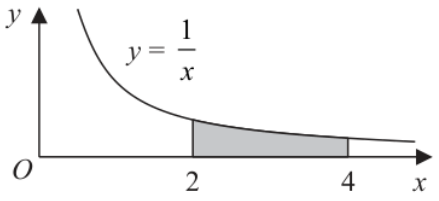


Figure 2

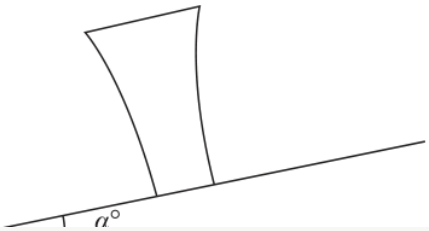
shown in Figure 2 is bounded by the curve with equati

ion $x = 2$, the line with equation $x = 4$, and the x-axis.
ted through 360° about the x-axis to form a solid.

to model a uniform solid pedestal of height 2 m, upper r
25 m.

ume of the pedestal is $\frac{\pi}{4}$ m^3

centre of mass of the pedestal is $(4 - 4\ln 2)$ m from its ba



(a) Show that its centre of mass is $(4 - 4\ln 2)$ m from its base. **(4)**

The pedestal stands on a sufficiently rough plane inclined at $\alpha°$ and is on the point of toppling. (b)
Find α. **(3)**

Model solution

(a) Use a thin circular disc of thickness $\mathrm{d}x$. Its volume, and hence its mass, is proportional to $\pi y^2 \mathrm{d}x$; moments about O therefore require an extra factor x.

$$\text{moment about O} = \pi\rho \int_2^4 x \left(\frac{1}{x}\right)^2 \mathrm{d}x = \pi\rho \int_2^4 \frac{1}{x}\,\mathrm{d}x = \pi\rho \ln 2.$$

$$\bar{x} = \frac{\pi\rho \ln 2}{\rho(\pi/4)} = 4\ln 2.$$

This is measured from O, whereas the base is at $x = 4$.

$$\text{distance from base} = 4 - \bar{x} = 4 - 4\ln 2 \text{ m}.$$

$(4 - 4\ln 2)$ *m from the base.*

(4)

(b) At impending toppling, the vertical line through the centre of mass passes through the downhill edge of the base. The right triangle has horizontal side 0.25 *m and perpendicular height* $4 - 4\ln 2$ *m.*

$$\tan\alpha = \frac{0.25}{4 - 4\ln 2}, \qquad \alpha = 11.512\ldots^\circ .$$

$\alpha = 11.5^\circ$ *(or* 12° *to 2 s.f.).*

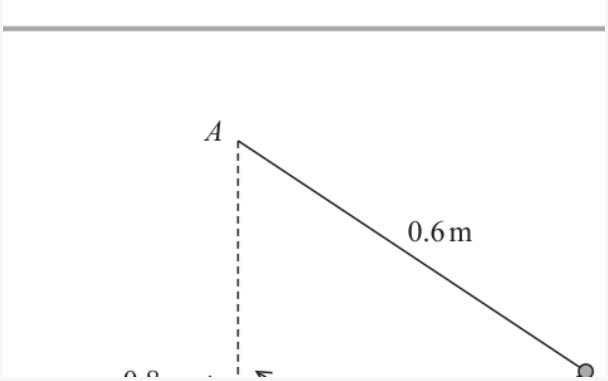
(3)

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Question 3

Question 3

A particle P of mass 0.2 kg is joined to fixed points A and B by light inextensible strings of length 0.6 m. B is vertically below A, with AB = 0.8 m. P moves in a horizontal circle with constant angular speed ω, both strings taut. The tension in PA is twice that in PB.



Find ω. **(7)**

Model solution

The equal strings put P midway in height between A and B. First find the circle radius from the isosceles-triangle geometry, then resolve vertically and horizontally towards the axis.

$r = \sqrt{0.6^2 - 0.4^2} = \sqrt{0.2}$, *vertical component factor* $= \dfrac{0.4}{0.6} = \dfrac{2}{3}$, *horizontal component factor* =

Let the lower-string tension be T, *so the upper-string tension is* 2T. *Vertical acceleration is zero.*

$$2T\left(\frac{2}{3}\right) - T\left(\frac{2}{3}\right) = 0.2g \quad \Rightarrow \quad T = 0.3g = 2.94 \text{ N}.$$

Both horizontal components point towards the vertical axis, providing the centripetal force.

$$2T\left(\frac{\sqrt{5}}{3}\right) + T\left(\frac{\sqrt{5}}{3}\right) = 0.2r\omega^2.$$

$$\sqrt{5}T = 0.2\sqrt{0.2}\,\omega^2 \quad \Rightarrow \quad \omega^2 = 25T = 73.5.$$

$$\omega = 8.57 \text{ rad s}^{-1} \text{ (or } 8.6 \text{ rad s}^{-1}\text{)}.$$

(7)

Question 4

Question 4

The uniform $15a \times 3a$ rectangle ACDF is folded along BE, where $AB = ED = 6a$, to form the lamina shown.

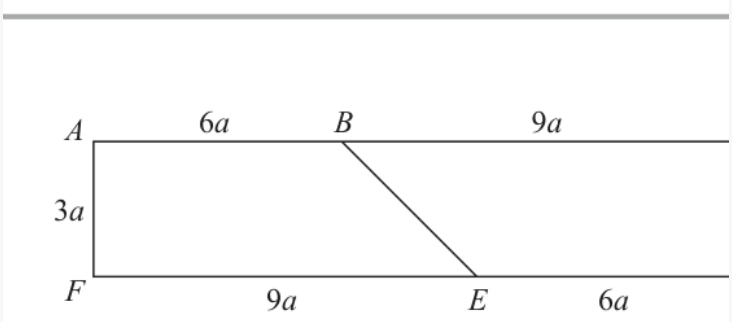
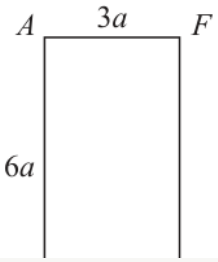


Figure 5

tangular lamina $ACDF$ shown in Figure 5 has $AC = FD =$
The point B on AC is such that $AB = 6a$. The point E on



(a) Show that the centre of mass is distance $\frac{2}{5}a$ from EF. **(5)**

(b) Explain why the centre of mass lies on the perpendicular bisector of BE. **(1)**

The lamina is freely suspended from F. (c) Find the angle between FE and the downward vertical. **(3)**

Model solution

(a) Split the folded shape into the left rectangle, two congruent triangular parts and the right rectangle. Uniform density means their masses are proportional to their areas. Take signed moments about EF.

part	*left rectangle*	*two triangles*	*right rectangle*
mass ratio	18	9	18
distance from EF	$-\frac{3a}{2}$	$-a$	$3a$

$$45d = 18\left(-\frac{3a}{2}\right) + 9(-a) + 18(3a) = 18a, \qquad d = \frac{2}{5}a.$$

The distance from EF *is* $\frac{2}{5}a$.

(5)

(b) The folded lamina has a line of symmetry: the perpendicular bisector of BE. *A uniform lamina's centre of mass must lie on every line of symmetry.*

It lies on the perpendicular bisector of BE.

(1)

(c) When suspended, the centre of mass is vertically below F. *Use the symmetry result and the distance in part (a) to form the fixed right triangle from* F *to the centre of mass: its components relative to* FE *are* $\frac{2}{5}a$ *and* $6a - \frac{2}{5}a = \frac{28}{5}a$.

$$\tan\theta = \frac{(2/5)a}{(28/5)a} = \frac{1}{14}, \qquad \theta = 4.0856\ldots^\circ.$$

$\theta = 4.09^\circ$ *(or* 4.1°*).*

(3)

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Question 5

Question 5

A 1000 kg car moves on a horizontal road. Its driving force is 800 N and its resistance has magnitude $2v^2$ N when its speed is v m s^{-1}.

(a) Show that the time T for its speed to increase from 5 to 15 m s^{-1} is $\frac{25}{2}\ln\left(\frac{21}{5}\right)$. **(7)**

When $t = 0$, $v = 2$. (b) Show that $\frac{20+v}{20-v} = pe^{qt}$, where p and q are rational numbers to be found. **(2)**

(c) Hence explain why $v < 20$ for all $t \geqslant 0$. **(1)**

Model solution

(a) The forces are along the line of motion, so Newton's second law gives a separable differential equation. Take forward as positive.

$$800 - 2v^2 = 1000\frac{dv}{dt} \quad \Rightarrow \quad \frac{dv}{dt} = \frac{400 - v^2}{500}.$$

$$\int \frac{500}{400 - v^2}\,dv = \int dt.$$

Factor $400 - v^2 = (20 - v)(20 + v)$*; partial fractions give the logarithmic antiderivative.*

$$\frac{500}{400 - v^2} = \frac{25}{2}\left(\frac{1}{20+v} + \frac{1}{20-v}\right),$$

$$t + C = \frac{25}{2}\ln\left(\frac{20+v}{20-v}\right).$$

$$T = \frac{25}{2}\left[\ln\left(\frac{20+v}{20-v}\right)\right]_5^{15} = \frac{25}{2}\ln\left(\frac{35/5}{25/15}\right) = \frac{25}{2}\ln\left(\frac{21}{5}\right).$$

$$T = \frac{25}{2}\ln\left(\frac{21}{5}\right) \text{ s.}$$

(7)

(b) Apply $v = 2$ *when* $t = 0$ *to the integrated relation.*

$$0 + C = \frac{25}{2}\ln\left(\frac{22}{18}\right) = \frac{25}{2}\ln\left(\frac{11}{9}\right).$$

$$\ln\left(\frac{20+v}{20-v}\right) = \frac{2t}{25} + \ln\left(\frac{11}{9}\right).$$

$$\frac{20+v}{20-v} = \frac{11}{9}e^{2t/25}.$$

$$p = \frac{11}{9} \text{ and } q = \frac{2}{25}.$$

(2)

(c) The right-hand side is positive for every $t \geqslant 0$. *Since* $20 + v > 0$, *the fraction can be positive only while* $20 - v > 0$.

$$v < 20 \text{ m s}^{-1} \text{ for all } t \geqslant 0.$$

(1)

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Question 6

Question 6

(a) A uniform semicircular lamina has radius r. Its centre of mass is distance d from the midpoint O of its diameter. Use algebraic integration to show that $d = \frac{4r}{3\pi}$. **(4)**

A uniform template T consists of a $8a \times 2a$ rectangle and a semicircular ring centred at the midpoint O of DC, with outer radius $4a$ and inner radius $2a$.

led as a uniform lamina.

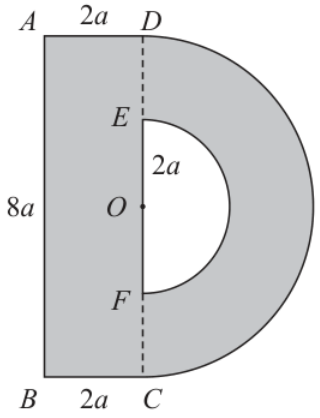


Figure 7

(b) Show that the distance of its centre of mass from DC is $\frac{32a}{3(8+3\pi)}$. **(5)**

T is suspended from A. A horizontal force of magnitude λW acts at B, holding it in equilibrium with AB vertical; the weight is W. (c) Find λ. **(3)**

Model solution

(a) Put the diameter on the x-axis and use vertical strips. A strip at x has area $2y\,dx$, where $y = \sqrt{r^2 - x^2}$. Symmetry means it is enough to integrate from 0 to r.

$$d\left(\frac{1}{2}\pi r^2\right) = \int_0^r x\left(2\sqrt{r^2 - x^2}\right) dx = 2\int_0^r x\sqrt{r^2 - x^2}\, dx.$$

$$2\int_0^r x\sqrt{r^2 - x^2}\, dx = \left[-\frac{2}{3}(r^2 - x^2)^{3/2}\right]_0^r = \frac{2}{3}r^3.$$

$$d = \frac{(2/3)r^3}{(1/2)\pi r^2} = \frac{4r}{3\pi}.$$

$$\boxed{d = \frac{4r}{3\pi}.}$$

(4)

(b) Use the rectangle plus the large semicircle minus the small semicircle. Areas give mass

ratios, and part (a) supplies each semicircle's centroid distance from its diameter DC.

part	*rectangle*	*large semicircle*	*removed small semicircle*
mass ratio	16	8π	2π
distance from DC	$-a$	$\frac{16a}{3\pi}$	$\frac{8a}{3\pi}$

$$(16+8\pi-2\pi)x = 16(-a)+8\pi\left(\frac{16a}{3\pi}\right)-2\pi\left(\frac{8a}{3\pi}\right).$$

$$(16+6\pi)x = \frac{64a}{3} \quad \Rightarrow \quad x = \frac{32a}{3(8+3\pi)}.$$

$$\textit{Distance from } DC = \frac{32a}{3(8+3\pi)}.$$

(5)

(c) Take moments about the suspension point A. *The weight acts vertically through the centre of mass, whose horizontal distance from* A *is* $2a+x$; *the horizontal force at* B *has perpendicular distance* $AB = 8a$.

$$W\left(2a+\frac{32a}{3(8+3\pi)}\right) = \lambda W(8a).$$

$$\lambda = \frac{2+32/[3(8+3\pi)]}{8} = \frac{9\pi+40}{12(8+3\pi)} = 0.3265\ldots$$

$$\lambda = 0.327 \textit{ (or } 0.33\textit{).}$$

(3)

Question 7

Question 7

A 0.4 kg particle moves in simple harmonic motion. Its maximum speed is 5 m s^{-1} and its maximum acceleration magnitude is 12.5 m s^{-2}.

(a) Show that the period is $T = \frac{4\pi}{5}$. **(4)**

(b) Find the time in each complete oscillation for which the particle is more than 0.5 m from the centre. **(4)**

(c) Find its kinetic energy when it is 0.2 m from the centre. **(3)**

Model solution

(a) For SHM of amplitude a, maximum speed is $a\omega$ and maximum acceleration is $a\omega^2$. Dividing the two equations isolates ω.

$$a\omega = 5, \qquad a\omega^2 = 12.5 \quad \Rightarrow \quad \omega = \frac{12.5}{5} = \frac{5}{2}.$$

$$T = \frac{2\pi}{\omega} = \frac{4\pi}{5}.$$

$$T = \frac{4\pi}{5} \text{ s.}$$

(4)

(b) First $a = 5/\omega = 2$. Taking $x = 2\sin(\frac{5}{2}t)$, the boundary of the required region is $|x| = 0.5$, so $|\sin(\frac{5}{2}t)| = 1/4$. In one period, the particle is within 0.5 m of the centre for four equal intervals, each of duration $\frac{\sin^{-1}(1/4)}{\omega}$.

$$\text{required time} = \frac{4\pi}{5} - \frac{4\sin^{-1}(1/4)}{5/2} = 2.108985\ldots \text{ s.}$$

$$2.11 \text{ s.}$$

(4)

(c) Use $v^2 = \omega^2(a^2 - x^2)$, which follows from the SHM energy relation.

$$v^2 = \left(\frac{5}{2}\right)^2 (2^2 - 0.2^2) = \frac{99}{4}.$$

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}(0.4)\left(\frac{99}{4}\right) = \frac{99}{20} \text{ J.}$$

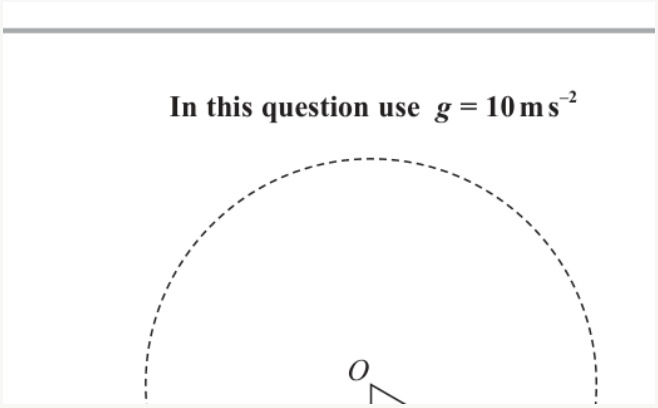
$$4.95 \text{ J.}$$

(3)

Question 8

Question 8

In this question use $g = 10\,\text{m s}^{-2}$. A 0.4 kg particle is attached to a light inextensible string of length 1.5 m, initially at rest vertically below O. It is projected horizontally at $6\,\text{m s}^{-1}$. When the string has turned through θ, it remains taut and the particle speed is v.



(a) Show that when $v = 3\sqrt{2}\,\text{m s}^{-1}$, the tension is 6.4 N. **(7)**
(b) Find the magnitude of the acceleration at this instant. **(5)**

Model solution

(a) Tension does no work, so conservation of mechanical energy relates the change in speed to the rise $r(1-\cos\theta)$.

$$\frac{1}{2}m(6)^2 = \frac{1}{2}m(3\sqrt{2})^2 + mg\{1.5(1-\cos\theta)\}.$$

$$18 = 9 + 15(1-\cos\theta) \quad \Rightarrow \quad \cos\theta = \frac{2}{5}.$$

Resolve radially towards O. The inward component of weight is $-mg\cos\theta$*, so Newton's second law is* $T - mg\cos\theta = mv^2/r$.

$$T - 4\left(\frac{2}{5}\right) = \frac{0.4(3\sqrt{2})^2}{1.5} = 4.8.$$

$$T = 4.8 + 1.6 = 6.4\ \textit{N}.$$

$$T = 6.4\ \textit{N}.$$

(7)

(b) The acceleration has perpendicular radial and tangential components. Circular motion gives $a_r = v^2/r$*, while the tangential component is the component of gravity along the tangent.*

$$\sin\theta = \sqrt{1-\left(\frac{2}{5}\right)^2} = \frac{\sqrt{21}}{5}, \qquad a_r = \frac{18}{1.5} = 12, \qquad a_t = 10\sin\theta = 2\sqrt{21}.$$

$$|\mathbf{a}| = \sqrt{12^2 + (2\sqrt{21})^2} = \sqrt{228} = 2\sqrt{57} = 15.099\ldots \ m\ s^{-2}.$$

15.1 *m* s^{-2} *(or* 15 *m* s^{-2} *to 2 s.f.).*

(5)

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