Maths with Matt

Physics and maths resources

Edexcel A Level Further Mathematics

9FM0/4D - Decision Mathematics 2

Monday 23 June 2025

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2025 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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Question 1

Question 1

Ravi chooses option A, B or C. The outcome profits and probabilities are shown in the decision tree.

avi can choose one of three options, A, B or C, when playing a game. The profit, in
ounds, associated with each outcome, and the corresponding probabilities, are shown
the decision tree in Figure 1.

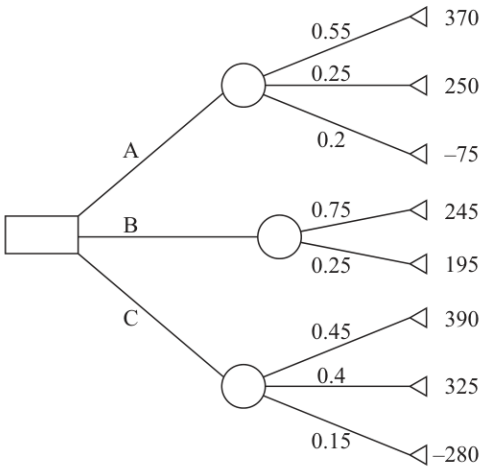


Figure 1

alculate the optimal EMV to determine Ravi's best course of action. You must make
our working clear.

(Total for Question 1 is 3 marks)

Calculate the optimal expected monetary value (EMV) and determine Ravi's best option.

Model solution

The outcomes are uncertain, so calculate the EMV of each option: multiply each profit by its probability and add. The option with the greatest EMV is preferred.

$$\begin{aligned}\text{EMV}(A) &= 0.55(370) + 0.25(250) + 0.20(-75) = 251.00,\\ \text{EMV}(B) &= 0.75(245) + 0.25(195) = 232.50,\\ \text{EMV}(C) &= 0.45(390) + 0.40(325) + 0.15(-280) = 263.50.\end{aligned}$$

Option C has the largest expected profit.

The optimal EMV is \$263.50, so Ravi should choose option C.

(3)

Question 2

Question 2

Five workers A–E are available for tasks P–S. Their times (minutes) are

	P	Q	R	S
A	25	32	43	28
B	41	37	–	38
C	43	35	37	39
D	40	33	37	41
E	37	38	43	40

Worker B cannot do R. Use the Hungarian algorithm to minimise total time.

Model solution

(a) The Hungarian algorithm needs a square assignment array. Add a dummy task X, with time 0 for every worker, to represent the one worker who receives no real task. Put a prohibitive large value, say 100, in BR; this prevents the forbidden assignment.

Add dummy column X of zeros, and replace BR by 100.

(2)

(b) Start with the augmented cost array and subtract the smallest entry in each row, then the smallest entry in each column.

	P	Q	R	S	X
A	0	7	18	3	75
B	4	0	63	1	63
C	8	0	2	4	65
D	7	0	4	8	67
E	0	1	6	3	63

$\longrightarrow$

	P	Q	R	S	X
A	0	7	16	2	12
B	4	0	61	0	0
C	8	0	0	3	2
D	7	0	2	7	4
E	0	1	4	2	0

Three lines are needed to cover all zeros, so this is not optimal. Using the smallest uncovered value and the Hungarian adjustment gives successively:

	P	Q	R	S	X
A	0	7	16	2	12
B	4	0	61	0	0
C	8	0	0	3	2
D	7	0	2	7	4
E	0	1	4	2	0

adjust by 1
cover by 3 lines $\longrightarrow$

	P	Q	R	S	X
A	0	5	12	0	10
B	6	0	59	0	0
C	10	0	0	3	2
D	9	0	0	7	4
E	2	1	4	2	0

adjust by 9
cover by 4 lines $\longrightarrow$

	P	Q	R	S	X
A	0	5	14	0	10
B	6	0	61	0	0
C	10	0	0	3	2
D	9	0	2	7	4
E	2	1	4	2	0

Now five lines are required, so an independent set of five zeros gives an optimal assignment. Choose A--P, B--S, C--R, D--Q and E--X.

A does P, B does S, C does R, D does Q; E is unassigned.

(6)

(c) Return to the original times, rather than the reduced array.

$$25 + 38 + 37 + 33 = 133.$$

$$\text{Least total time} = 133 \text{ minutes}.$$

(1)

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Question 3

Question 3

The transportation costs per unit, supplies and demands are

	P	Q	R	Supply
A	27	23	25	35
B	29	30	28	41
C	29	33	26	29
D	32	34	36	45
Demand	57	31	62	

(a) Find a north-west corner initial solution. (b) Do one stepping-stone iteration. (c) Formulate an LP.

Model solution

(a) The north-west corner method starts at AP and allocates as much as possible, then moves right when a demand is met and down when a supply is exhausted.

	P	Q	R
A	35		
B	22	19	
C		12	17
D			45

The initial basic feasible solution is as displayed.

(1)

(b) Use the occupied cells to find shadow costs r_i, c_j*, taking* $r_A = 0$*, so that* $r_i + c_j$ *equals the cost in every occupied cell.*

	A	B	C	D
r_i	0	2	5	15

	P	Q	R
c_j	27	28	21

For an empty cell the improvement index is cost $- r_i - c_j$.

	P	Q	R
A	$\times$	-5	4
B	$\times$	$\times$	5
C	-3	$\times$	$\times$
D	-10	-9	$\times$

DP has the most negative index, so it enters. Its closed stepping-stone route is $DP \to BP \to BQ \to CQ \to CR \to DR \to DP$*. Add* θ *at DP, BQ and CR and subtract* θ *at BP, CQ and DR.*

$$\theta = \min(22, 12, 45) = 12.$$

Thus CQ is the exiting cell and DP is the entering cell.

	P	Q	R
A	35		
B	10	31	
C			29
D	12		33

Enter DP, exit CQ; the improved allocation is as shown.

(4)

(c) Let x_{ij} be the number of units transported from supply point $i \in \{A, B, C, D\}$ to demand point $j \in \{P, Q, R\}$. Every unit has a stated transport cost, so minimise total transport cost subject to supply and demand limits.

Minimise $$C = 27x_{AP} + 23x_{AQ} + 25x_{AR} + 29x_{BP} + 30x_{BQ} + 28x_{BR} + 29x_{CP} + 33x_{CQ} + 26x_{CR} + 32x_{DP} + 34x_{DQ} + 36x_{DR}.$$

$$x_{AP} + x_{AQ} + x_{AR} \leqslant 35, \quad x_{BP} + x_{BQ} + x_{BR} \leqslant 41,$$
$$x_{CP} + x_{CQ} + x_{CR} \leqslant 29, \quad x_{DP} + x_{DQ} + x_{DR} \leqslant 45,$$
$$x_{AP} + x_{BP} + x_{CP} + x_{DP} = 57,$$
$$x_{AQ} + x_{BQ} + x_{CQ} + x_{DQ} = 31,$$
$$x_{AR} + x_{BR} + x_{CR} + x_{DR} = 62, \qquad x_{ij} \geqslant 0.$$

The LP is the objective function and constraints above.

(5)

Question 4

Question 4

Figure 2 is a directed network whose arc labels are (lower capacity, upper capacity).

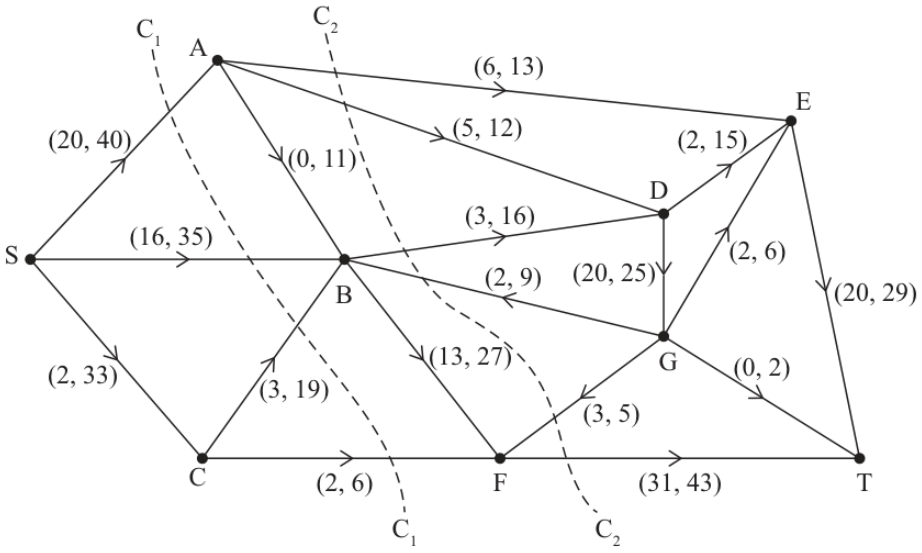


Figure 2

Figure 2 shows a capacitated, directed network. The network represents a system of pipes through which fluid can flow.

The weights on the arcs show the lower and upper capacities for the corresponding pipes, in litres per second.

(a) Calculate the capacity of

(i) cut C_1

For the initial flow, the excess and possible backflow values are shown in Figure 3.

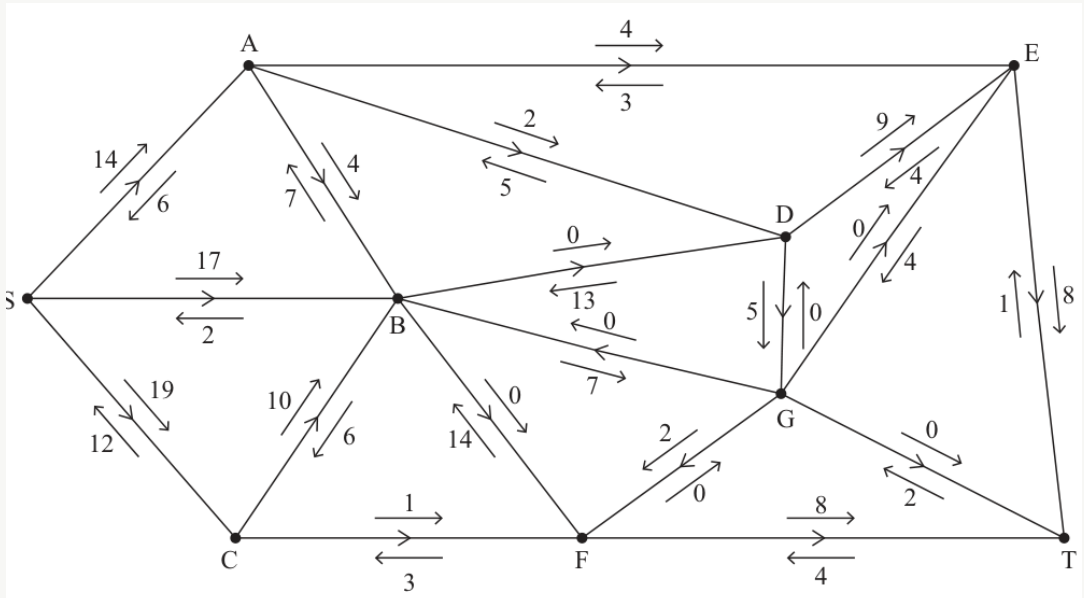


Figure 3

A scientist monitors the system of pipes in which a fluid now flows from the source, S, to the sink, T.

The scientist initialises the labelling procedure for this system. The excess and potential backflows for this are shown on the arrows either side of each arc in Figure 3.

(c) State the value of the initial flow.

(1)

Find and justify a maximum flow.

Model solution

(a) A cut capacity is the sum of upper capacities on arcs crossing from the source side to the sink side, less the lower capacities on arcs crossing in the reverse direction.

$$C_1 = 40 + 35 + 19 + 6 = 100, \qquad C_2 = 13 + 12 + 16 - 2 - 3 + 43 = 79.$$

$$(\text{i})\, C_1 = 100 \text{litress}^{-1}; (\text{ii})\, C_2 = 79 \text{litress}^{-1}.$$

(2)

(b) No flow can exceed the capacity of any cut. The smaller displayed cut therefore gives the useful bound.

$$\text{Themaximumflowisatmost} 79 \text{litress}^{-1}.$$

(1)

(c) Add the flows leaving S (or equivalently entering T) in the initial pattern.

$$\text{Initialflow} = 58 \text{litress}^{-1}.$$

(1)

(d) In the residual network, follow arcs with positive excess capacity; a reverse traversal uses available backflow. Augment each route by its bottleneck flow.

$$\text{SAET}: 4, \quad \text{SADET}: 2, \quad \text{SBGFT}: 2, \quad \text{SCFT}: 1.$$

The total increase is $4+2+2+1=9$.

$$\text{Maximumflowfoundbylabellingis} 58+9=67\text{litress}^{-1}.$$

(3)

(e) Applying those augmentations gives the following flow pattern (one value per arc). This is the requested maximum-flow drawing in tabular form.

arc	SA	SB	SC	AB	AD	AE	CB	CF	BF	BD	DG	DE	GB	GE	GF	GT	FT	ET
flow	32	20	15	7	12	13	9	6	27	16	20	8	7	6	5	2	38	27

$$\text{Amaximum}-\text{flowpatternistheonelistedabove, ofvalue67litress}^{-1}.$$

(1)

(f) To prove optimality, use the max-flow min-cut theorem. The cut $\{S, A, B, C, G\} \mid \{D, E, F, T\}$ *crosses the saturated arcs AE, AD, BD, DG, GE, GT, GF, BF and CF. Its capacity is*

$$13+12+16-20+6+2+5+27+6=67.$$

value of the flow $=67=$ *capacity of a cut.*

$$\text{Bythemax}-\text{flowmin}-\text{cuttheorem, theflowof67litress}^{-1}\text{ismaximal.}$$

(3)

Question 5

Question 5

Monthly orders from November to March are $2, 5, 6, 4, 3$. Up to five boats may be made monthly; making more than three incurs a $400 assistant cost, production overhead is $60 per boat, and storage is $70 per boat per month, with capacity three. Initial and final stock are zero. Use dynamic programming to minimise cost.

Model solution

(a) Work backwards. A state is the stock available at the start of a month, the action is the number made, and the destination is the stock after that month's delivery. For each feasible action, add production cost $60a$, 400 *if* $a > 3$, *storage cost* $70d$, *and the best future value. An asterisk marks the least value for that state.*

month	*state*	*action*	*destination*	*total value*
Mar	3	0	0	210^*
	2	1	0	200^*
	1	2	0	190^*
	0	3	0	180^*
Feb	3	1, 2, 3, 4	0, 1, 2, 3	$450^*, 520, 590, 1060$
	2	2, 3, 4, 5	0, 1, 2, 3	$440^*, 510, 980, 1050$
	1	3, 4, 5	0, 1, 2	$430^*, 900, 970$
	0	4, 5	0, 1	$820^*, 890$
Jan	3	3, 4, 5	0, 1, 2	$1210^*, 1280, 1350$
	2	4, 5	0, 1	$1600, 1270^*$
	1	5	0	1590^*
Dec	3	3, 4, 5	1, 2, 3	$1980^*, 2120, 2120$
	2	4, 5	1, 2	$2370, 2110^*$
	1	5	1	2360^*
Nov	0	3	1	2540^*
	0	4	2	2750
	0	5	3	2680

For example, from February state 2, making 2 boats leaves no stock and costs $140 + 120 + 180 = 440$; *making 5 leaves stock 3 and costs* $140 + 300 + 400 + 210 = 1050$. *The same calculation gives every entry in the table. Following the starred decisions from the initial state gives the production schedule.*

month	*Nov*	*Dec*	*Jan*	*Feb*	*Mar*
boats made	3	5	5	4	3

$$\text{Minimum cost} = \$2540, \text{with production } 3, 5, 5, 4, 3.$$

(12)

(b) November is now forced to use more than three boats. Of the feasible November actions, making four is the cheaper alternative: its total cost is 2750. *Compare this with the unconstrained minimum.*

$$2750 - 2540 = 140.$$

$$\text{Minimum additional cost} = \$140.$$

(1)

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Question 6

Question 6

Player A's pay-off matrix is $\begin{pmatrix} 4 & -2 & -5 \\ -1 & 1 & 3 \end{pmatrix}$. A third row $(4, -1, 1)$ is then added. Analyse the two games as specified.

Model solution

(a) A stable solution requires the row maximin to equal the column minimax.

row minima: $-5, -1,$ *so row maximin* $= -1;$ *column maxima:* $4, 1, 3,$ *so column minimax* $= 1.$

$$\text{Since} -1 \neq 1, \text{thereisnostablesolution.}$$

(2)

(b) If X is used with probability p*, Y is used with probability* $1 - p$*. For each pure reply by B, form A's expected pay-off; B will choose the lowest of these lines.*

$$P : -1 + 5p, \qquad Q : 1 - 3p, \qquad R : 3 - 8p.$$

The lower envelope is maximised where P and Q meet; R is above neither determining line there.

$$1 - 3p = -1 + 5p \implies p = \frac{1}{4}.$$

$$\text{AshouldplayXwithprobability}\frac{1}{4}\text{andYwithprobability}\frac{3}{4}.$$

(6)

(c) The entries include negative values, so add 5 to every pay-off before forming a simplex tableau. Let V *be the value of the augmented game and use slack variables* r, s, t, u*. The augmented matrix is* $\begin{pmatrix} 9 & 3 & 0 \\ 4 & 6 & 8 \\ 9 & 4 & 6 \end{pmatrix}$*. The constraints are written so that every column of B has expected pay-off at least* V*, and probabilities sum to one.*

b.v.	V	x	y	z	r	s	t	u	*Value*
r	1	−9	−4	−9	1	0	0	0	0
s	1	−3	−6	−4	0	1	0	0	0
t	1	0	−8	−6	0	0	1	0	0
u	0	1	1	1	0	0	0	1	1
P	−1	0	0	0	0	0	0	0	0

An initial simplex tableau is as displayed.

(4)

(d) The optimal tableau gives $x = 0$, $y = \frac{5}{7}$, *so the probability condition gives* $z = \frac{2}{7}$. *Substitute into the three augmented column pay-offs to find the value:*

$$V \leqslant \frac{38}{7}, \frac{38}{7}, \frac{52}{7} \implies V = \frac{38}{7}.$$

Subtracting the augmentation 5 gives the original 3-by-3 game value $\frac{38}{7} - 5 = \frac{3}{7}$. *From part (b), the 2-by-3 game value is* $1 - 3(\frac{1}{4}) = \frac{1}{4}$.

$$\frac{3}{7} - \frac{1}{4} = \frac{12-7}{28} = \frac{5}{28}.$$

$$\text{The } 3-\text{by}-3 \text{ game is } \frac{5}{28} \text{ better for A (values } \frac{3}{7} \text{ and } \frac{1}{4}).$$

(3)

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Question 7

Question 7

For non-zero constants α, β, k, with $\alpha \neq k$, $u_{n+1} = \alpha u_n - \beta k^n$. Find the general solution; use $u_0 = L, u_T = 0$; then apply the result to Taylor's loan.

Model solution

(a) The homogeneous recurrence has trial solution $u_n = m^n$, giving $m = \alpha$, so its complementary function is $A\alpha^n$. The forcing term is proportional to k^n, so try a particular solution Bk^n.

$$Bk^{n+1} = \alpha Bk^n - \beta k^n \quad \Longrightarrow \quad Bk - \alpha B = -\beta \quad \Longrightarrow \quad B = \frac{\beta}{\alpha - k}.$$

$$u_n = A\alpha^n + \frac{\beta k^n}{\alpha - k}.$$

(4)

(b) Apply each endpoint condition to the general solution, then eliminate the two unknown constants A and β.

$$u_0 = L: \quad A + \frac{\beta}{\alpha - k} = L, \qquad u_T = 0: \quad A\alpha^T + \frac{\beta k^T}{\alpha - k} = 0.$$

$$\beta = (L - A)(\alpha - k), \\ qquadA\alpha^T + (L - A)k^T = 0,$$

$$A = \frac{Lk^T}{k^T - \alpha^T}.$$

Substitute this value and $\beta = (L - A)(\alpha - k)$ back into the general solution and factor.

$$u_n = \frac{Lk^n}{1 - \left(\frac{k}{\alpha}\right)^T}\left[1 - \left(\frac{k}{\alpha}\right)^{T-n}\right].$$

$$u_n = \frac{Lk^n}{1 - (k/\alpha)^T}\left[1 - (k/\alpha)^{T-n}\right], as required.$$

(5)

(c) Each year the balance is first multiplied by 1.06 because of interest, while the repayments grow geometrically by 4%. The term βk^n is therefore the repayment in year $n + 1$.

$$(i)\alpha = 1.06; (ii)k = \\ 1.04; (iii)\beta is the repayment on the first anniversary of the loan.$$

(3)

(d) Use the particular solution with $T = 40$, $n = 30$, $k = 1.04$, $\alpha = 1.06$.

$$u_{30} = \frac{L(1.04)^{30}}{1 - \left(\frac{1.04}{1.06}\right)^{40}} \left[1 - \left(\frac{1.04}{1.06}\right)^{10}\right] \approx 1.05L.$$

Since $u_{30} \approx 1.05L >$ L, after 30 years the outstanding debt is larger than the original loan.

(2)

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