



Edexcel AS Further Mathematics

8FM0/01 - Core Pure Mathematics

Tuesday 14 May 2019

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2019 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

Contents

Question 1	2
Question 8	3
Question 9	4
Question 10	5
Question 2	6
Question 3	7
Question 4	8
Question 5	9
Question 6	10
Question 7	11

www.mathswithmatt.co.uk

Question 1

Question 1

$\mathbf{M} = \begin{pmatrix} 4 & -5 \\ 2 & -7 \end{pmatrix}$. (a) Show that $\mathbf{M}$ is non-singular. The transformation with matrix $\mathbf{M}$ maps triangle R to triangle S, whose area is 63 square units. (b) Find the area of R. (c) Show that $y = 2x$ is invariant.

Model solution

(a) $\det \mathbf{M} = 4(-7) - (-5)(2) = -18 \neq 0$, so $\mathbf{M}$ is non-singular.
multiplied by $|\det \mathbf{M}| = 18$. Hence $63 = 18 \text{ area}(\mathbf{R})$.

(2) (b) Areas are

$$\text{area}(\mathbf{R}) = \frac{7}{2} \text{ square units.}$$

(2) (c) A general point on $y = 2x$ is $\begin{pmatrix} x \\ 2x \end{pmatrix}$. Its image is $\begin{pmatrix} 4 & -5 \\ 2 & -7 \end{pmatrix} \begin{pmatrix} x \\ 2x \end{pmatrix} = \begin{pmatrix} -6x \\ -12x \end{pmatrix}$, which satisfies $y' = 2x'$. Thus every point on the line maps to the line, so it is invariant. (2)

www.mathswithmatt.co.uk

Question 8

Question 8

A pipeline passes through $P(-300, 400, -150)$ and $Q(300, 300, -50)$, in metres. (a) Find its vector equation. The mountainside is $2x + 3y - 5z = 300$. A new accessway is a shortest tunnel from the pipeline to $M(100, k, 100)$ on the mountainside. (b) Find where it meets the pipeline and its length. (c) Decide whether it should be built, given existing accessways OP, OQ . (d) Give one limitation of the model.

Model solution

(a) $\overrightarrow{PQ} = (600, -100, 100)$, so $\mathbf{r} = \begin{pmatrix} -300 \\ 400 \\ -150 \end{pmatrix} + \lambda \begin{pmatrix} 600 \\ -100 \\ 100 \end{pmatrix}$. (2) (b) As M is on the plane,

$200 + 3k - 500 = 300$, hence $k = 200$. Let $X = (-300, 400, -150) + \lambda(600, -100, 100)$. The shortest tunnel is perpendicular to the pipeline, so

$$((-400 + 600\lambda), (200 - 100\lambda), (-250 + 100\lambda)) \cdot (600, -100, 100) = 0.$$

Thus $\lambda = 3/4$, $X = (150, 325, -75)$, and $|MX| = \sqrt{50^2 + 125^2 + (-175)^2} = 25\sqrt{78}$.

$$(b)(i) X = (150, 325, -75).$$

$$(5) (b)(ii) |MX| = 25\sqrt{78} \text{ m} \approx 221 \text{ m}.$$

The tunnel length is $25\sqrt{78} \text{ m} \approx 221 \text{ m}$.

(2) (c) $|OP| \approx 522 \text{ m}$ and $|OQ| \approx 427 \text{ m}$, both significantly greater than 221 m.

Build the new accessway.

(2) (d) The mountainside need not be planar, so a plane may be an inadequate model. (1)

Question 9

Question 9

$f(x) = 2x^{1/3} + x^{-2/3}$, $x > 0$. The region bounded by $y = f(x)$, $x = 1/8$, the x -axis and $x = 8$ is rotated through θ radians about the x -axis. Its volume is $461/2$ units³. Find θ .

Model solution

The volume is $V = \frac{\theta}{2} \int_{1/8}^8 f(x)^2 dx$. Now

$$\int f(x)^2 dx = \int (4x^{2/3} + 4x^{-1/3} + x^{-4/3}) dx = \frac{12}{5}x^{5/3} + 6x^{2/3} - 3x^{-1/3}.$$

At 8 this is $993/10$, and at $1/8$ it is $-177/40$, so the definite integral is $4149/40$. Therefore

$$\frac{461}{2} = \frac{\theta}{2} \cdot \frac{4149}{40}.$$

$$\theta = \frac{40}{9} \text{ radians.}$$

(8)

Question 10

Question 10

A chimpanzee population is modelled by $\begin{pmatrix} J_{n+1} \\ A_{n+1} \end{pmatrix} = \begin{pmatrix} a & 0.15 \\ 0.08 & 0.82 \end{pmatrix} \begin{pmatrix} J_n \\ A_n \end{pmatrix}$. Initially there are 64000 chimpanzees; after one year $J_1 = 15360$, $A_1 = 43008$. (a) Interpret a . (b) Find the inverse matrix, then a , then the first-year change in juveniles. (c) Comment on short-term suitability if juveniles are known to be declining. (d) Refine the model if adults cease reproducing at age 40.

Model solution

(a) a is the proportion of juveniles that survive and remain juvenile next year. (1) (b)(i) The determinant is $0.82a - 0.012$, hence $\begin{pmatrix} a & 0.15 \\ 0.08 & 0.82 \end{pmatrix}^{-1} = \frac{1}{0.82a - 0.012} \begin{pmatrix} 0.82 & -0.15 \\ -0.08 & a \end{pmatrix}$. (3) (b)(ii) Applying this inverse to $\begin{pmatrix} 15360 \\ 43008 \end{pmatrix}$ gives $\begin{pmatrix} J_0 \\ A_0 \end{pmatrix} = \frac{1}{0.82a - 0.012} \begin{pmatrix} 6144 \\ 43008a - 1228.8 \end{pmatrix}$. Using $J_0 + A_0 = 64000$,

$$4915.2 + 43008a = 64000(0.82a - 0.012), \quad a = \frac{5683.2}{9472} = 0.60.$$

$$a = 0.60.$$

$$(3) (b)(iii) J_0 = 6144/0.48 = 12800, \text{ so } J_1 - J_0 = 15360 - 12800 = 2560.$$

An increase of 2560 juvenile chimpanzees.

(2) (c) It predicts an increase when juveniles are known to be declining, so it is not suitable in the short term.

(1) (d) Introduce mature, non-reproducing chimpanzees M_n : $\begin{pmatrix} J_{n+1} \\ A_{n+1} \\ M_{n+1} \end{pmatrix} = \begin{pmatrix} a & b & 0 \\ 0.08 & c & 0 \\ 0 & d & e \end{pmatrix} \begin{pmatrix} J_n \\ A_n \\ M_n \end{pmatrix}$. The zero prevents mature chimpanzees producing juveniles. (2)

Question 2

Question 2

The equation $2x^3 + 6x^2 - 3x + 12 = 0$ has roots α, β, γ . Without solving it, find the cubic with roots $\alpha + 3, \beta + 3, \gamma + 3$, in the form $pw^3 + qw^2 + rw + s = 0$.

Model solution

Put $w = x + 3$, so $x = w - 3$. Substitution gives

$$2(w - 3)^3 + 6(w - 3)^2 - 3(w - 3) + 12 = 0.$$

Expanding and collecting terms,

$$2w^3 - 18w^2 + 54w - 54 + 6w^2 - 36w + 54 - 3w + 9 + 12 = 0.$$

$$2w^3 - 12w^2 + 15w + 21 = 0.$$

(5)

www.mathswithmatt.co.uk

Question 3

Question 3

Prove by mathematical induction that, for $n \in \mathbb{N}$, $\sum_{r=1}^n \frac{1}{(2r-1)(2r+1)} = \frac{n}{2n+1}$.

Model solution

For $n = 1$, the left side is $1/(1 \cdot 3) = 1/3$, and the right side is $1/(2 + 1) = 1/3$. Thus the result is true for $n = 1$. Assume it is true for $n = k$, so $\sum_{r=1}^k \frac{1}{(2r-1)(2r+1)} = \frac{k}{2k+1}$.

Then

$$\sum_{r=1}^{k+1} \frac{1}{(2r-1)(2r+1)} = \frac{k}{2k+1} + \frac{1}{(2k+1)(2k+3)} = \frac{k(2k+3) + 1}{(2k+1)(2k+3)}.$$

Since $k(2k+3) + 1 = 2k^2 + 3k + 1 = (k+1)(2k+1)$, this is $\frac{k+1}{2k+3} = \frac{k+1}{2(k+1)+1}$. Hence the statement is true for $k+1$, and therefore for every $n \in \mathbb{N}$. (6)

Question 4

Question 4

The line l is $\frac{x+2}{1} = \frac{y-5}{-1} = \frac{z-4}{-3}$, and the plane Π is $\mathbf{r} \cdot (\mathbf{i} - 2\mathbf{j} + \mathbf{k}) = -7$. Determine whether l intersects Π once, lies in Π , or is parallel to Π without intersecting it.

Model solution

A point on l is $(-2, 5, 4)$, and its direction vector is $(1, -1, -3)$. The normal to Π is $(1, -2, 1)$.

$$(1, -1, -3) \cdot (1, -2, 1) = 1 + 2 - 3 = 0,$$

so the line is parallel to the plane. For its point, $(-2, 5, 4) \cdot (1, -2, 1) = -8 \neq -7$, so it is not in the plane.

l is parallel to Π and does not intersect it.

(5)

www.mathswithmatt.co.uk

Question 5

Question 5

In an Argand diagram, $z_1 = -2$, $z_2 = -1 + 2i$, $z_3 = 1 + i$. (a) Explain why they cannot all be roots of a quartic with real coefficients. (b) Show that $\arg\left(\frac{z_2 - z_1}{z_3 - z_1}\right) = \pi/4$. (c) Hence show $\arctan 2 - \arctan(1/3) = \pi/4$. (d) State the region satisfying $|z + 2| \leq |z - 1 - i|$.

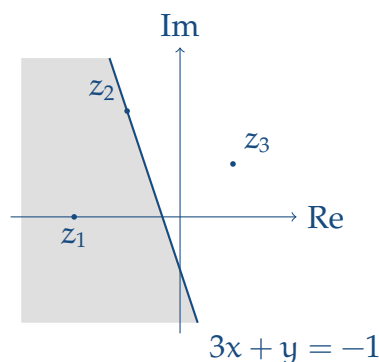
Model solution

(a) Non-real roots of a polynomial with real coefficients occur in conjugate pairs. Since both $-1 + 2i$ and $1 + i$ are roots, $-1 - 2i$ and $1 - i$ would also be roots, together with -2 : five roots. A quartic cannot have these. (2) (b) $z_2 - z_1 = 1 + 2i$ and $z_3 - z_1 = 3 + i$, so

$$\frac{z_2 - z_1}{z_3 - z_1} = \frac{1 + 2i}{3 + i} = \frac{(1 + 2i)(3 - i)}{10} = \frac{1 + i}{2}.$$

Its argument is $\pi/4$.

(3) (c) Taking arguments in part (b) gives $\arg(1 + 2i) - \arg(3 + i) = \arctan 2 - \arctan(1/3) = \pi/4$. (2) (d) Writing $z = x + iy$ and squaring both sides gives $(x + 2)^2 + y^2 \leq (x - 1)^2 + (y - 1)^2$, hence $3x + y \leq -1$. This is the closed half-plane on the $(-2, 0)$ side of the perpendicular bisector of the two points.



$$3x + y \leq -1.$$

(2)

Question 6

Question 6

The r th marble has mass $(7 + 3r)$ grams, $r = 1, \dots, n$. (a) Show that the mean is $\frac{1}{2}(3n + 17)$. Given $n = 85$, (b) use the standard summation formulae to find the standard deviation, in grams, to one decimal place.

Model solution

$$(a) \bar{x} = \frac{1}{n} \sum_{r=1}^n (7 + 3r) = \frac{1}{n} \left(7n + 3 \frac{n(n+1)}{2} \right) = \frac{1}{2}(3n + 17). (3) (b) \text{ With } n = 85, \bar{x} = 136.$$

Also

$$\begin{aligned} \sum x^2 &= \sum_{r=1}^{85} (7 + 3r)^2 = 49(85) + 42 \sum_{r=1}^{85} r + 9 \sum_{r=1}^{85} r^2 \\ &= 4165 + 42(3655) + 9 \left(\frac{85 \cdot 86 \cdot 171}{6} \right) = 2032690. \end{aligned}$$

$$\text{Thus } \sigma^2 = \frac{2032690}{85} - 136^2 = 5418.$$

$$\sigma = \sqrt{5418} = 73.6 \text{ g (1 d.p.)}$$

(6)

Question 7

Question 7

$f(z) = z^3 - 8z^2 + pz - 24$, where p is real. Its distinct roots are $\alpha, \beta, \alpha + \frac{12}{\alpha} - \beta$. (a) Solve $f(z) = 0$ completely. (b) Hence find p .

Model solution

By the sum of roots, $\alpha + \beta + (\alpha + 12/\alpha - \beta) = 8$. Therefore $2\alpha + 12/\alpha = 8$, so $\alpha^2 - 4\alpha + 6 = 0$, giving $\alpha = 2 \pm i\sqrt{2}$. Since f has real coefficients, the conjugate root is $2 \mp i\sqrt{2}$. Hence $z^2 - 4z + 6$ is a factor of $f(z)$. The remaining root is 4, since the product of the roots is 24.

$$z = 2 + i\sqrt{2}, 2 - i\sqrt{2}, 4.$$

$$(6) \text{ Thus } f(z) = (z - 4)(z^2 - 4z + 6) = z^3 - 8z^2 + 22z - 24.$$

$$p = 22.$$

(2)

www.mathswithmatt.co.uk