



Edexcel AS Further Mathematics

8FM0/01 - Core Pure Mathematics

Tuesday 17 May 2022

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2022 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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Question 1

Question 1

$A = \begin{pmatrix} 4 & -1 \\ 7 & 2 \\ -5 & 8 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 3 & 2 \\ -1 & 6 & 5 \end{pmatrix}$, $C = \begin{pmatrix} -5 & 2 & 1 \\ 4 & 3 & 8 \\ -6 & 11 & 2 \end{pmatrix}$. Given I is the 3×3 identity matrix: (a) (i) show $AB - 3C + kI = 0$, stating k ; (ii) explain why there is no m for which $BA - 3C + mI = 0$. (b) Use C to solve $-5x + 2y + z = -14$, $4x + 3y + 8z = 3$, $-6x + 11y + 2z = 7$.

Model solution

(a)(i) Multiply in the specified order, then compare with $3C$; a scalar multiple of I reveals the required constant.

$$AB = \begin{pmatrix} 9 & 6 & 3 \\ 12 & 33 & 24 \\ -18 & 33 & 30 \end{pmatrix}, \quad 3C = \begin{pmatrix} -15 & 6 & 3 \\ 12 & 9 & 24 \\ -18 & 33 & 6 \end{pmatrix}, \quad AB - 3C = 24I.$$

$$AB - 3C - 24I = 0, \text{ so } k = -24.$$

(3) (a)(ii) BA is 2×2 , whereas $3C$ and I are 3×3 . Matrices of different dimensions cannot be subtracted, so no such m exists. (1) (b) The coefficient matrix is C , so use its inverse to isolate the column of unknowns.

$$C \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -14 \\ 3 \\ 7 \end{pmatrix} \implies \begin{pmatrix} x \\ y \\ z \end{pmatrix} = C^{-1} \begin{pmatrix} -14 \\ 3 \\ 7 \end{pmatrix} = \begin{pmatrix} 7/2 \\ 3 \\ -5/2 \end{pmatrix}.$$

$$x = \frac{7}{2}, \quad y = 3, \quad z = -\frac{5}{2}.$$

(3)

Question 6

Question 6

A tennis ball follows $\mathbf{r} = (-4.1 + 9\lambda - 2.3\lambda^2)\mathbf{i} + (-10.25 + 15\lambda)\mathbf{j} + (0.84 + 0.8\lambda - \lambda^2)\mathbf{k}$, $\lambda \geq 0$, in metres. Its direction is $(9 - 4.6\lambda)\mathbf{i} + 15\mathbf{j} + (0.8 - 2\lambda)\mathbf{k}$. Find (a) when it hits the ground, (b) its direction then, (c) its angle with the ground, (d) its position at the net $\mathbf{r} \cdot \mathbf{j} = 0$, (e) whether it clears a 0.9 m net in the model, and (f) whether it actually will.

Model solution

(a) Ground level means that the vertical, $\mathbf{k}$, component is zero.

$$0.84 + 0.8\lambda - \lambda^2 = 0 \implies \lambda = -0.6, 1.4.$$

The parameter is non-negative.

$$\lambda = 1.4.$$

(2) (b) Substitute this parameter into the supplied direction vector.

$$\mathbf{v} = (9 - 4.6(1.4))\mathbf{i} + 15\mathbf{j} + (0.8 - 2(1.4))\mathbf{k} = 2.56\mathbf{i} + 15\mathbf{j} - 2\mathbf{k}.$$

$$2.56\mathbf{i} + 15\mathbf{j} - 2\mathbf{k}.$$

(1) (c) The ground is horizontal, so find the angle to the horizontal component (2.56, 15, 0).

$$\sin \theta = \frac{|-2|}{\sqrt{2.56^2 + 15^2 + (-2)^2}}, \quad \theta = 7.48 \dots^\circ.$$

The ball hits the ground at 7.5° (to 1 d.p.).

(4) (d) Being in the net plane means the $\mathbf{j}$ -component is zero.

$$-10.25 + 15\lambda = 0 \implies \lambda = \frac{41}{60}.$$

$$\mathbf{r} = \frac{35137}{36000}\mathbf{i} + 0\mathbf{j} + \frac{3311}{3600}\mathbf{k}.$$

$$\mathbf{r} \approx 0.976\mathbf{i} + 0.920\mathbf{k}.$$

(3) (e) At the net the ball's height is 0.920 m, greater than the modelled net height 0.9 m.

According to the model, it passes over the net.

(1) (f) The particle model ignores the ball's diameter. Its clearance is only about 2 cm, smaller than a tennis ball's diameter, so the real ball may hit or clip the net.

It cannot reliably be concluded that the ball actually passes over.

(2)

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Question 7

Question 7

Prove by mathematical induction that, for $n \in \mathbb{N}$, $\begin{pmatrix} -5 & 9 \\ -4 & 7 \end{pmatrix}^n = \begin{pmatrix} 1-6n & 9n \\ -4n & 1+6n \end{pmatrix}$.

Model solution

Induction requires a base case and an inductive step.

$$\text{For } n = 1, \quad \begin{pmatrix} 1-6 & 9 \\ -4 & 1+6 \end{pmatrix} = \begin{pmatrix} -5 & 9 \\ -4 & 7 \end{pmatrix}.$$

Assume the result holds for $n = k$. Then

$$\begin{aligned} M^{k+1} &= \begin{pmatrix} 1-6k & 9k \\ -4k & 1+6k \end{pmatrix} \begin{pmatrix} -5 & 9 \\ -4 & 7 \end{pmatrix} = \begin{pmatrix} -5-6k & 9k+9 \\ -4k-4 & 7+6k \end{pmatrix} \\ &= \begin{pmatrix} 1-6(k+1) & 9(k+1) \\ -4(k+1) & 1+6(k+1) \end{pmatrix}. \end{aligned}$$

This is the required form for $k+1$; hence it holds for all $n \in \mathbb{N}$.

(6)

Question 8

Question 8

A 16 cm vase has base and opening diameter 8 cm; its halfway cross-section also has diameter 8 cm. It is modelled by rotating a curve about the x -axis for $-8 \leq x \leq 8$. Model A is $y = a - 2 \sin\left(\frac{45x}{2}^\circ\right)$; Model B is $y = a + x(x-8)(x+8)/100$. (a) Find a . (b) For each model find the distance from the base to the widest point and its diameter. (c) Choose a model given that the widest point is 12 cm across just over 3 cm from the base. (d) Find Model B's volume. (e) Comment given that the measured capacity is 900 ml.

Model solution

(a) The radius at the base, top and halfway point is 4 cm.

$$a = 4.$$

(1) (b) For Model A, the maximum radius occurs when the sine is -1 , at $x = -4$. This is 4 cm from the base $x = -8$, and $y = 6$, so its diameter is 12 cm. (2) For Model B, differentiate to locate the widest stationary point.

$$y = 4 + \frac{x^3 - 64x}{100}, \quad \frac{dy}{dx} = \frac{3x^2 - 64}{100} = 0 \implies x = \pm \frac{8}{\sqrt{3}}.$$

The negative value gives the largest radius.

$$\text{distance} = 8 - \frac{8}{\sqrt{3}} = 3.38 \dots \text{ cm}, \quad y\left(-\frac{8}{\sqrt{3}}\right) = 5.97 \dots, \quad \text{diameter} = 11.9 \text{ cm}.$$

Model A: 4 cm, 12 cm; Model B: 3.38 cm, 11.9 cm.

(5) (c) Both diameters are close to 12 cm, but Model B places the widest point much closer to just over 3 cm from the base.

Model B is more appropriate.

(1) (d) Rotation about the x -axis calls for $V = \pi \int y^2 dx$. Expand before integrating algebraically.

$$\begin{aligned} V &= \pi \int_{-8}^8 \left(4 - \frac{16}{25}x + \frac{x^3}{100}\right)^2 dx \\ &= \pi \int_{-8}^8 \left(16 - \frac{128}{25}x + \frac{256}{625}x^2 + \frac{2}{25}x^3 - \frac{8}{625}x^4 + \frac{x^6}{10000}\right) dx \\ &= \pi \left[16x - \frac{64}{25}x^2 + \frac{256}{1875}x^3 + \frac{x^4}{50} - \frac{8x^5}{3125} + \frac{x^7}{70000}\right]_{-8}^8. \end{aligned}$$

$$V = 287.9566 \dots \pi = 904.6 \dots \text{ cm}^3 \approx 905 \text{ cm}^3.$$

(5) (e) $1 \text{ cm}^3 = 1 \text{ ml}$, so the observed capacity is $1000 - 100 = 900 \text{ ml}$. The prediction is about 905 ml, which is very close.

Model B is suitable in light of this measurement.

(1)

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Question 2

Question 2

(a) Express $w = 4\sqrt{3} - 4i$ as $r(\cos \theta + i \sin \theta)$, where $r > 0$ and $-\pi < \theta \leq \pi$. (b) On one Argand diagram show w and $\arg(z + 10i) = \pi/3$. (c) Hence find the minimum distance of w from this locus.

Model solution

(a) *Modulus--argument form comes from the radius and direction of the complex number.*

$$|w| = \sqrt{(4\sqrt{3})^2 + (-4)^2} = 8, \quad \tan \theta = -\frac{1}{\sqrt{3}}.$$

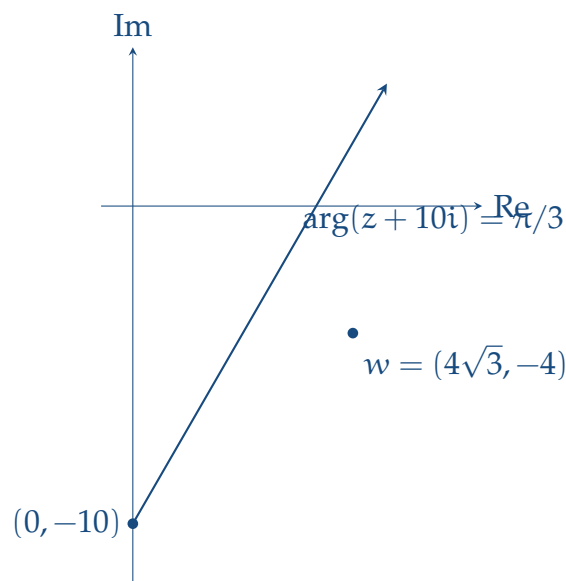
The point is in quadrant IV, hence $\theta = -\pi/6$.

$$w = 8(\cos(-\pi/6) + i \sin(-\pi/6)).$$

(4) (b) *Plot w at $(4\sqrt{3}, -4)$. Since $z + 10i$ is the vector from $(0, -10)$ to z , the locus is the half-line*

$$y + 10 = \sqrt{3}x, \quad x \geq 0,$$

starting at $(0, -10)$, at angle $\pi/3$ to the positive real axis.



(3) (c) *The shortest distance is perpendicular to the locus. If X is the foot of the perpendicular, the right triangle with hypotenuse 10 gives $OX = 10 \sin(\pi/6) = 5$. Since $OW = 8$, $WX = 8 - 5$.*

$$\text{Minimum distance} = 3.$$

(3)

Question 3

Question 3

$P = (8, 3, 2)$. Q is its reflection in $y = 0$, and R is its rotation through 120° anticlockwise about the y -axis using the stated rotation matrix. Find Q , R , $|PR|$, prove $PR \perp PQ$, and find the area of PQR .

Model solution

(a) Reflection in $y = 0$ reverses only the y -coordinate.

$$Q = (8, -3, 2).$$

(1) (b) Substitute $\theta = 120^\circ$ in the given matrix.

$$R = \begin{pmatrix} -1/2 & 0 & \sqrt{3}/2 \\ 0 & 1 & 0 \\ -\sqrt{3}/2 & 0 & -1/2 \end{pmatrix} \begin{pmatrix} 8 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} -4 + \sqrt{3} \\ 3 \\ -1 - 4\sqrt{3} \end{pmatrix}.$$

$$R = (-4 + \sqrt{3}, 3, -1 - 4\sqrt{3}).$$

(2) (c) Use the magnitude of the displacement vector.

$$\vec{PR} = (-12 + \sqrt{3}, 0, -3 - 4\sqrt{3}), \quad |PR|^2 = (-12 + \sqrt{3})^2 + (-3 - 4\sqrt{3})^2 = 204.$$

$$|PR| = 2\sqrt{51}.$$

(2) (d) A zero scalar product proves perpendicularity.

$$\vec{PQ} = (0, -6, 0), \quad \vec{PR} \cdot \vec{PQ} = 0.$$

$$PR \perp PQ.$$

(1) (e) The triangle is right-angled at P , so take half the product of its perpendicular sides.

$$\text{Area} = \frac{1}{2}(6)(2\sqrt{51}).$$

$$6\sqrt{51}.$$

(2)

Question 4

Question 4

The roots of $3x^4 + 5x^3 - 7x + 6 = 0$ are $\alpha, \beta, \gamma, \delta$. Without solving, find (i) $\sum \alpha^2$, (ii) $\sum 2/\alpha$, and (iii) $(3 - \alpha)(3 - \beta)(3 - \gamma)(3 - \delta)$.

Model solution

Vieta's relations are appropriate because every requested expression is symmetric in the roots. Dividing by 3 gives

$$\sum \alpha = -\frac{5}{3}, \quad \sum_{i < j} \alpha_i \alpha_j = 0, \quad \sum_{i < j < k} \alpha_i \alpha_j \alpha_k = \frac{7}{3}, \quad \alpha\beta\gamma\delta = 2.$$

(i) Square the root sum and remove twice the pair-product sum.

$$\sum \alpha^2 = \left(-\frac{5}{3}\right)^2 - 2(0) = \frac{25}{9}.$$

$$\frac{25}{9}.$$

(3) (ii) Adding reciprocals gives the triple-product sum divided by the total product.

$$\sum \frac{1}{\alpha} = \frac{7/3}{2} = \frac{7}{6}.$$

$$\sum \frac{2}{\alpha} = \frac{7}{3}.$$

(3) (iii) Evaluate the polynomial at 3; its leading coefficient is 3.

$$3 \prod (3 - \alpha) = 3(3)^4 + 5(3)^3 - 7(3) + 6 = 363.$$

$$121.$$

(3)

Question 5

Question 5

(a) Show $\sum_{r=1}^n (3r^2 - 17r - 25) = n(n^2 - An - B)$. (b) Explain why $\sum_{r=1}^{3k} r \tan(60r)^\circ = -k\sqrt{3}$. (c) Solve $\sum_{r=5}^n (3r^2 - 17r - 25) = 15 \left[\sum_{r=6}^{3n} r \tan(60r)^\circ \right]^2$.

Model solution

(a) Split the sum and apply the standard formulae for $\sum r$ and $\sum r^2$.

$$3 \frac{n(n+1)(2n+1)}{6} - 17 \frac{n(n+1)}{2} - 25n = n(n^2 - 7n - 33).$$

$$A = 7, B = 33.$$

(4) (b) Tangent repeats every 180° . Each block of three has sum $\sqrt{3} - 2\sqrt{3} + 0 = -\sqrt{3}$, and there are k blocks.

$$\sum_{r=1}^{3k} r \tan(60r)^\circ = -k\sqrt{3}.$$

(2) (c) Subtract the initial terms from the results above.

$$\sum_{r=5}^n (3r^2 - 17r - 25) = n(n^2 - 7n - 33) + 180.$$

$$\sum_{r=1}^5 r \tan(60r)^\circ = -2\sqrt{3}, \quad \sum_{r=6}^{3n} r \tan(60r)^\circ = (2 - n)\sqrt{3}.$$

$$n(n^2 - 7n - 33) + 180 = 45(n - 2)^2 \implies n(n - 3)(n - 49) = 0.$$

The sum beginning at $r = 5$ rules out $n = 0, 3$.

$$n = 49.$$

(6)