



Edexcel AS Further Mathematics

8FM0/21 - Further Pure Mathematics 1

Friday 16 May 2025

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2025 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

Contents

Question 1	2
Question 2	4
Question 3	5
Question 4	7
Question 5	8

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Question 1

Question 1

The surface temperature of a lake is modelled by

$$S = 12 - \frac{15}{2} \cos x^\circ - \frac{27}{10} \sin x^\circ,$$

where S is in degrees Celsius and x is the number of days after the start of the year.

(a) Find the temperature at the start of the year.

Using $t = \tan\left(\frac{x}{2}\right)$, (b) show that $S = \frac{At^2 + Bt + C}{10(1 + t^2)}$, where A , B and C are integers.

(c) Hence determine the number of days after the start of the year when the temperature is 10°C for the second time, to the nearest day.

Model solution

(a) At the start of the year $x = 0$. Substitute this directly into the model.

$$S = 12 - \frac{15}{2} \cos 0^\circ - \frac{27}{10} \sin 0^\circ = 12 - \frac{15}{2} = \frac{9}{2}.$$

$$\frac{9}{2}^\circ\text{C} = 4.5^\circ\text{C}.$$

(b) The stated half-angle substitution tells us to use $\cos x = \frac{1-t^2}{1+t^2}$ and $\sin x = \frac{2t}{1+t^2}$. This puts both trigonometric terms over a common denominator. (1)

$$\begin{aligned} S &= 12 - \frac{15}{2} \left(\frac{1-t^2}{1+t^2} \right) - \frac{27}{10} \left(\frac{2t}{1+t^2} \right) \\ &= \frac{120(1+t^2) - 75(1-t^2) - 54t}{10(1+t^2)} \\ &= \frac{120 + 120t^2 - 75 + 75t^2 - 54t}{10(1+t^2)} \\ &= \frac{195t^2 - 54t + 45}{10(1+t^2)}. \end{aligned}$$

$$A = 195, B = -54, C = 45.$$

(c) Set the model equal to 10. Part (b) gives a quadratic in t , which can then be converted back to x . (3)

$$\frac{195t^2 - 54t + 45}{10(1+t^2)} = 10 \Rightarrow 195t^2 - 54t + 45 = 100 + 100t^2 \Rightarrow 95t^2 - 54t - 55 = 0.$$

$$t = \frac{54 \pm \sqrt{(-54)^2 - 4(95)(-55)}}{190} = \frac{54 \pm \sqrt{23816}}{190},$$

so $t = 1.096\dots$ or $t = -0.528\dots$. Since $t = \tan(x/2)$, these give

$$x = 2 \tan^{-1}(1.096\dots) = 95.2\dots^\circ,$$

and, for the second occurrence in the cycle,

$$x = 2 \tan^{-1}(-0.528\dots) + 360^\circ = 304.3\dots^\circ.$$

The latter is the second time after the start of the year at which the temperature is 10°C .

304 days.

(5)

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Question 2

Question 2

Water leaks from a hole in the base of a large spherical tank. Its depth H metres is modelled by

$$3(5H - H^2) \frac{dH}{dt} = -4\sqrt{H}, \quad 0 < H < 5,$$

where t is in hours after the leak started. The depth after 10 minutes was 2 m.

Use two applications of $\left(\frac{dy}{dx}\right)_n \approx \frac{y_{n+1} - y_n}{h}$ to estimate the depth one hour after the leak started.

Model solution

Two equal Euler steps take us from 10 minutes to 60 minutes, so each step is 25 minutes. Euler's formula uses the gradient at the start of each step. The depth gradients should be negative because water is leaking.

$$h = \frac{25}{60} = \frac{5}{12} \text{ hours}, \quad H_0 = 2.$$

First rearrange the differential equation to obtain the rate of change of depth.

$$\frac{dH}{dt} = \frac{-4\sqrt{H}}{3(5H - H^2)} = \frac{-4}{3\sqrt{H}(5 - H)}.$$

$$\left(\frac{dH}{dt}\right)_0 = \frac{-4}{3\sqrt{2}(5 - 2)} = -0.314\dots$$

$$H_1 = H_0 + h \left(\frac{dH}{dt}\right)_0 = 2 + \frac{5}{12}(-0.314\dots) = 1.86905\dots$$

This estimates the depth after 35 minutes. Repeat the Euler step using the gradient at this new depth.

$$\left(\frac{dH}{dt}\right)_1 = \frac{-4}{3\sqrt{1.86905\dots}(5 - 1.86905\dots)} = -0.311\dots$$

$$H_2 = 1.86905\dots + \frac{5}{12}(-0.311\dots) = 1.739\dots$$

The estimated depth one hour after the leak started is 1.74 m.

(7)

Question 3

Question 3

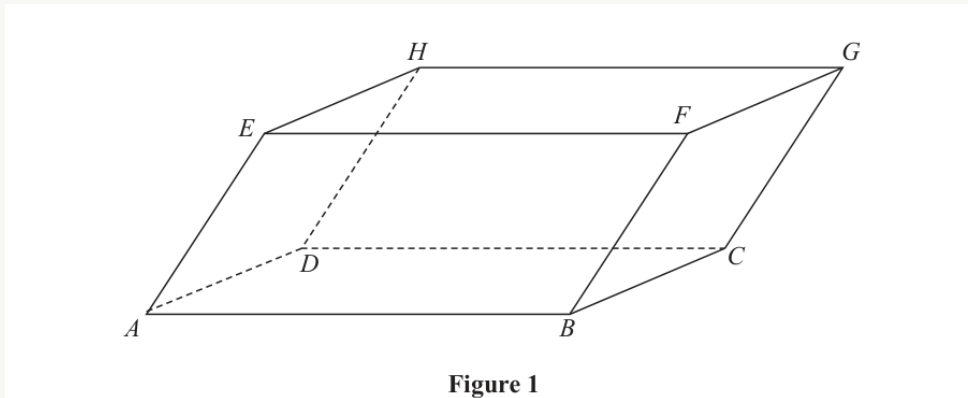


Figure 1

The figure shows the parallelepiped ABCDHEFG. Relative to an origin O,

$$A = (3, 2, 7), \quad B = (4, 4, 3), \quad D = (4, 1, t), \quad E = (t, -1, 10).$$

(a) Determine $\vec{AB} \times \vec{AD}$ in simplest form in terms of t .

Given that the volume of the parallelepiped is 9, (b) determine the possible values of t .

Model solution

(a) The cross product is appropriate because it is perpendicular to the parallelogram spanned by two edges. Form the edge vectors by subtracting the coordinates of A.

$$\vec{AB} = \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix}, \quad \vec{AD} = \begin{pmatrix} 1 \\ -1 \\ t-7 \end{pmatrix}.$$

$$\begin{aligned} \vec{AB} \times \vec{AD} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & -4 \\ 1 & -1 & t-7 \end{vmatrix} \\ &= (2(t-7) - 4)\mathbf{i} - ((t-7) + 4)\mathbf{j} + (-1 - 2)\mathbf{k} \\ &= (2t - 18)\mathbf{i} + (3 - t)\mathbf{j} - 3\mathbf{k}. \end{aligned}$$

$$\vec{AB} \times \vec{AD} = \begin{pmatrix} 2t - 18 \\ 3 - t \\ -3 \end{pmatrix}.$$

(b) The magnitude of a scalar triple product is the volume. The third edge from A is $\vec{AE}$,⁽³⁾ and the signed triple product may be 9 or -9.

$$\vec{AE} = \begin{pmatrix} t-3 \\ -3 \\ 3 \end{pmatrix},$$

$$\begin{aligned}\vec{AE} \cdot (\vec{AB} \times \vec{AD}) &= (t-3)(2t-18) - 3(3-t) + 3(-3) \\ &= 2t^2 - 21t + 36.\end{aligned}$$

$$2t^2 - 21t + 36 = 9 \Rightarrow 2t^2 - 21t + 27 = 0 \Rightarrow t = 9 \text{ or } \frac{3}{2},$$

$$2t^2 - 21t + 36 = -9 \Rightarrow 2t^2 - 21t + 45 = 0 \Rightarrow t = \frac{15}{2} \text{ or } 3.$$

$$t \in \left\{ \frac{3}{2}, 3, \frac{15}{2}, 9 \right\}.$$

(6)

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Question 4

Question 4

Determine the values of x for which

$$\frac{x-8}{x} \leq \frac{7}{x(x-2)},$$

giving the answer in set notation.

Model solution

Use a common denominator and then a sign diagram. A rational expression can change sign at a numerator zero or a denominator zero; the original expression is undefined at $x = 0$ and $x = 2$.

$$\frac{x-8}{x} - \frac{7}{x(x-2)} \leq 0 \Rightarrow \frac{(x-8)(x-2) - 7}{x(x-2)} \leq 0$$

$$\Rightarrow \frac{x^2 - 10x + 9}{x(x-2)} = \frac{(x-1)(x-9)}{x(x-2)} \leq 0.$$

The critical values are 0, 1, 2, 9. Test one point in each resulting interval. Zeros of the numerator are included for a non-strict inequality, but denominator zeros are excluded.

x	$(-\infty, 0)$	$(0, 1)$	$(1, 2)$	$(2, 9)$	$(9, \infty)$
$\frac{(x-1)(x-9)}{x(x-2)}$	+	-	+	-	+

$$\{x : 0 < x \leq 1\} \cup \{x : 2 < x \leq 9\}.$$

(6)

Question 5

Question 5

The parabola C has equation $y^2 = 16x$, and $P(4t^2, 8t)$ lies on C .

(a) Show that the tangent to C at P is $yt - x = 4t^2$.

The line l passes through the origin and is perpendicular to this tangent. It intersects the tangent at Q .

(b) Determine the coordinates of Q in terms of t .

(c) Hence show that, as t varies, Q lies on $y^2 = \frac{Ax^3}{x+B}$, where A and B are integers.

Model solution

(a) As the curve is given implicitly, differentiate both sides with respect to x to find the tangent gradient.

$$2y \frac{dy}{dx} = 16 \Rightarrow \frac{dy}{dx} = \frac{8}{y}.$$

For $t \neq 0$, at P , $y = 8t$, so the gradient is $1/t$. Point--gradient form through $(4t^2, 8t)$ gives

$$y - 8t = \frac{1}{t}(x - 4t^2) \Rightarrow ty - 8t^2 = x - 4t^2 \Rightarrow ty - x = 4t^2.$$

If $t = 0$, then $P = (0, 0)$. Writing the curve as $x = y^2/16$ shows that its tangent there is $x = 0$, which is also $yt - x = 4t^2$ when $t = 0$.

$$yt - x = 4t^2.$$

(3)

(b) For $t \neq 0$, a perpendicular gradient to $1/t$ is $-t$. Since l passes through the origin, $y = -tx$. Solve this with the tangent equation.

$$y = -tx, \quad yt - x = 4t^2.$$

$$(-tx)t - x = 4t^2 \Rightarrow -x(t^2 + 1) = 4t^2 \Rightarrow x = -\frac{4t^2}{1 + t^2}.$$

$$y = -t \left(-\frac{4t^2}{1 + t^2} \right) = \frac{4t^3}{1 + t^2}.$$

If $t = 0$, the tangent is $x = 0$ and the perpendicular line through the origin is $y = 0$, so $Q = (0, 0)$, as given by the same coordinate formula.

$$Q \left(-\frac{4t^2}{1 + t^2}, \frac{4t^3}{1 + t^2} \right).$$

(3)

(c) Eliminate t . The normal equation $y = -tx$ gives $y^2 = t^2x^2$, so rearrange the expression for x to find t^2 .

$$x = -\frac{4t^2}{1 + t^2} \Rightarrow x(1 + t^2) = -4t^2 \Rightarrow x = -t^2(x + 4) \Rightarrow t^2 = -\frac{x}{x + 4}.$$

$$y^2 = t^2 x^2 = \left(-\frac{x}{x+4} \right) x^2 = -\frac{x^3}{x+4}.$$

This is the stated form with integer constants $A = -1$ and $B = 4$.

$$y^2 = -\frac{x^3}{x+4}, \quad A = -1, \quad B = 4.$$

(3)

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