



Edexcel AS Further Mathematics

8FM0/22 - Further Pure Mathematics 2

Friday 16 May 2025

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2025 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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Question 1

Question 1

- (i)(a) Use the Euclidean algorithm to determine the highest common factor h of 105 and 24. (3)
 (i)(b) Hence determine integers a and b such that $105a + 24b = h$. (3)
 (ii) Determine the remainder when 179^5 is divided by 11. (2)

Model solution

(i)(a) The Euclidean algorithm repeatedly writes the larger number as a multiple of the smaller number plus a remainder. The last non-zero remainder is the highest common factor.

$$105 = 4 \times 24 + 9, \quad 24 = 2 \times 9 + 6,$$

$$9 = 1 \times 6 + 3, \quad 6 = 2 \times 3 + 0.$$

The last non-zero remainder is 3, so it divides both original numbers and is their greatest common factor.

$$h = 3.$$

(3)

(i)(b) Work backwards through the Euclidean algorithm so that the hcf is written as a linear combination of 105 and 24.

$$3 = 9 - 6 = 9 - (24 - 2 \times 9) = 3 \times 9 - 24.$$

$$9 = 105 - 4 \times 24,$$

$$3 = 3(105 - 4 \times 24) - 24 = 3 \times 105 - 13 \times 24.$$

$$a = 3, \quad b = -13.$$

(3)

(ii) Modular arithmetic is appropriate because only the remainder is required. First reduce the base modulo 11, then raise the smaller remainder to the fifth power.

$$179 = 16 \times 11 + 3 \implies 179 \equiv 3 \pmod{11}.$$

$$179^5 \equiv 3^5 = 243 = 22 \times 11 + 1 \equiv 1 \pmod{11}.$$

The remainder is 1.

(2)

Question 2

Question 2

- (i) Using a suitable algorithm and without performing any division, determine whether 13 306 617 is divisible by 9. (2)
- (ii) The group $G = \{1, 3, 7, 9, 11, 13, 17, 19\}$ has multiplication modulo 20 as its operation.
- (a) Complete its Cayley table. (3)
- (b) State the inverse of 7. (1)
- (c) Determine the order of 13. (1)
- (d) Write down a subgroup of G of order 4. (1)

Model solution

(i) The digit-sum test is the required algorithm: a whole number is divisible by 9 exactly when its digit sum is divisible by 9.

$$1 + 3 + 3 + 0 + 6 + 6 + 1 + 7 = 27.$$

Since 27 is a multiple of 9, the original number is also a multiple of 9.

13 306 617 is divisible by 9.

(2)

(ii)(a) Each entry is the product of its row and column headings, reduced modulo 20. For example, $3 \times 7 = 21 \equiv 1 \pmod{20}$.

$\times_{20}$	1	3	7	9	11	13	17	19
1	1	3	7	9	11	13	17	19
3	3	9	1	7	13	19	11	17
7	7	1	9	3	17	11	19	13
9	9	7	3	1	19	17	13	11
11	11	13	17	19	1	3	7	9
13	13	19	11	17	3	9	1	7
17	17	11	19	13	7	1	9	3
19	19	17	13	11	9	7	3	1

The completed Cayley table is shown above.

(3)

(ii)(b) The inverse is the element which gives the identity 1 when multiplied by 7. In the row for 7, the entry 1 occurs under the column headed 3.

$$7^{-1} = 3.$$

(1)

(ii)(c) The order is the least positive power which gives the identity. The table gives $13^2 \equiv 9 \pmod{20}$ and hence $13^4 \equiv 9^2 \equiv 1 \pmod{20}$; neither 13 nor 13^2 is 1.

The order of 13 is 4.

(1)

(ii)(d) A subgroup must contain the identity and be closed under the operation. The powers of 3 provide four such elements: $3^0, 3^1, 3^2, 3^3$.

$\{1, 3, 7, 9\}$ is a subgroup of order 4.

(1)

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Question 3

Question 3

A loan of £180 000 is taken out to buy a house. The monthly interest rate is 0.15%, added at the end of each month. A repayment of £900 is made at the end of each month immediately after interest is added. Let B_n be the balance in thousands of pounds at the end of month n , after interest and repayment.

- (a) Explain why $B_n = 1.0015B_{n-1} - 0.9$, with $B_0 = 180$. (2)
- (b) State an assumption needed for this model to be valid. (1)
- (c) Solve the recurrence relation to obtain a closed form for B_n . (5)
- (d) Hence determine the time to repay the loan, in years and months to the nearest month. (2)

Model solution

(a) The balance is measured in thousands of pounds. Start with the balance from the previous month, increase it by the stated monthly interest, then subtract the repayment because the repayment is made after the interest is added.

$$B_0 = 180.$$

$$\text{Interest multiplier} = 100.15\% = 1.0015, \quad \$900 = 0.9 \text{ thousand pounds.}$$

$$B_n = 1.0015B_{n-1} - 0.9.$$

$$B_n = 1.0015B_{n-1} - 0.9, \quad B_0 = 180.$$

(2)

(b) The recurrence uses the same multiplier and subtraction every month, so those quantities must remain unchanged.

For example, assume that the monthly interest rate remains 0.15% throughout the loan.

(1)

(c) This is a first-order linear recurrence with a constant term. Use "complementary function plus a particular solution": try $B_n = a(1.0015)^n + b$, where the constant b accounts for the fixed monthly repayment.

$$b = 1.0015b - 0.9.$$

$$0.0015b = 0.9 \implies b = 600.$$

The constant 600 is the equilibrium value of the recurrence. Now use the initial balance to find the remaining constant.

$$B_n = a(1.0015)^n + 600,$$

$$180 = a(1.0015)^0 + 600 = a + 600, \quad a = -420.$$

$$B_n = 600 - 420(1.0015)^n.$$

(5)

(d) The loan is repaid when the modelled balance reaches zero. Set the closed form equal to zero and solve for the number of months.

$$600 - 420(1.0015)^n = 0$$

$$(1.0015)^n = \frac{10}{7}, \quad n = \frac{\ln(10/7)}{\ln(1.0015)} = 237.96 \dots$$

This is approximately 238 months. Since 228 months is 19 years, the remaining 10 months give the required time.

The loan takes 19 years and 10 months to repay, to the nearest month.

(2)

Question 4

Question 4

$$A = \begin{pmatrix} 6 & -1 \\ 2 & 3 \end{pmatrix}.$$

Determine a matrix P and a diagonal matrix D such that $P^{-1}AP = D$.

(7)

Model solution

Diagonalisation is possible by finding two independent eigenvectors. Put those eigenvectors as the columns of P ; their corresponding eigenvalues, in the same order, form the diagonal entries of D .

$$\det(A - \lambda I) = \begin{vmatrix} 6 - \lambda & -1 \\ 2 & 3 - \lambda \end{vmatrix} = (6 - \lambda)(3 - \lambda) + 2.$$

$$\det(A - \lambda I) = \lambda^2 - 9\lambda + 20 = (\lambda - 5)(\lambda - 4).$$

$$\lambda = 5 \quad \text{or} \quad \lambda = 4.$$

For $\lambda = 5$, solve $(A - 5I)\mathbf{v} = \mathbf{0}$.

$$\begin{pmatrix} 1 & -1 \\ 2 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \implies x = y, \quad \mathbf{v}_5 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

For $\lambda = 4$, solve $(A - 4I)\mathbf{v} = \mathbf{0}$.

$$\begin{pmatrix} 2 & -1 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \implies y = 2x, \quad \mathbf{v}_4 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$

Use the eigenvectors in the order 5, 4. This ensures that the first and second diagonal entries are respectively 5 and 4.

$$P = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}, \quad D = \begin{pmatrix} 5 & 0 \\ 0 & 4 \end{pmatrix}.$$

$$P = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}, \quad D = \begin{pmatrix} 5 & 0 \\ 0 & 4 \end{pmatrix}.$$

(7)

Question 5

Question 5

In an Argand diagram, the curve C is given by

$$\arg\left(\frac{z+4}{z-2i}\right) = \frac{\pi}{4}.$$

Given that $z = x + iy$, where x and y are real,

(a) show that the circle containing this arc has equation $x^2 + y^2 + ax + by + c = 0$, where a , b and c are constants to be determined. (4)

(b) For the curve C, determine the exact minimum value of $|z|$. (3)

Model solution

(a) Substitute $z = x + iy$ and rationalise the complex fraction. This exposes its real and imaginary parts; an argument of $\pi/4$ means that these parts are equal (and positive on the specified arc).

$$\frac{z+4}{z-2i} = \frac{x+4+iy}{x+i(y-2)} \times \frac{x-i(y-2)}{x-i(y-2)}.$$

$$\frac{z+4}{z-2i} = \frac{(x+4+iy)(x-i(y-2))}{x^2+(y-2)^2} = \frac{x^2+y^2+4x-2y+i(2x-4y+8)}{x^2+(y-2)^2}.$$

The denominator is real and positive away from $z = 2i$, so equality of the real and imaginary parts of the numerator gives

$$x^2 + y^2 + 4x - 2y = 2x - 4y + 8.$$

$$x^2 + y^2 + 2x + 2y - 8 = 0.$$

$$x^2 + y^2 + 2x + 2y - 8 = 0, \text{ so } a = 2, b = 2 \text{ and } c = -8.$$

(4)

(b) Complete the square to identify the circle's centre and radius. Here the origin is inside the circle, so the least distance to its circumference is the radius minus the distance from the centre to the origin.

$$x^2 + 2x + y^2 + 2y - 8 = 0$$

$$(x+1)^2 + (y+1)^2 = 10.$$

$$\text{centre} = (-1, -1), \quad r = \sqrt{10}, \quad \text{distance of centre from origin} = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}.$$

$$\min |z| = \sqrt{10} - \sqrt{2}.$$

The nearest point has equal positive coordinates, so the real and imaginary parts in part (a) are equal and positive. It is therefore on the specified arc, not merely on the full circle.

$$\min |z| = \sqrt{10} - \sqrt{2}$$

(3)

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