



Edexcel AS Further Mathematics

8FM0/25 - Further Mechanics 1

Thursday 17 May 2018

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2018 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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Question 1

Question 1

A small ball of mass 0.3 kg is released from rest from a point 3.6 m above horizontal ground. It falls freely under gravity, hits the ground and rebounds vertically upwards. In its first impact with the ground, the ball receives an impulse of magnitude 4.2 N s. The ball is modelled as a particle.

- (a) Find the speed of the ball immediately after its first impact with the ground.
 (b) Find the kinetic energy lost by the ball as a result of the impact with the ground.

Model solution

(a) Take upwards as positive. Before the impact, the ball has fallen 3.6 m from rest, so using $v^2 = u^2 + 2as$ with downwards positive for this calculation gives

$$v^2 = 0 + 2(9.8)(3.6) = 70.56, \quad v = 8.4 \text{ m s}^{-1}.$$

Its velocity immediately before impact is therefore -8.4 m s^{-1} . Let its velocity immediately after impact be $w \text{ m s}^{-1}$. The upward impulse is 4.2 N s, so impulse equals change in momentum:

$$4.2 = 0.3(w - (-8.4)).$$

$$w + 8.4 = 14, \quad w = 5.6.$$

$$5.6 \text{ m s}^{-1} \text{ upwards.}$$

(5)

(b) The kinetic energy lost is the kinetic energy immediately before impact minus that immediately afterwards:

$$\frac{1}{2}(0.3)(8.4)^2 - \frac{1}{2}(0.3)(5.6)^2 = 5.88.$$

$$5.88 \text{ J.}$$

(3)

Question 2

Question 2

A ramp is inclined at an angle θ to the horizontal, where $\sin \theta = \frac{2}{7}$. A parcel of mass 4 kg is projected up a line of greatest slope from a point A on the ramp with speed 5 m s^{-1} . It first comes to instantaneous rest at B, where $AB = 2.5 \text{ m}$. The parcel is modelled as a particle.

The total resistance from non-gravitational forces is modelled as a constant force of magnitude $R \text{ N}$.

(a) Use the work–energy principle to show that $R = 8.8$.

(b) After coming to rest at B, the parcel slides back down the ramp. The total resistance is modelled as a constant force of magnitude 8.8 N . Find its speed when it returns to A.

(c) Suggest two improvements that could be made to the model.

Model solution

(a) During the upward journey, the parcel loses all of its initial kinetic energy. This equals the gain in gravitational potential energy plus the work done against resistance:

$$\frac{1}{2}(4)(5)^2 = 4(9.8)(2.5) \sin \theta + R(2.5).$$

Since $\sin \theta = \frac{2}{7}$,

$$50 = 4(9.8)(2.5) \left(\frac{2}{7}\right) + 2.5R = 28 + 2.5R.$$

$$2.5R = 22, \quad R = 8.8.$$

$R = 8.8 \text{ N}$, as required.

(4)

(b) From B to A, the loss of gravitational potential energy is $4(9.8)(2.5)\frac{2}{7} = 28 \text{ J}$. The resistance does negative work $8.8(2.5) = 22 \text{ J}$. Thus, if the returning speed is v ,

$$\frac{1}{2}(4)v^2 = 28 - 22 = 6.$$

$$v^2 = 3, \quad v = \sqrt{3} = 1.73 \dots$$

1.7 m s^{-1} (to 2 s.f.).

(3)

(c) The resistance should not necessarily be constant; for example, air resistance could be modelled as depending on the parcel's speed. Also, rather than treating the parcel as a particle, it could be modelled as a body with dimensions, allowing any rotational effects to be considered.

For example: use a speed-dependent resistance and model the parcel with dimensions/possible rotation.

(2)

Question 3

Question 3

A van of mass 750 kg moves along a straight horizontal road. When its speed is $v \text{ m s}^{-1}$, the resistance to its motion is modelled as $\lambda v \text{ N}$, where λ is constant. The engine works at a constant rate of 18 kW. When $v = 15 \text{ m s}^{-1}$, the acceleration is 0.6 m s^{-2} .

(a) Show that $\lambda = 50$.

(b) The van now moves up a straight road with $\sin \alpha = \frac{1}{15}$. The non-gravitational resistance is modelled as $50v \text{ N}$. When the engine works at a constant rate of 12 kW, the van moves at a constant speed $V \text{ m s}^{-1}$. Find V .

Model solution

(a) If F is the driving force, then power = Fv . Hence, at $v = 15$,

$$18000 = 15F, \quad F = 1200 \text{ N}.$$

Taking forwards as positive and applying Newton's second law on the horizontal road,

$$F - \lambda v = 750(0.6).$$

$$1200 - 15\lambda = 450, \quad 15\lambda = 750, \quad \lambda = 50.$$

$$\lambda = 50, \text{ as required.}$$

(4)

(b) At speed V , the driving force is obtained from $P = FV$:

$$F = \frac{12000}{V}.$$

Take up the slope as positive. Since the speed is constant, the acceleration is zero. The forces opposing the motion are the resistance $50V$ and the component of weight $750(9.8) \sin \alpha = 490 \text{ N}$. Therefore,

$$\frac{12000}{V} - 50V - 490 = 0.$$

$$12000 - 50V^2 - 490V = 0 \implies 5V^2 + 49V - 1200 = 0.$$

$$V = \frac{-49 \pm \sqrt{49^2 + 4(5)(1200)}}{10}.$$

The negative root is not physically possible, so

$$V = 11.3 \dots$$

$$V = 11.3 \text{ m s}^{-1} \text{ (3 s.f.)}$$

(5)

Question 4

Question 4

Particle P, of mass $3m$, moves on a smooth horizontal floor in a straight line. Particle Q, of mass $5m$, moves along the same line in the opposite direction. They collide directly. Immediately before the collision, the speeds of P and Q are $2u$ and u respectively, and the coefficient of restitution between them is e .

- (a) Show that the speed of Q immediately after the collision is $\frac{u}{8}(9e + 1)$.
 (b) Find the range of values of e for which the direction of motion of P is not changed by the collision.
 (c) At their first collision, the particles are distance d from a smooth fixed vertical wall perpendicular to their line of motion. Particle Q then collides directly with the wall and rebounds, and there is a second collision between P and Q, at distance x from the wall. Given that $e = \frac{1}{18}$ and that the coefficient of restitution between Q and the wall is $\frac{1}{3}$, find x in terms of d .

Model solution

(a) Take the initial direction of motion of P as positive. Thus, immediately before impact, the velocities are $2u$ for P and $-u$ for Q. Let the velocities immediately after the collision be v for P and w for Q. Conservation of linear momentum gives

$$3m(2u) + 5m(-u) = 3mv + 5mw,$$

$$u = 3v + 5w. \quad (1)$$

The relative speed of separation is $w - v$, and the relative speed of approach was $2u - (-u) = 3u$. Hence the restitution equation is

$$w - v = 3ue. \quad (2)$$

Using $v = w - 3ue$ in (1),

$$u = 3(w - 3ue) + 5w = 8w - 9ue,$$

$$w = \frac{u}{8}(9e + 1).$$

The speed of Q is $\frac{u}{8}(9e + 1)$, as required.

(6)

(b) From (1), using the value of w ,

$$v = \frac{u - 5w}{3} = \frac{u - 5\left(\frac{u}{8}(9e + 1)\right)}{3} = \frac{u}{8}(1 - 15e).$$

For P to continue in its original positive direction, $v > 0$. Since $u > 0$,

$$1 - 15e > 0 \implies e < \frac{1}{15}.$$

As a coefficient of restitution satisfies $0 \leq e \leq 1$, the required range is

$$0 \leq e < \frac{1}{15}.$$

(2)

(c) With $e = \frac{1}{18}$, the velocities after the first collision are

$$w = \frac{u}{8} \left(9 \cdot \frac{1}{18} + 1 \right) = \frac{3u}{16}, \quad v = \frac{u}{8} \left(1 - 15 \cdot \frac{1}{18} \right) = \frac{u}{48}.$$

Thus Q travels towards the wall at speed $\frac{3u}{16}$, while P follows it at speed $\frac{u}{48}$. The time for Q to reach the wall is

$$\frac{d}{3u/16} = \frac{16d}{3u}.$$

In this time, P travels

$$\frac{u}{48} \cdot \frac{16d}{3u} = \frac{d}{9}.$$

Therefore, when Q reaches the wall, P is $d - \frac{d}{9} = \frac{8d}{9}$ from it. On rebounding from the wall, Q has speed

$$\frac{1}{3} \left(\frac{3u}{16} \right) = \frac{u}{16}$$

towards P. If the second collision is at distance x from the wall, then from the rebound to that collision Q travels x , while P travels $x/3$ (its speed is one third of Q's speed). Hence

$$x + \frac{x}{3} = \frac{8d}{9}.$$

$$\frac{4x}{3} = \frac{8d}{9}, \quad x = \frac{2d}{3}.$$

$$x = \frac{2d}{3}.$$

(6)