



Edexcel AS Further Mathematics

8FM0/25 - Further Mechanics 1

Thursday 16 May 2019

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2019 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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Question 1

Question 1

A lorry of mass 16 000 kg moves along a straight horizontal road at a constant speed of 25 m s^{-1} . Initially, the resistance to its motion is modelled as a constant force of magnitude 16 000 N.

(a) Show that the engine is working at a rate of 400 kW.

The model is refined: at speed $V \text{ m s}^{-1}$, the resistance has magnitude $640V \text{ N}$. The engine continues to work at 400 kW.

(b) Find the speed when the lorry accelerates at 2.1 m s^{-2} .

Model solution

(a) Take the direction of motion as positive. Since the lorry moves at constant speed, $a = 0$. Resolving parallel to the road,

$$F - 16\,000 = 16\,000(0),$$

so the driving force is $F = 16\,000 \text{ N}$. The power developed is

$$P = Fv = 16\,000 \times 25 = 400\,000 \text{ W} = 400 \text{ kW}.$$

At constant speed, the driving force balances the resistance; power is force times speed.

400 kW, as required.

(4)

(b) For constant engine power,

$$400\,000 = FV \Rightarrow F = \frac{400\,000}{V} \text{ N}.$$

Taking forward as positive and applying Newton's second law,

$$\frac{400\,000}{V} - 640V = 16\,000(2.1).$$

Thus

$$400\,000 - 640V^2 = 33\,600V,$$

$$640V^2 + 33\,600V - 400\,000 = 0,$$

or

$$2V^2 + 105V - 1250 = 0.$$

Hence

$$V = \frac{-105 \pm \sqrt{105^2 + 4(2)(1250)}}{4} = \frac{-105 \pm 145}{4}.$$

The negative root is not a possible speed, so $V = 10$. With a fixed power, the driving force is $400\,000/V$; the resistive force acts opposite to the motion.

10 m s^{-1}

(6)

Question 2

Question 2

Particles A and B, of masses $2m$ and $3m$ respectively, move on a smooth horizontal plane in opposite directions along the same line and collide directly. Immediately before collision, their speeds are $2u$ and u , respectively. The impulse of A on B has magnitude $5mu$.

- (a) Find the coefficient of restitution.
 (b) Find the total loss of kinetic energy due to the collision.

Model solution

Take positive as the initial direction of A. Thus $u_A = 2u$ and $u_B = -u$.

(a) Let v_A, v_B be the velocities immediately after the collision. The impulse on B is positive, so

$$5mu = 3m(v_B - (-u)),$$

giving

$$v_B = \frac{2u}{3}.$$

Conservation of linear momentum gives

$$2m(2u) + 3m(-u) = 2mv_A + 3mv_B.$$

Therefore

$$mu = 2mv_A + 3m\left(\frac{2u}{3}\right), \quad v_A = -\frac{u}{2}.$$

By Newton's law of restitution,

$$e = \frac{\text{speed of separation}}{\text{speed of approach}} = \frac{v_B - v_A}{u_A - u_B} = \frac{\frac{2u}{3} - (-\frac{u}{2})}{2u - (-u)} = \frac{7}{18}.$$

There is no external horizontal impulse, so momentum is conserved. The given impulse fixes v_B , and restitution uses the relative speeds.

$$e = \frac{7}{18}$$

(9)

(b) The kinetic-energy loss is

$$\left[\frac{1}{2}(2m)(2u)^2 + \frac{1}{2}(3m)u^2 \right] - \left[\frac{1}{2}(2m)\left(-\frac{u}{2}\right)^2 + \frac{1}{2}(3m)\left(\frac{2u}{3}\right)^2 \right] = \frac{11}{2}mu^2 - \left(\frac{1}{4} + \frac{2}{3}\right)mu^2 = \frac{55}{12}mu^2.$$

Kinetic energy depends on velocity squared, so the signs of the final velocities do not affect these terms.

$$\text{Total loss of kinetic energy} = \frac{55}{12}mu^2 \text{ J.}$$

(4)

Question 3

Question 3

A particle P, of mass m kg, is projected at speed 5 m s^{-1} down a line of greatest slope of a rough plane. The plane is inclined at angle α to the horizontal, where $\sin \alpha = \frac{3}{5}$. The total resistance to the motion has magnitude $\frac{1}{5}mg$.

Use the work–energy principle to find the speed of P after it has travelled 8 m down the plane.

Model solution

The work done against the resistance over 8 m is

$$\frac{1}{5}mg \times 8 = \frac{8}{5}mg.$$

The vertical drop is $8 \sin \alpha$, so the loss of gravitational potential energy is

$$mg(8 \sin \alpha) = mg \left(8 \times \frac{3}{5} \right) = \frac{24}{5}mg.$$

Let $v \text{ m s}^{-1}$ be the final speed. By work–energy,

$$\frac{24}{5}mg = \left(\frac{1}{2}mv^2 - \frac{1}{2}m(5)^2 \right) + \frac{8}{5}mg.$$

Hence

$$\frac{1}{2}mv^2 - \frac{25}{2}m = \frac{16}{5}mg,$$

so, with $g = 9.8 \text{ m s}^{-2}$,

$$v^2 = 25 + \frac{32}{5}(9.8) = 87.72.$$

Thus $v = 9.365 \dots$. The loss of gravitational potential energy supplies both the increase in kinetic energy and the work done against resistance.

$$9.4 \text{ m s}^{-1} \text{ (to 2 significant figures).}$$

(7)

Question 4

Question 4

Three particles P, Q, R are initially at rest on a smooth horizontal plane, in that order along a straight line, with Q between P and R. The masses of Q and R are m and km , where k is a constant. Particle Q is projected towards R with speed u , and Q and R collide directly. The coefficient of restitution for each collision is e .

(a) Find, in terms of e , the range of values of k for which there is a second collision.

Given that the mass of P is km and that there is a second collision,

(b) write down, in terms of u, k, e , the speed of Q after this second collision.

Model solution

Take the initial direction of Q, towards R, as positive. Let v_Q, v_R be the velocities immediately after the collision of Q with R.

(a) Conservation of linear momentum gives

$$mu = mv_Q + kmv_R.$$

Newton's law of restitution gives

$$v_R - v_Q = eu.$$

Using $v_R = v_Q + eu$ in the momentum equation,

$$u = (1 + k)v_Q + keu,$$

so

$$v_Q = \frac{u(1 - ke)}{1 + k}.$$

For a second collision, Q must rebound towards P, i.e. it must have $v_Q < 0$. Since $u > 0$ and $k + 1 > 0$,

$$\frac{u(1 - ke)}{1 + k} < 0 \Rightarrow ke > 1 \Rightarrow k > \frac{1}{e}.$$

A second collision occurs only if Q reverses direction after colliding with R, so it can travel back to P.

$$k > \frac{1}{e}$$

(9)

(b) Before its collision with P, Q's speed is

$$w = -v_Q = \frac{u(ke - 1)}{k + 1}.$$

For the second collision, Q has mass m , P has mass km , and P is initially at rest. Applying momentum and restitution (or the direct-collision result) gives the rebound speed of Q as

$$\frac{ke - 1}{k + 1}w.$$

Therefore

$$\frac{ke - 1}{k + 1} \cdot \frac{u(ke - 1)}{k + 1} = \frac{u(ke - 1)^2}{(k + 1)^2}.$$

The second collision has the same mass ratio as the first, with Q approaching an initially stationary particle.

$$\frac{u(ke - 1)^2}{(k + 1)^2} m s^{-1}$$

(1)

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