



Edexcel AS Further Mathematics

8FM0/25 - Further Mechanics 1

Tuesday 24 May 2022

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2022 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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Question 1

Question 1

A car of mass 1200 kg moves up a straight road inclined at an angle α to the horizontal, where $\sin \alpha = \frac{1}{15}$. The total non-gravitational resistance is a constant force of magnitude R N. When the engine is working at 32 kW and the car speed is 20 m s^{-1} , its acceleration is 0.5 m s^{-2} . Find R .

Model solution

The engine power and speed give the driving force, using $P = Fv$. We then resolve forces up the slope, taking uphill as positive.

$$32000 = F(20), \quad F = 1600 \text{ N.}$$

Along the slope, the forces opposing the upward motion are the resistance R and the component of weight $1200g \sin \alpha$. Newton's second law therefore gives

$$1600 - R - 1200g \sin \alpha = 1200(0.5).$$

$$1600 - R - 1200(9.8) \left(\frac{1}{15} \right) = 600,$$

$$1600 - R - 784 = 600, \quad R = 216.$$

$$R = 216 \text{ N.}$$

(5)

Question 2

Question 2

Particles A and B, of masses m and $3m$ respectively, move in opposite directions along the same straight line on a smooth horizontal plane and collide directly. Immediately before collision, A has speed $2u$ and B has speed u . Each reverses direction. The impulse exerted on A by B has magnitude $\frac{9}{2}mu$.

- (a) Find the coefficient of restitution.
 (b) Hence write down the total loss in kinetic energy, giving a reason.

Model solution

(a) Take the original direction of A as positive. After collision, A has velocity $-v$ and B has velocity w , where $v, w > 0$. The given impulse is particularly useful because impulse equals change in momentum.

$$-\frac{9}{2}mu = m(-v) - m(2u).$$

$$-\frac{9}{2}u = -v - 2u \implies v = \frac{5}{2}u.$$

Applying the same principle to B gives its post-collision speed. Its impulse is equal and opposite, so it is positive in the chosen direction.

$$\frac{9}{2}mu = 3mw - 3m(-u) = 3m(w + u),$$

$$\frac{3}{2}u = w + u \implies w = \frac{1}{2}u.$$

The coefficient of restitution is the speed of separation divided by the speed of approach.

$$e = \frac{v + w}{2u + u} = \frac{\frac{5}{2}u + \frac{1}{2}u}{3u} = 1.$$

$$e = 1.$$

(7)

(b) A coefficient of restitution of 1 means the collision is perfectly elastic, so kinetic energy is conserved.

$$\text{Total loss in kinetic energy} = 0 \text{ J.}$$

(1)

Question 3

Question 3

A plane is inclined at angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$. A particle P is projected from rest point A up a line of greatest slope with speed 25 m s^{-1} . It travels $\frac{25}{6} \text{ m}$ up the plane.

(a) With the plane smooth and air resistance negligible, use conservation of mechanical energy to find its speed.

(b) With coefficient of friction $\frac{3}{5}$ and air resistance still negligible, use work-energy to find its speed.

Model solution

(a) The clue "smooth" means there is no frictional energy loss. From the 3--4--5 triangle implied by $\tan \alpha = \frac{3}{4}$, $\sin \alpha = \frac{3}{5}$. The vertical rise is distance along the plane times $\sin \alpha$.

$$h = \frac{25}{6} \sin \alpha = \frac{25}{6} \cdot \frac{3}{5} = \frac{5}{2} \text{ m}.$$

Conservation of mechanical energy equates the initial kinetic energy to final kinetic energy plus the gain in gravitational potential energy.

$$\frac{1}{2}m(25)^2 = \frac{1}{2}mv^2 + mg \left(\frac{5}{2} \right).$$

$$\frac{625}{2} = \frac{1}{2}v^2 + \frac{49}{2}, \quad v^2 = 576.$$

$$v = 24 \text{ m s}^{-1}.$$

(4)

(b) Work-energy is now needed because friction does negative work. First find the normal reaction by resolving perpendicular to the plane; $\cos \alpha = \frac{4}{5}$.

$$R = mg \cos \alpha = \frac{4}{5}mg, \quad F = \mu R = \frac{3}{5} \cdot \frac{4}{5}mg = \frac{12}{25}mg.$$

Over $\frac{25}{6} \text{ m}$, the work done against friction is $F \times \frac{25}{6} = 2mg$. The energy equation includes both this loss and the gain $mg \times \frac{5}{2}$ in potential energy.

$$\frac{1}{2}m(25)^2 = \frac{1}{2}mv^2 + mg \left(\frac{5}{2} \right) + 2mg.$$

$$\frac{625}{2} = \frac{1}{2}v^2 + \frac{49}{2} + \frac{98}{5}, \quad v^2 = \frac{2684}{5}.$$

$$v = \sqrt{\frac{2684}{5}} = 23.2 \text{ m s}^{-1} \quad (3 \text{ s.f.})$$

$$v = 23.2 \text{ m s}^{-1}.$$

(8)

Question 4

Question 4

On a smooth horizontal plane, particle P of mass $2m$ kg and speed $2u$ m s⁻¹ collides with stationary particle Q of mass $3m$ kg. Their coefficient of restitution is e . Immediately after collision Q has speed v m s⁻¹.

(a) Show that $v = \frac{4}{5}u(1 + e)$.

(b) Show that $\frac{4}{5}u \leq v \leq \frac{8}{5}u$.

Given that P reverses direction in the collision:

(c) Find, in terms of u and e , the speed of P immediately after collision.

Afterwards Q strikes a wall fixed at right angles to its motion and rebounds. The coefficient of restitution between Q and the wall is $\frac{1}{6}$. Given that P and Q collide again:

(d) Find the full range of possible values of e .

Model solution

(a) Take the initial direction of P as positive. Since P reverses, write its velocity after collision as $-w$, where $w > 0$; Q has velocity v . Conservation of linear momentum applies because the plane is smooth, so there is no external horizontal impulse.

$$2m(2u) = 2m(-w) + 3mv,$$

$$4u = -2w + 3v. \quad (1)$$

The restitution equation uses relative speed of separation over relative speed of approach. Here those speeds are $v + w$ and $2u$.

$$v + w = 2ue. \quad (2)$$

Substitute $w = 2ue - v$ from (2) into momentum equation (1), leaving an equation only in v .

$$4u = -2(2ue - v) + 3v = -4ue + 5v,$$

$$5v = 4u(1 + e).$$

$$v = \frac{4}{5}u(1 + e), \text{ as required.}$$

(6)

(b) A coefficient of restitution must satisfy $0 \leq e \leq 1$. Since $u > 0$, multiplying this range by $\frac{4}{5}u$ and then adding $\frac{4}{5}u$ preserves the inequality.

$$\frac{4}{5}u(1 + 0) \leq v \leq \frac{4}{5}u(1 + 1),$$

$$\frac{4}{5}u \leq v \leq \frac{8}{5}u, \text{ as required.}$$

(2)

(c) Use the relative-speed equation already formed in part (a), then substitute the expression for v . This gives the magnitude w of P's reversed velocity.

$$w = 2ue - v = 2ue - \frac{4}{5}u(1 + e) = \frac{2}{5}u(3e - 2).$$

Because P is stated to reverse direction, this quantity is positive; it is its speed.

$$\text{speed of P} = \frac{2}{5}u(3e - 2) \text{ m s}^{-1}.$$

(2)

(d) On rebounding from a fixed wall, Q reverses and its speed becomes $\frac{1}{6}v$. Particle P is travelling towards the wall with speed w , so for a second collision Q must be moving back faster than P: $\frac{1}{6}v > w$.

$$\frac{1}{6} \left[\frac{4}{5}u(1 + e) \right] > \frac{2}{5}u(3e - 2).$$

Since $u > 0$, cancel $\frac{2}{5}u$ to solve the condition.

$$\frac{1}{3}(1 + e) > 3e - 2,$$

$$1 + e > 9e - 6, \quad 7 > 8e, \quad e < \frac{7}{8}.$$

The reversal in part (c) also requires $w > 0$, so $3e - 2 > 0$ and $e > \frac{2}{3}$. Combining both strict conditions gives

$$\frac{2}{3} < e < \frac{7}{8}.$$

(5)