



Edexcel AS Further Mathematics

8FM0/25 - Further Mechanics 1

Friday 19 May 2023

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2023 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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Question 1

Question 1

Two particles P and Q, of masses $3m$ and $2m$ respectively, move in opposite directions along the same straight line on a smooth horizontal plane and collide directly. Immediately before the collision, P has speed $2u$. The magnitude of the impulse exerted on P by Q is $\frac{9}{2}mu$.

(a) Find the speed of P immediately after the collision.

The coefficient of restitution between P and Q is e . Given that Q has speed u immediately before the collision:

(b) find e .

Model solution

(a) Take the initial direction of P as positive. The impulse on P is then negative because it acts opposite to this direction. Impulse equals change in momentum, so it gives P's velocity immediately after the collision. Let this velocity be v .

$$-\frac{9}{2}mu = 3m(v - 2u).$$

$$-\frac{9}{2}u = 3v - 6u \implies 3v = \frac{3}{2}u \implies v = \frac{1}{2}u.$$

The positive value confirms that P is still moving in its original direction.

The speed of P is $\frac{1}{2}u$.

(3)

(b) The impulses on the two particles are equal and opposite. Let y be the velocity of Q after collision, using the same positive direction as before. Since Q initially has velocity $-u$, its change in momentum is $2m(y - (-u))$.

$$\frac{9}{2}mu = 2m(y + u).$$

$$\frac{9}{2}u = 2y + 2u \implies 2y = \frac{5}{2}u \implies y = \frac{5}{4}u.$$

Now use Newton's experimental law: coefficient of restitution is relative speed of separation divided by relative speed of approach. The particles separate with speed $y - v$, and approached with speed $2u - (-u) = 3u$.

$$e = \frac{\frac{5}{4}u - \frac{1}{2}u}{2u + u} = \frac{\frac{3}{4}u}{3u} = \frac{1}{4}.$$

$e = \frac{1}{4}.$

(5)

Question 2

Question 2

A racing car of mass 750 kg moves along a straight horizontal road at a constant speed of $U \text{ km h}^{-1}$. Its engine works at a constant rate of 60 kW. Resistance is modelled by a force of magnitude $37.5v \text{ N}$, where $v \text{ m s}^{-1}$ is the car's speed.

(a) Find U .

Later, the car accelerates up a straight road inclined at angle α to the horizontal, where $\sin \alpha = \frac{5}{49}$. The engine still works at 60 kW and non-gravitational resistance is $37.5v \text{ N}$. At an instant when its acceleration is 2 m s^{-2} , its speed is $V \text{ m s}^{-1}$.

(b) Find V .

Model solution

(a) At constant speed the resultant force is zero. First convert engine power into a driving force using $P = Fv$, with $P = 60000 \text{ W}$.

$$60000 = Fv \implies F = \frac{60000}{v}.$$

Taking the direction of motion as positive, the driving force and the resistance are the only horizontal forces.

$$\frac{60000}{v} - 37.5v = 0.$$

$$60000 = 37.5v^2, \text{ so } v^2 = 1600, \text{ so } v = 40 \text{ m s}^{-1}.$$

The requested unit is kilometres per hour, so multiply by 3.6.

$$U = 40(3.6) = 144.$$

$$U = 144 \text{ km h}^{-1}.$$

(4)

(b) Again find the driving force from the power, but this time the resultant force is ma . Take up the slope as positive. The resistance and the component $750g \sin \alpha$ of the weight both act down the slope.

$$\frac{60000}{V} - 37.5V - 750g \sin \alpha = 750(2).$$

Substituting $g = 9.8$ and $\sin \alpha = \frac{5}{49}$ gives an equation in V only.

$$\frac{60000}{V} - 37.5V - 750 = 1500,$$

$$60000 - 37.5V^2 = 2250V.$$

$$V^2 + 60V - 1600 = 0 \implies (V + 100)(V - 20) = 0.$$

A speed cannot be negative, so use the positive root.

$$V = 20 \text{ m s}^{-1}.$$

(4)

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Question 3

Question 3

A stone of mass 0.5 kg is projected vertically upwards from point A, which is 2.5 m above horizontal ground, with speed $U \text{ m s}^{-1}$. It hits the ground with speed 25 m s^{-1} . From projection until it reaches the ground it is modelled as a particle moving freely under gravity.

- Use conservation of mechanical energy to find U .
- State how air resistance in reality would affect your answer to part (a).
- The ground is soft and the stone sinks vertically a distance $d \text{ cm}$ into it. The ground exerts a constant resistive force of magnitude 2000 N. Use the work-energy principle to find d , to 3 significant figures.

Model solution

(a) The phrase "moving freely under gravity" tells us that there is no air resistance in the model, so mechanical energy is conserved. Take gravitational potential energy to be zero at ground level.

$$\frac{1}{2}(0.5)U^2 + (0.5)g(2.5) = \frac{1}{2}(0.5)(25)^2.$$

The left side is the energy at A; the right side is the kinetic energy just before impact. Cancelling the common factor 0.5 gives

$$\frac{1}{2}U^2 + 9.8(2.5) = \frac{1}{2}(625),$$

$$\frac{1}{2}U^2 = 288, \text{ so } U^2 = 576.$$

$$U = 24 \text{ m s}^{-1}.$$

(4)

(b) Air resistance removes mechanical energy during the journey. Therefore a greater initial speed would be needed to give the same impact speed.

The value of U would be larger.

(1)

(c) Apply work-energy from just before impact to the instant the stone stops. The distance sunk is $0.01d$ metres. The upward resistive force does work $2000(0.01d)$ against the motion; this absorbs the initial kinetic energy and also the energy supplied by the stone's weight as it moves down.

$$2000(0.01d) = \frac{1}{2}(0.5)(25)^2 + (0.5)(9.8)(0.01d).$$

$$20d = 156.25 + 0.049d,$$

$$19.951d = 156.25, \text{ so } d = 7.8317 \dots$$

$$d = 7.83 \text{ cm (3 s.f.)}$$

(5)

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Question 4

Question 4

Particles P, Q and R lie at rest on a smooth horizontal plane in a straight line, with Q between P and R. Particle P, of mass m , is projected towards Q with speed u . At the same time, R is projected with speed $\frac{1}{2}u$ away from Q in the direction QR. Particle Q has mass $2m$. The coefficient of restitution between P and Q is e .

(a) Show that the speed of Q immediately after the collision between P and Q is $\frac{u(1+e)}{3}$.

Given that $e > \frac{1}{2}$:

(b) determine whether Q collides with R;

(c) determine the direction of motion of P immediately after its collision with Q;

(d) find, in terms of m , u and e , the total kinetic energy lost in the collision between P and Q, simplifying your answer;

(e) explain how $e = 1$ can be used to check the answer to part (d).

Model solution

(a) Take the direction from P to R as positive. The plane is smooth, so there is no external horizontal impulse and linear momentum is conserved. Let the velocities of P and Q after their collision be v_P and v_Q .

$$mu = mv_P + 2mv_Q,$$

$$u = v_P + 2v_Q. \quad (1)$$

Newton's experimental law uses the relative speed of separation. Initially Q is stationary, so the approach speed is u ; after collision the separation speed is $v_Q - v_P$.

$$v_Q - v_P = eu. \quad (2)$$

From (2), $v_P = v_Q - eu$. Substitution in (1) eliminates v_P .

$$u = (v_Q - eu) + 2v_Q = 3v_Q - eu,$$

$$3v_Q = u(1 + e).$$

$$v_Q = \frac{u(1 + e)}{3}, \text{ as required.}$$

(6)

(b) The clue $e > \frac{1}{2}$ lets us compare Q's speed with the speed $\frac{1}{2}u$ at which R moves away.

$$v_Q = \frac{u(1 + e)}{3} > \frac{u(1 + \frac{1}{2})}{3} = \frac{1}{2}u.$$

Thus Q is moving in the direction of R faster than R, so it catches R.

There is a collision between Q and R.

(2)

(c) Use momentum equation (1) with the value of v_Q already found. This identifies the sign of P's velocity, which determines its direction.

$$v_P = u - 2 \left(\frac{u(1+e)}{3} \right) = \frac{u(1-2e)}{3}.$$

Since $e > \frac{1}{2}$, $1 - 2e < 0$, so $v_P < 0$. It therefore moves opposite to the positive direction, which is opposite to its original direction.

P reverses direction (it moves away from Q towards its original side).

(2)

(d) The total kinetic energy lost is kinetic energy before the collision minus kinetic energy after it. Use $v_P = \frac{u(1-2e)}{3}$ and $v_Q = \frac{u(1+e)}{3}$.

$$KE \text{ before} = \frac{1}{2}mu^2.$$

$$\begin{aligned} KE \text{ after} &= \frac{1}{2}m \left(\frac{u(1-2e)}{3} \right)^2 + \frac{1}{2}(2m) \left(\frac{u(1+e)}{3} \right)^2 \\ &= \frac{mu^2}{18} ((1-2e)^2 + 2(1+e)^2) = \frac{mu^2}{6}(1+2e^2). \end{aligned}$$

Subtracting this from the initial kinetic energy gives

$$\text{loss} = \frac{1}{2}mu^2 - \frac{1}{6}mu^2(1+2e^2) = \frac{1}{3}mu^2(1-e^2).$$

$$\text{Total kinetic energy lost} = \frac{1}{3}mu^2(1-e^2).$$

(3)

(e) Setting $e = 1$ represents a perfectly elastic collision, for which kinetic energy is conserved. The expression in part (d) should therefore become zero, and it does:

$$\frac{1}{3}mu^2(1-1^2) = 0.$$

Using $e = 1$ checks that the loss is 0, as required for an elastic collision.

(1)