



Edexcel AS Further Mathematics

8FM0/26 - Further Mechanics 2

Thursday 17 May 2018

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2018 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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Question 1

Question 1

Figure 1 shows a thin uniform rod of total length $30a$ and mass M , bent to form the triangular frame ABC , where $AB = 12a$, $BC = 5a$ and $CA = 13a$.

(a) Show that the centre of mass of the frame is $\frac{3}{2}a$ from AB .

The frame is freely suspended from A . A horizontal force of magnitude kMg is applied at B , in the vertical plane containing the frame. It hangs in equilibrium with AB vertical.

(b) Find k .

Model solution

(a) The rod is uniform, so the masses of its three sides are proportional to their lengths. Taking moments about AB , the side AB has zero moment. The centres of the straight rods BC and CA are each $\frac{5}{2}a$ from AB . Hence, writing d for the distance of the centre of mass from AB ,

$$\left(\frac{5M}{30}\right) \left(\frac{5}{2}a\right) + \left(\frac{13M}{30}\right) \left(\frac{5}{2}a\right) = Md.$$

$$d = \frac{18}{30} \cdot \frac{5}{2}a = \frac{3}{2}a.$$

The centre of mass is $\frac{3}{2}a$ from AB , as required.

(4)

(b) With AB vertical, take moments about the suspension point A . The weight Mg , acting through the centre of mass, has perpendicular distance $\frac{3}{2}a$ from A . The horizontal force at B has perpendicular distance $12a$. Equilibrium gives

$$Mg \left(\frac{3}{2}a\right) = kMg(12a).$$

$$k = \frac{3}{24} = \frac{1}{8}.$$

$$k = \frac{1}{8}.$$

(3)

Question 2

Question 2

A car moves round a bend banked at a constant angle θ to the horizontal. At a constant speed of 80 km h^{-1} , there is no sideways frictional force. The car is modelled as a particle moving in a horizontal circle of radius 500 m .

(a) Find θ .

(b) Identify one limitation of this model.

The speed is increased to 90 km h^{-1} , with the same particle model and radius.

(c) Describe the extra force now acting on the car, including its direction.

Model solution

(a) Let R be the normal reaction from the road. Since there is no sideways friction, the only forces are R and the weight mg . Resolve vertically, taking upward as positive:

$$R \cos \theta = mg.$$

Resolving horizontally towards the centre of the circular path gives the centripetal-force equation

$$R \sin \theta = \frac{mv^2}{r}.$$

Convert the speed before substitution:

$$v = 80 \times \frac{1000}{3600} = \frac{200}{9} \text{ m s}^{-1}.$$

Dividing the horizontal equation by the vertical equation, with $r = 500$,

$$\tan \theta = \frac{v^2}{rg} = \frac{(200/9)^2}{500(9.8)} = 0.1008 \dots$$

$$\theta = 5.76 \dots^\circ.$$

$$\theta = 5.8^\circ \text{ (to 2 s.f.)}$$

(7)

(b) Modelling the car as a particle treats all its mass as acting at one point; for example, it ignores its dimensions and the separate tyre--road contact points.

For example: the car is treated as a single point of contact with the road.

(1)

(c) The original banking angle is too small for the higher speed without assistance. A frictional force must therefore act down the slope, providing an additional horizontal component towards the centre of the bend.

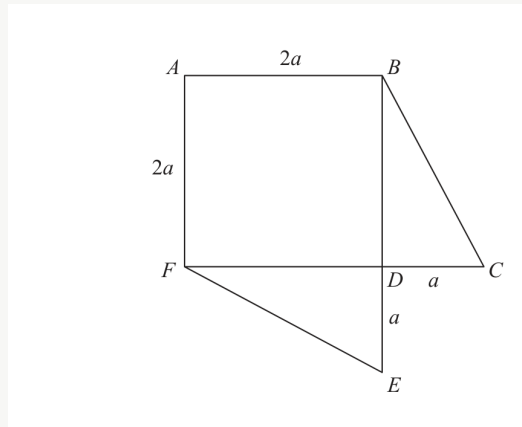
Friction acts down the banked slope.

(1)

Question 3

Question 3

The lamina L consists of a uniform square lamina $ABDF$, of side $2a$, and identical uniform triangular laminas BDC and FDE . The lines BDE and FDC are perpendicular, with $BD = DF = 2a$ and $DC = DE = a$. The square has mass per unit area M and each triangle has mass per unit area $3M$. Its centre of mass is G .



- (a) Without calculation, explain why G lies on AD .
 (b) Show that the distance of G from D is $\frac{\sqrt{2}}{2}a$.
 The lamina is freely suspended from B and hangs in equilibrium.
 (c) Find the angle between BE and the downward vertical.

Model solution

(a) The two triangular parts are mirror images in the line AD , and the square is also symmetric in AD . Thus the complete lamina is symmetric about AD , so its centre of mass lies on AD .

G lies on the axis of symmetry AD .

(1)

(b) Take moments about BE . The masses and signed distances of the centres of mass from BE are:

part	mass	signed distance from BE
$ABDF$	$4a^2M$	$-a$
BDC	$3a^2M$	$\frac{a}{3}$
FDE	$3a^2M$	$-\frac{2a}{3}$

The total mass is $10a^2M$. If the centre of mass has signed distance x from BE ,

$$(4a^2M)(-a) + (3a^2M)\left(\frac{a}{3}\right) + (3a^2M)\left(-\frac{2a}{3}\right) = (10a^2M)x,$$

$$-4a^3M + a^3M - 2a^3M = 10a^2Mx, \quad x = -\frac{a}{2}.$$

Since G lies on AD , its displacement from D has perpendicular components $\frac{a}{2}$ and $\frac{a}{2}$.

Therefore

$$DG = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2} = \frac{\sqrt{2}}{2}a.$$

$$DG = \frac{\sqrt{2}}{2}a, \text{ as required.}$$

(7)

(c) When freely suspended, the line BG is vertically downward. From the geometry of the lamina and $DG = \frac{\sqrt{2}}{2}a$, the right triangle relating BE to BG has horizontal side $\frac{1}{2}a$ and vertical side $\frac{3}{2}a$. Hence, if θ is the angle between BE and the downward vertical,

$$\tan \theta = \frac{1}{3}.$$

$$\theta = 18.434\dots^\circ.$$

$$18.4^\circ \text{ (3 s.f.)}$$

(3)

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Question 4

Question 4

A particle P moves on the x-axis. At time $t \geq 0$, its velocity is $v \text{ m s}^{-1}$ and its acceleration is $a \text{ m s}^{-2}$, both in the direction of increasing x. Initially it is at rest at the origin. Given that

$$a = \frac{5}{2}(5 - v),$$

(a) show that $v = 5(1 - e^{-2.5t})$.

(b) State the limiting value of v as t increases.

When $v = 2.5$, the particle is d metres from the origin.

(c) Show that $d = 2 \ln 2 - 1$.

Model solution

(a) Since $a = \frac{dv}{dt}$, separate variables:

$$\frac{dv}{5-v} = \frac{5}{2} dt.$$

Integrating and using $v = 0$ when $t = 0$,

$$\int_0^v \frac{dv}{5-v} = \int_0^t \frac{5}{2} dt,$$

$$-\ln(5-v) + \ln 5 = \frac{5}{2}t.$$

$$\ln\left(\frac{5}{5-v}\right) = \frac{5}{2}t \implies 5-v = 5e^{-2.5t}.$$

$$v = 5(1 - e^{-2.5t}).$$

$$v = 5(1 - e^{-2.5t}), \text{ as required.}$$

(5)

(b) As $t \rightarrow \infty$, $e^{-2.5t} \rightarrow 0$.

$$5 \text{ m s}^{-1}.$$

(1)

(c) Since $v = \frac{dx}{dt}$, use the expression found in part (a):

$$\frac{dx}{dt} = 5(1 - e^{-2.5t}).$$

$$x = \int 5(1 - e^{-2.5t}) dt = 5t + 2e^{-2.5t} + C.$$

When $v = 2.5$,

$$2.5 = 5(1 - e^{-2.5t}) \implies e^{-2.5t} = \frac{1}{2} \implies t = \frac{\ln 2}{2.5}.$$

Use $x = 0$ at $t = 0$, then evaluate at this time:

$$d = [5t + 2e^{-2.5t}]_0^{\ln 2/2.5} = 5\left(\frac{\ln 2}{2.5}\right) + 2\left(\frac{1}{2}\right) - 2 = 2 \ln 2 - 1.$$

$d = 2 \ln 2 - 1$, as required.

(7)

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