



Edexcel AS Further Mathematics

8FM0/26 - Further Mechanics 2

Thursday 16 May 2019

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2019 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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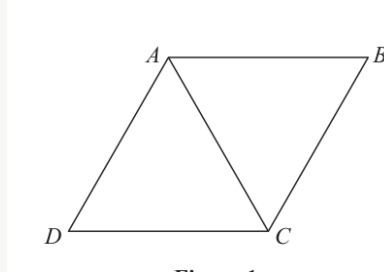
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Question 1

Question 1

Five identical uniform rods, each of length $4a$ and weight W , form the rigid framework ABCD shown in Figure 1. The centre of mass is G.



(a) Explain why G is the midpoint of AC.

The framework is suspended by vertical light strings attached at A and B, and is in equilibrium with AB horizontal.

(b) Find (i) the tension at A, and (ii) the tension at B.

A particle of weight kW is attached at D and a particle of weight $2kW$ at C. The strings remain vertical and either will break if its tension exceeds $6W$.

(c) Find the greatest possible value of k .

Model solution

(a) The clue is that all five rods are identical and uniform. The framework has reflectional symmetry about the midpoint of AC: the rod AC lies on this symmetry line, while the other rods occur in matching positions. Therefore the mass distribution is symmetric about this point.

G is the midpoint of AC.

(1)

(b) Use equilibrium of a rigid body. Taking moments about A eliminates the unknown tension at A. Since the rods form equilateral triangles of side $4a$, the midpoint of AC is horizontally a from A. Thus the total framework weight $5W$ acts at a horizontal distance a from A.

$$T_B(4a) = 5W(a).$$

$$T_B = \frac{5W}{4}.$$

The vertical forces must also balance, since the framework has no vertical acceleration.

$$T_A + T_B = 5W \implies T_A = 5W - \frac{5W}{4} = \frac{15W}{4}.$$

(i) $T_A = \frac{15W}{4}$, (ii) $T_B = \frac{5W}{4}$.

(4)

(c) With the added particles, take moments about B so that the tension at B is removed. The tension at A is already larger than the tension at B, and the added weight at D increases it further. It is therefore the first to reach the breaking value. At the limiting value of k , set $T_A = 6W$.

$$6W(4a) = 5W(3a) + kW(6a) + 2kW(2a).$$

The framework weight acts $3a$ to the left of B; D is $6a$ to the left, and C is $2a$ to the left. Cancelling aW gives

$$24 = 15 + 6k + 4k = 15 + 10k,$$

$$10k = 9, \quad k = \frac{9}{10} = 0.9.$$

At this value, vertical equilibrium gives $T_B = 5W + 3(0.9W) - 6W = 1.7W$, which is below $6W$. Thus the tension at A is indeed the limiting one.

$$k_{\max} = 0.9.$$

(4)

Question 2

Question 2

A car moves in a straight line on a horizontal road. At time $t \geq 0$, its speed is $v \text{ m s}^{-1}$. It passes point A at $t = 0$ with speed 2 m s^{-1} . Its acceleration, in the direction of motion, is

$$a = \frac{4}{v+2} \text{ m s}^{-2}.$$

- (a) Use algebraic integration to show that $v = \sqrt{8t+16} - 2$.
 (b) When it passes point B, its speed is 4 m s^{-1} . Use algebraic integration to find AB.

Model solution

(a) The acceleration is given in terms of v and t , so use $a = \frac{dv}{dt}$. Separating variables puts all the v terms on one side before integrating.

$$\frac{dv}{dt} = \frac{4}{v+2} \implies (v+2) dv = 4 dt.$$

$$\int (v+2) dv = \int 4 dt \implies \frac{1}{2}(v+2)^2 = 4t + C.$$

At $t = 0$, the stated initial speed is $v = 2$, so this fixes the integration constant.

$$\frac{1}{2}(2+2)^2 = C = 8.$$

$$\frac{1}{2}(v+2)^2 = 4t + 8 \implies (v+2)^2 = 8t + 16.$$

Speed is non-negative, so $v+2$ is the positive square root.

$$v+2 = \sqrt{8t+16} \implies v = \sqrt{8t+16} - 2.$$

$$v = \sqrt{8t+16} - 2.$$

(6)

(b) First find the time at B from the speed formula. This is needed because distance is obtained by integrating $\frac{dx}{dt} = v$.

$$4 = \sqrt{8t+16} - 2 \implies \sqrt{8t+16} = 6 \implies t = \frac{5}{2}.$$

$$\frac{dx}{dt} = \sqrt{8t+16} - 2.$$

Integrating gives displacement from A; use $x = 0$ when $t = 0$.

$$x = \int (\sqrt{8t+16} - 2) dt = \frac{1}{12}(8t+16)^{3/2} - 2t + C.$$

$$0 = \frac{1}{12}(16)^{3/2} + C = \frac{16}{3} + C \implies C = -\frac{16}{3}.$$

$$AB = x \Big|_{t=5/2} = \frac{1}{12}(36)^{3/2} - 5 - \frac{16}{3} = 18 - 5 - \frac{16}{3} = \frac{23}{3}.$$

$$AB = \frac{23}{3} \text{ m.}$$

(6)

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Question 3

Question 3

A light inextensible string of length $8a$ is fixed at A and B, where A is vertically above B and $AB = 4a$. A ball of mass m is attached at P, with $AP = 5a$. It moves in a horizontal circle at constant speed v , with both AP and BP taut. The string breaks if its tension exceeds $\frac{3}{2}mg$.

(a) Show that

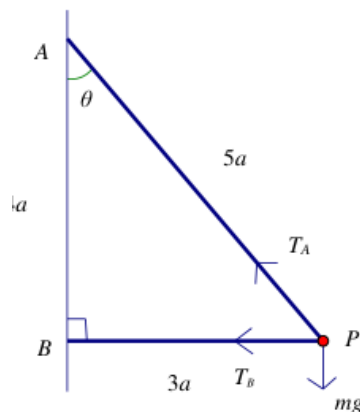
$$\frac{9}{4}ag < v^2 \leq \frac{27}{4}ag.$$

(b) Find the least possible time for one complete revolution.

Model solution

(a) The two string lengths are $AP = 5a$ and $BP = 8a - 5a = 3a$. Together with $AB = 4a$, these make a 3--4--5 right triangle, right-angled at B. Thus the horizontal radius of the circle is $BP = 3a$, and if θ is the angle between AP and the vertical,

$$\cos \theta = \frac{4}{5}, \quad \sin \theta = \frac{3}{5}.$$



Let the tensions in AP and BP be T_A and T_B . There is no vertical motion, so vertical equilibrium gives

$$T_A \cos \theta = mg \implies T_A \left(\frac{4}{5} \right) = mg \implies T_A = \frac{5}{4}mg.$$

Horizontally, both tension components point towards the centre of the circular path. Newton's second law in this radial direction gives

$$T_A \sin \theta + T_B = \frac{mv^2}{3a}.$$

$$\frac{5}{4}mg \left(\frac{3}{5} \right) + T_B = \frac{mv^2}{3a} \implies T_B = \frac{mv^2}{3a} - \frac{3}{4}mg.$$

Both parts of the string are taut, so $T_B > 0$. This supplies the lower bound.

$$\frac{mv^2}{3a} - \frac{3}{4}mg > 0 \implies v^2 > \frac{9}{4}ag.$$

Also, neither tension may exceed $\frac{3}{2}mg$. The fixed value $T_A = \frac{5}{4}mg$ is safe, so apply the breaking condition to T_B .

$$\frac{mv^2}{3a} - \frac{3}{4}mg \leq \frac{3}{2}mg \implies \frac{v^2}{3a} \leq \frac{9}{4}g \implies v^2 \leq \frac{27}{4}ag.$$

$$\frac{9}{4}ag < v^2 \leq \frac{27}{4}ag.$$

(b) The shortest period occurs at the greatest permitted speed. Use $v^2 = \frac{27}{4}ag$ and circumference $2\pi r = 2\pi(3a)$. (7)

$$T_{\min} = \frac{2\pi(3a)}{\sqrt{27ag/4}} = \frac{4\pi}{3} \sqrt{\frac{3a}{g}}.$$

$$T_{\min} = \frac{4\pi}{3} \sqrt{\frac{3a}{g}} \text{ s.}$$

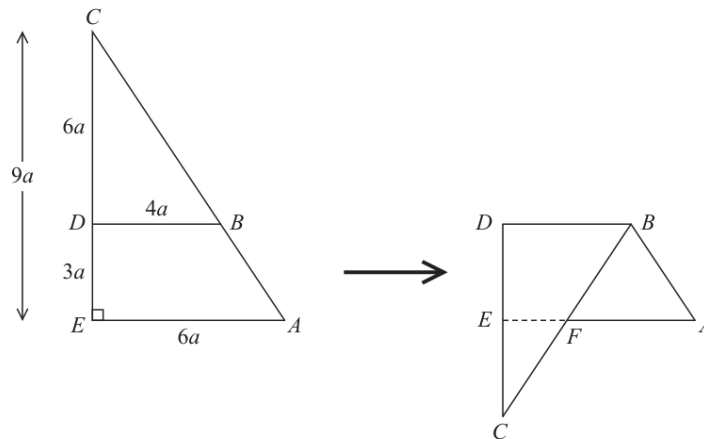
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Question 4

Question 4

The uniform triangular lamina ACE has $\angle CEA = 90^\circ$, $CE = 9a$ and $EA = 6a$. Point D lies on CE with $DE = 3a$. Point B lies on CA such that $DB \parallel EA$ and $DB = 4a$. The lamina is folded along DB to form ABDECF.



The distance of the original lamina's centre of mass from DC is d_1 , and that of the folded lamina's centre of mass from DC is d_2 .

(a) Explain why $d_1 = d_2$.

The folded lamina is freely suspended from B and hangs with BA inclined at angle α to the downward vertical through B.

(b) Find α , to the nearest degree.

Model solution

(a) Folding is a reflection in the line DB. Since DB is parallel to DC, each point stays the same perpendicular distance from DC when it is reflected. Hence the whole mass distribution has the same moment about DC.

$$d_1 = d_2.$$

(1)

(b) When a lamina is freely suspended, its centre of mass lies on the downward vertical through the suspension point. We therefore find the centre of mass G of the folded shape, then use the angle between BA and BG. Take E as the origin, EA as the positive x -axis and EC as the positive y -axis. Thus

$$A = (6a, 0), \quad C = (0, 9a), \quad D = (0, 3a), \quad B = (4a, 3a).$$

The centre of mass of the original triangle ACE has x -coordinate $2a$. From part (a), folding does not change the distance of the centre of mass from DC (the y -axis), so

$$\bar{x} = 2a.$$

To find $\bar{y}$, treat the folded lamina as the large triangle ACE, with triangle BCD removed on one side of DB and its reflected copy added on the other. The areas are proportional

to

$$\text{area}(\text{ACE}) : \text{area}(\text{BCD}) : \text{area}(\text{BCD}) = 27a^2 : 12a^2 : 12a^2.$$

Their perpendicular distances from EA are respectively $3a$, $5a$ and a . Taking moments about EA gives the y -coordinate of G.

$$27a^2(3a) - 12a^2(5a) + 12a^2(a) = 27a^2y.$$

$$81a^3 - 60a^3 + 12a^3 = 27a^2y \implies y = \frac{11a}{9}.$$

Hence $G = (2a, 11a/9)$. Relative to B,

$$\overrightarrow{BA} = a \begin{pmatrix} 2 \\ -3 \end{pmatrix}, \quad \overrightarrow{BG} = \frac{2a}{9} \begin{pmatrix} -9 \\ -8 \end{pmatrix}.$$

The downward vertical through B is the line BG. Use a scalar product to find the angle α between BA and BG.

$$\overrightarrow{BA} \cdot \overrightarrow{BG} = \frac{2a^2}{9} ((-9)(2) + (-8)(-3)) = \frac{4a^2}{3}.$$

$$|BA| = a\sqrt{13}, \quad |BG| = \frac{2a}{9}\sqrt{145},$$

$$\cos \alpha = \frac{4a^2/3}{a\sqrt{13} (2a\sqrt{145}/9)} = 0.138\dots, \quad \alpha = 82.0\dots^\circ.$$

$$\alpha = 82^\circ \text{ to the nearest degree.}$$

(9)