



Edexcel AS Further Mathematics

8FM0/26 - Further Mechanics 2

Tuesday 24 May 2022

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2022 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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Question 1

Question 1

A uniform lamina is the isosceles trapezium shown. BCEF is a square of side $3a$, and $AB = CD = a$. The mass is M .

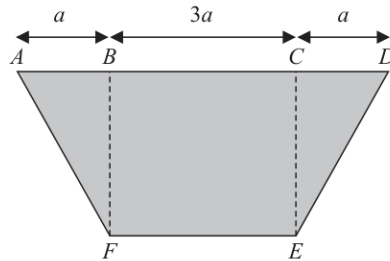


Figure 1

A uniform plane lamina is in the shape of an isosceles trapezium $ABCDEF$, as shown

- (a) Show that its centre of mass is $\frac{11a}{8}$ from AD .
 (b) It is suspended by light vertical strings at A and F , hanging with BF horizontal. Find the tension at A .

Model solution

(a) Divide the lamina into the square and its two congruent triangular ends. Taking moments of area about AD is appropriate because all three centroid distances are then simple.

piece	area	centroid distance from AD
$\triangle ABF$	$\frac{3}{2}a^2$	a
$\square BCEF$	$9a^2$	$\frac{3}{2}a$
$\triangle CDE$	$\frac{3}{2}a^2$	a

If y is the distance of the combined centre of mass from AD , equating moments gives

$$\left(\frac{3}{2}a^2\right)a + (9a^2)\left(\frac{3}{2}a\right) + \left(\frac{3}{2}a^2\right)a = \left(\frac{3}{2}a^2 + 9a^2 + \frac{3}{2}a^2\right)y.$$

$$\frac{33}{2}a^3 = 12a^2y \Rightarrow y = \frac{11a}{8}.$$

The centre of mass is $\frac{11a}{8}$ from AD .

(5)

(b) Take moments about F to remove the other unknown tension. The weight acts vertically through the centre of mass, whose horizontal distance from F is $3a - \frac{11a}{8} = \frac{13a}{8}$.

$$T_A(3a) = Mg\left(\frac{13a}{8}\right) \Rightarrow T_A = \frac{13}{24}Mg.$$

$$T_A = \frac{13Mg}{24}.$$

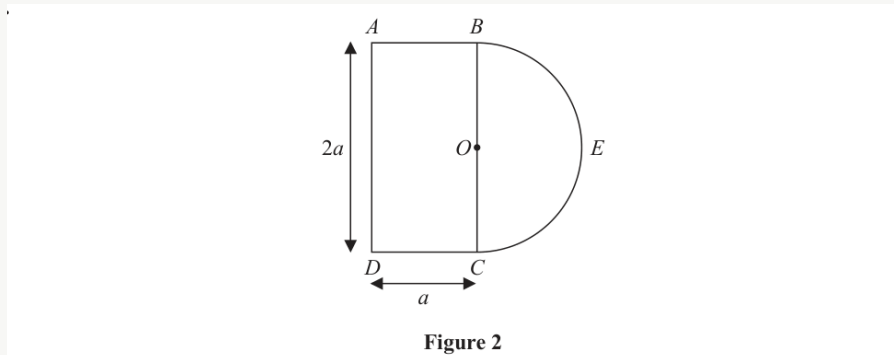
(2)

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Question 2

Question 2

Uniform wire forms the framework shown. ABCD is a rectangle with $AD = 2a$ and $DC = a$. BEC is a semicircular arc of radius a , with diameter BC and centre O; $OE \perp BC$.



- (a) Show that the centre of mass is $\frac{a}{6 + \pi}$ from BC.
 (b) Suspended from A, the framework hangs with AE at angle θ to the downward vertical. Find θ .
 (c) The framework has mass M . A particle of mass kM is attached at B, and the loaded centre of mass lies on OA. Find k .

Model solution

(a) Since the wire is uniform, mass is proportional to length. Take moments about BC: the rectangular wire has its centre on this line, so gives no moment.

$$\text{rectangular-wire length} = 6a, \quad \text{arc length} = \pi a.$$

The centroid of a semicircular arc is $\frac{2a}{\pi}$ from its diameter. The rectangular wire's centre is $\frac{a}{2}$ to the left of BC, whereas the arc's centre is to its right. Taking moments about BC, with leftward moments positive, gives

$$(6a) \left(\frac{a}{2} \right) - (\pi a) \left(\frac{2a}{\pi} \right) = (6a + \pi a)x.$$

$$a^2 = a(6 + \pi)x \Rightarrow x = \frac{a}{6 + \pi}.$$

The centre of mass is $\frac{a}{6 + \pi}$ to the left of BC.

(5)

(b) A freely suspended body has its centre of mass vertically beneath its suspension point. Let that centre be G, so the downward vertical is AG. The required rotation is the angle between AE and AG, so calculate both angles to the horizontal.

$$\angle(\text{AE, horizontal}) = \tan^{-1} \left(\frac{a}{2a} \right) = \tan^{-1} \left(\frac{1}{2} \right).$$

From A to G, the horizontal displacement is $a - \frac{a}{6+\pi}$ and the downward displacement is a . Hence

$$\angle(AG, \text{horizontal}) = \tan^{-1} \left(\frac{a}{a - \frac{a}{6+\pi}} \right) = \tan^{-1} \left(\frac{6+\pi}{5+\pi} \right).$$

$$\theta = \angle(AG, \text{horizontal}) - \angle(AE, \text{horizontal}) = \tan^{-1} \left(\frac{6+\pi}{5+\pi} \right) - \tan^{-1} \left(\frac{1}{2} \right) = 21.746 \dots^\circ.$$

$$\theta = 21.7^\circ \text{ (approximately).}$$

(4)

(c) Take moments about OA. The framework's centre is at perpendicular distance $x \sin 45^\circ$ from OA, and B is on the other side at distance $a \sin 45^\circ$. The opposing moments balance.

$$M \left(\frac{a}{6+\pi} \right) \sin 45^\circ = kM(a) \sin 45^\circ.$$

$$k = \frac{1}{6+\pi}.$$

$$k = \frac{1}{6+\pi} \text{ (about 0.109).}$$

(3)

Question 3

Question 3

A cyclist moves at constant speed around a horizontal circle of radius r on a track banked at angle α , where $\tan \alpha = \frac{3}{4}$. In a rough-track model the coefficient of sideways friction is $\mu < \frac{4}{3}$.

(a) Show that the maximum speed without sideways slipping is

$$V = \sqrt{\frac{3+4\mu}{4-3\mu}}rg.$$

- (b) In a frictionless model find the corresponding speed U .
 (c) Show that $U < V$.

Model solution

(a) At maximum speed the cyclist tends to slide up the bank, so limiting friction acts down the slope. Let R be the normal reaction and F the friction. Resolve vertically and horizontally towards the centre.

$$R \cos \alpha - F \sin \alpha = mg, \quad R \sin \alpha + F \cos \alpha = \frac{mV^2}{r}.$$

At limiting equilibrium, $F = \mu R$. Since $\tan \alpha = 3/4$, $\sin \alpha = 3/5$ and $\cos \alpha = 4/5$, so

$$R \left(\frac{4}{5} - \frac{3\mu}{5} \right) = mg, \quad R \left(\frac{3}{5} + \frac{4\mu}{5} \right) = \frac{mV^2}{r}.$$

$$R(4 - 3\mu) = 5mg, \quad R(3 + 4\mu) = \frac{5mV^2}{r}.$$

Dividing eliminates the unknown reaction.

$$\frac{V^2}{rg} = \frac{3+4\mu}{4-3\mu} \Rightarrow V = \sqrt{\frac{3+4\mu}{4-3\mu}}rg.$$

$$V = \sqrt{\frac{3+4\mu}{4-3\mu}}rg.$$

(7)

(b) With no sideways friction, put $\mu = 0$ in the result above.

$$U = \sqrt{\frac{3}{4}}rg.$$

$$U = \sqrt{\frac{3rg}{4}}.$$

(2)

(c) As $\mu > 0$ and $\mu < \frac{4}{3}$, $3 + 4\mu > 3$ and $4 - 3\mu < 4$, with positive denominators. Thus

$$\frac{3 + 4\mu}{4 - 3\mu} > \frac{3}{4}.$$

Multiplying by positive rg and taking positive square roots gives $V > \sqrt{3rg/4} = U$.

$$U < V.$$

(2)

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Question 4

Question 4

A particle moves on the x-axis. At time $t \geq 0$, its velocity is

$$v = \frac{1}{2} (3e^{2t} - 1) \text{ m s}^{-1}.$$

Its acceleration is $a \text{ m s}^{-2}$.

- Show that $a = 2v + 1$.
- Find the acceleration when $t = 0$.
- Find the exact distance travelled while the speed increases from 1 m s^{-1} to 4 m s^{-1} .

Model solution

(a) Acceleration is the rate of change of velocity, so differentiate with respect to time, then express the result using v .

$$a = \frac{dv}{dt} = \frac{1}{2} (3 \cdot 2e^{2t}) = 3e^{2t}.$$

$$2v + 1 = 2 \left[\frac{1}{2} (3e^{2t} - 1) \right] + 1 = 3e^{2t} = a.$$

$$a = 2v + 1.$$

(2)

(b) Put $t = 0$ into the acceleration expression.

$$a = 3e^0 = 3.$$

$$3 \text{ m s}^{-2}.$$

(1)

(c) Use $v = \frac{dx}{dt}$ to find position. The velocity is positive during this interval, so displacement equals distance travelled.

$$\frac{dx}{dt} = \frac{1}{2} (3e^{2t} - 1), \quad x = \frac{3}{4} e^{2t} - \frac{1}{2} t + C.$$

Use the two speeds to find the endpoints in time:

$$\frac{1}{2} (3e^{2t} - 1) = 1 \Rightarrow t = 0, \quad \frac{1}{2} (3e^{2t} - 1) = 4 \Rightarrow t = \frac{1}{2} \ln 3.$$

The constant cancels on subtracting positions.

$$\begin{aligned} \text{distance} &= \left[\frac{3}{4} e^{2t} - \frac{1}{2} t \right]_0^{\frac{1}{2} \ln 3} \\ &= \left(\frac{9}{4} - \frac{1}{4} \ln 3 \right) - \frac{3}{4} \\ &= \frac{3}{2} - \frac{1}{4} \ln 3. \end{aligned}$$

$$\frac{3}{2} - \frac{1}{4} \ln 3 \, m.$$

(7)

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