



Edexcel AS Further Mathematics

8FM0/26 - Further Mechanics 2

Friday 19 May 2023

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2023 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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Question 1

Question 1

Three particles of masses $4m$, $2m$ and km are at $(-3, -1)$, $(6, 1)$ and $(-1, 5)$ respectively. Their centre of mass is $(\bar{x}, \bar{y})$.

(a) Show that $\bar{x} = -\frac{k}{k+6}$. (3)

(b) Find $\bar{y}$ in terms of k . (2)

The centre of mass lies on $y = 2x + 3$.

(c) Find k . (2)

A fourth particle of mass λm is at $(\lambda, 4)$. The centre of mass of all four particles also lies on $y = 2x + 3$.

(d) Find λ . (2)

Model solution

(a) A centre-of-mass coordinate is a mass-weighted average. Taking moments about the y -axis gives the required x -coordinate.

$$(4m)(-3) + (2m)(6) + (km)(-1) = (k+6)m\bar{x}.$$

$$-12m + 12m - km = (k+6)m\bar{x} \implies \bar{x} = -\frac{k}{k+6}.$$

$$\bar{x} = -\frac{k}{k+6}.$$

(3)

(b) Take moments about the x -axis, so that the ordinates are used.

$$(4m)(-1) + (2m)(1) + (km)(5) = (k+6)m\bar{y}.$$

$$-4m + 2m + 5km = (k+6)m\bar{y} \implies \bar{y} = \frac{5k-2}{k+6}.$$

$$\bar{y} = \frac{5k-2}{k+6}.$$

(2)

(c) The given line is the clue to substitute the centre-of-mass coordinates into its equation.

$$\frac{5k-2}{k+6} = 2\left(-\frac{k}{k+6}\right) + 3.$$

$$5k - 2 = -2k + 3(k+6) = -2k + 3k + 18,$$

$$5k - 2 = k + 18 \implies 4k = 20 \implies k = 5.$$

$$k = 5.$$

(2)

(d) The original centre of mass is already on the line. For the new centre of mass to remain there, the added particle must itself lie on the line.

$$4 = 2\lambda + 3 \implies \lambda = \frac{1}{2}.$$

$$\lambda = \frac{1}{2}.$$

(2)

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Question 2

Question 2

A particle P moves along the x-axis. At time t seconds, $t \geq 0$, its velocity in the direction of increasing x is

$$v = e^{2t} + 6e^t - kt,$$

where k is positive. When $t = \ln 2$, $a = 0$.

(a) Find k . (4)

When $t = 0$, P passes through fixed point A. When $t = \ln 2$, it is d metres from A.

(b) Show all stages and find d , correct to 2 significant figures. (4)

Model solution

(a) Acceleration is the rate of change of velocity, so differentiate before applying $a = 0$.

$$a = \frac{dv}{dt} = 2e^{2t} + 6e^t - k.$$

At $t = \ln 2$, $e^t = 2$ and $e^{2t} = 4$, so

$$0 = 2(4) + 6(2) - k = 20 - k.$$

$$k = 20.$$

(4)

(b) Displacement is the integral of velocity. Limits from 0 to $\ln 2$ use the fact that the particle starts at A.

$$d = \int_0^{\ln 2} (e^{2t} + 6e^t - 20t) dt = \left[\frac{1}{2}e^{2t} + 6e^t - 10t^2 \right]_0^{\ln 2}.$$

$$d = \frac{1}{2}(4 - 1) + 6(2 - 1) - 10(\ln 2)^2 = 7.5 - 10(\ln 2)^2 = 2.695 \dots$$

The displacement is positive, so it is also the distance from A.

$$d = 2.7 \text{ m (2 significant figures).}$$

(4)

Question 3

Question 3

A girl and her bicycle, of combined mass 55 kg, cycle round a circular track. The coefficient of friction between the track and tyres is μ . The track is banked at 15° to the horizontal. They are modelled as a particle moving in a horizontal circle of radius 50 m. The minimum speed without slipping is 4.5 m s^{-1} .

Using the model, find μ .

(9)

Model solution

At the minimum speed the bicycle tends to slip down the bank, so friction F acts up the slope. Let R be the normal reaction, and take horizontal inward as positive: it is the direction of the centripetal acceleration.

Resolve horizontally to provide centripetal force; the horizontal components of R and F oppose each other:

$$\frac{55(4.5)^2}{50} = R \sin 15^\circ - F \cos 15^\circ.$$

There is no vertical acceleration, so

$$55g = R \cos 15^\circ + F \sin 15^\circ.$$

With $g = 9.8 \text{ m s}^{-2}$,

$$22.275 = R \sin 15^\circ - F \cos 15^\circ, \quad 539 = R \cos 15^\circ + F \sin 15^\circ.$$

Combining these equations resolves the two contact forces.

$$R = 22.275 \sin 15^\circ + 539 \cos 15^\circ = 526.55 \dots \text{ N},$$

$$F = 539 \sin 15^\circ - 22.275 \cos 15^\circ = 117.96 \dots \text{ N}.$$

At limiting equilibrium $F = \mu R$, hence

$$\mu = \frac{F}{R} = \frac{117.96 \dots}{526.55 \dots} = 0.224 \dots$$

$$\mu = 0.224 \text{ (or 0.22 to 2 significant figures).}$$

(9)

Question 4

Question 4

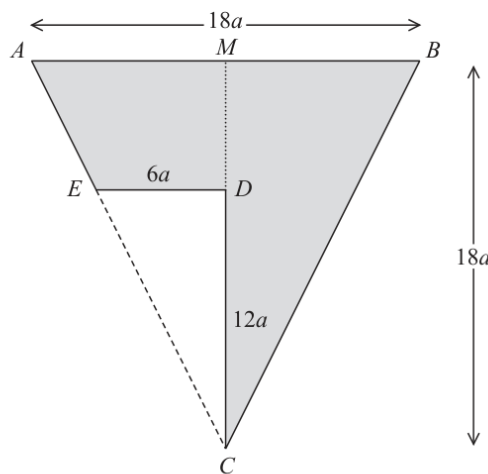


Figure 1

A uniform isosceles triangular lamina ABC is isosceles, with $AC = BC$. The midpoint of AB is M . The length of AB is $18a$ and the length of CM is $18a$.

A uniform isosceles triangular lamina ABC has $AB = 18a$, $CM = 18a$, and M is the midpoint of AB . Triangle CDE , where $DE = 6a$ and $CD = 12a$, is removed to form lamina L . Here $ED \parallel AB$ and M, D, C are collinear. The distance of the centre of mass of L from MC is d .

(a) Show that $d = \frac{4}{7}a$. (4)

Lamina L is suspended by vertical light strings at A and B , with AB horizontal. Its weight is W .

(b) Find, in terms of W , the tension in the string at B . (3)

The string at B breaks. Lamina L is then suspended from A . In equilibrium, the angle between AB and the downward vertical through A is θ° .

(c) Find θ . (7)

Model solution

(a) Regard the removed triangle as negative mass. Moments about CM are convenient because the centre of mass of the large isosceles triangle lies on CM .

$$\text{Area of } ABC = \frac{1}{2}(18a)(18a) = 162a^2, \quad \text{area of } CDE = \frac{1}{2}(6a)(12a) = 36a^2.$$

The centroid of CDE is one third of the way from CD towards E , hence $2a$ left of CM . The remaining area is $126a^2$, so

$$36a^2(2a) = 126a^2d.$$

$$d = \frac{72a^3}{126a^2} = \frac{4}{7}a.$$

$$d = \frac{4}{7}a.$$

(4)

(b) Take moments about A, removing the unknown tension at A. The centre of mass is $9a + \frac{4}{7}a$ horizontally from A.

$$T_B(18a) = W \left(9a + \frac{4}{7}a \right),$$

$$T_B = \frac{\frac{67}{7}a}{18a} W = \frac{67}{126} W.$$

$$T_B = \frac{67}{126} W.$$

(3)

(c) When suspended from A, the centre of mass must lie vertically below A. Find its perpendicular distance below AB by taking moments of the two triangular areas about AB. The centroid of ABC is $6a$ below AB. The centroid of removed triangle CDE is $\frac{18a+6a+6a}{3} = 10a$ below AB.

$$126a^2\bar{y} = 162a^2(6a) - 36a^2(10a),$$

$$\bar{y} = \frac{612a^3}{126a^2} = \frac{34}{7}a.$$

The centre of mass is $\frac{67}{7}a$ from A along AB. The right triangle formed when it is directly below A gives

$$\tan \theta = \frac{\frac{34}{7}a}{\frac{67}{7}a} = \frac{34}{67}.$$

$$\theta = \tan^{-1} \left(\frac{34}{67} \right) = 26.9 \dots^\circ.$$

$$\theta = 27.0^\circ \text{ (3 significant figures).}$$

(7)