



Edexcel AS Further Mathematics

8FM0/26 - Further Mechanics 2

Friday 17 May 2024

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2024 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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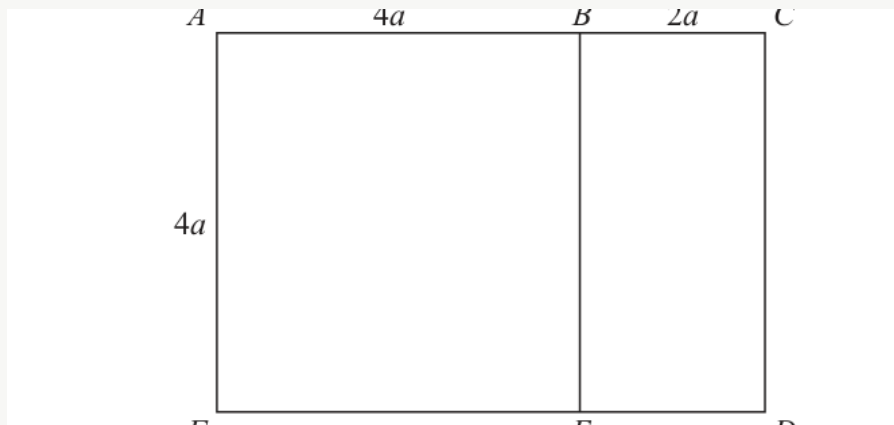
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Question 1

Question 1

A uniform rod of length $24a$ is cut into seven pieces to make the planar framework ABCDEF in Figure 1. The vertical rods AF, BE, CD have length $4a$; $AB = FE = 4a$ and $BC = ED = 2a$.



The distance of the centre of mass from AF is d .

(a) Show that $\frac{d}{a} = \frac{19}{6}$.

(b) Find the distance of the centre of mass from A.

Model solution

(a) Take moments about AF. This is efficient because every point on AF has zero perpendicular distance. Since the rod is uniform, mass is proportional to length, so lengths may be used in the moments equation.

$$24a d = (4a)(2a) + (4a)(4a) + (2a)(5a) + (4a)(6a) + (2a)(5a) + (4a)(2a).$$

The distances are those of the midpoints of AB, BE, BC, CD, DE and EF from AF. Only the rod AF itself has zero moment about this axis.

$$24ad = 8a^2 + 16a^2 + 10a^2 + 24a^2 + 10a^2 + 8a^2 = 76a^2, \quad d = \frac{19a}{6}.$$

$$\frac{d}{a} = \frac{19}{6}.$$

((a) 4 marks)

(b) The framework is symmetric about its horizontal midline, so its centre of mass is $2a$ vertically below A. Use Pythagoras with the horizontal distance from part (a).

$$D^2 = \left(\frac{19a}{6}\right)^2 + (2a)^2 = \frac{361a^2}{36} + \frac{144a^2}{36} = \frac{505a^2}{36}.$$

$$D = \frac{\sqrt{505}}{6}a.$$

The distance from A is $\frac{\sqrt{505}}{6}a$ (approximately $3.75a$).

((b) 3 marks)

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Question 2

Question 2

A thin hollow hemisphere of radius a , with centre O and vertical axis, is fixed as shown. A ball B of mass m moves in a horizontal circle of radius r on its smooth inner surface. Its circle has centre C , where $OC = h$, and it has constant angular speed ω . Air resistance is negligible.

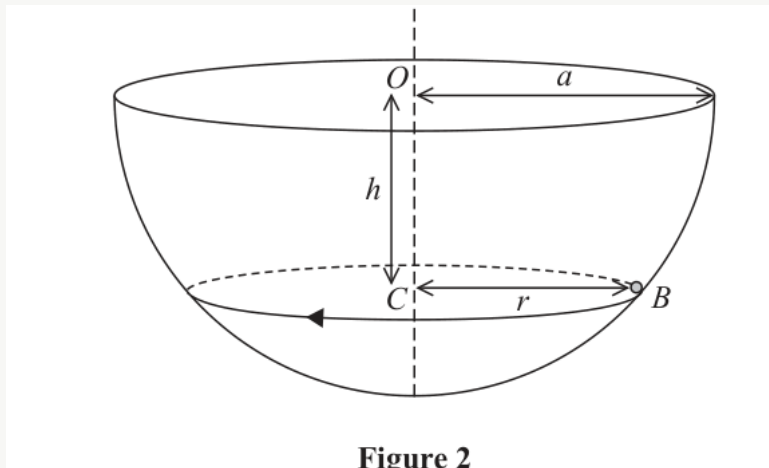


Figure 2

(a) Show that $\omega^2 = \frac{g}{h}$.

Given that the normal reaction has magnitude $3mg$:

(b) find ω in terms of g and a ;

(c) state how, apart from ignoring air resistance, the particle model for B has been used.

Model solution

(a) Resolve the forces horizontally towards the centre of the circular path and vertically. The only forces are the weight mg and normal reaction R ; the horizontal resultant supplies centripetal acceleration $r\omega^2$. Let θ be the angle made by R with the horizontal.

$$R \cos \theta = mr\omega^2, \quad R \sin \theta = mg.$$

Dividing eliminates the unknown reaction. From the geometry, $\tan \theta = h/r$.

$$\frac{r\omega^2}{g} = \frac{\cos \theta}{\sin \theta} = \frac{r}{h} \implies \omega^2 = \frac{g}{h}.$$

$$\omega^2 = \frac{g}{h}.$$

((a) 6 marks)

(b) Use the given reaction in the vertical balance. This fixes the height h of the circle.

$$3mg \sin \theta = mg \implies \sin \theta = \frac{1}{3}.$$

Since $OB = a$ and $OC = h$ is its vertical component, $h = a \sin \theta = a/3$. Substitute in part (a).

$$\omega^2 = \frac{g}{a/3} = \frac{3g}{a}, \quad \omega = \sqrt{\frac{3g}{a}}.$$

$$\omega = \sqrt{\frac{3g}{a}}.$$

((b) 3 marks)

(c) A particle has no dimensions in the model; hence its size, and any rotational effects due to its size, are ignored.

The dimensions of B have been ignored.

((c) 1 mark)

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Question 3

Question 3

A particle P moves along the positive x-axis. At time t it has velocity $v \text{ m s}^{-1}$ and acceleration $a \text{ m s}^{-2}$, where

$$a = 4 - 3v, \quad v = 0 \text{ when } t = 0.$$

(a) Use integration to show that $v = k(1 - e^{-3t})$, finding k .

When $t = 0$, P is at the origin O.

(b) Find, in terms of t only, the distance of P from O at time t .

Model solution

(a) The acceleration is given in terms of velocity, so use $a = dv/dt$ and separate variables before integrating.

$$\frac{dv}{dt} = 4 - 3v \implies \frac{dv}{4 - 3v} = dt.$$

$$\int \frac{dv}{4 - 3v} = \int dt \implies -\frac{1}{3} \ln(4 - 3v) = t + C.$$

The initial condition determines the integration constant.

$$v = 0 \text{ when } t = 0 \implies C = -\frac{1}{3} \ln 4.$$

$$-\frac{1}{3} \ln(4 - 3v) = t - \frac{1}{3} \ln 4 \implies \ln\left(\frac{4}{4 - 3v}\right) = 3t.$$

$$\frac{4}{4 - 3v} = e^{3t} \implies 4 - 3v = 4e^{-3t} \implies v = \frac{4}{3}(1 - e^{-3t}).$$

$$v = \frac{4}{3}(1 - e^{-3t}), \text{ so } k = \frac{4}{3}.$$

((a) 7 marks)

(b) Velocity is dx/dt . Integrate the velocity and then use $x = 0$ when $t = 0$.

$$\frac{dx}{dt} = \frac{4}{3}(1 - e^{-3t}) \implies x = \frac{4}{3}\left(t + \frac{1}{3}e^{-3t}\right) + C.$$

$$0 = \frac{4}{3}\left(\frac{1}{3}\right) + C \implies C = -\frac{4}{9}.$$

$$x = \frac{4}{3}t + \frac{4}{9}e^{-3t} - \frac{4}{9} = \frac{4}{3}t - \frac{4}{9}(1 - e^{-3t}).$$

The velocity is non-negative for $t \geq 0$, so this displacement is also the distance from the origin.

$$\text{distance} = \frac{4}{3}t - \frac{4}{9}(1 - e^{-3t}) \text{ m.}$$

((b) 4 marks)

Question 4

Question 4

The uniform right-angled triangular lamina ABC has $AB = 9a$, $AC = 6a$ and $AB \perp AC$. Point D lies on AB with $AD = a$. A $2a$ by $3a$ rectangle $DEFG$ is removed to form the template in Figure 3. The distance of its centre of mass from AC is d .

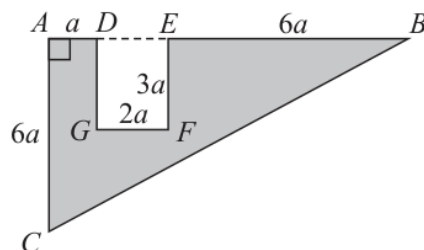


Figure 3

triangular lamina ABC has AB perpendicular to AC , $AB = 9a$ and $AC = 6a$. Point D lies on AB such that $AD = a$.

A rectangle $DEFG$, with $DE = 2a$ and $EF = 3a$, is removed from the lamina. The remaining shape is shown shaded in Figure 3.

The distance of the centre of mass of the template from AC is d .

$$d = \frac{23}{7}a$$

The template is freely suspended from A and hangs in equilibrium with AB at an angle θ° to the downward vertical through A .

Find the value of θ .

(a) Show that $\frac{d}{a} = \frac{23}{7}$.

The template is suspended from A and hangs with AB at an angle θ° to the downward vertical through A .

(b) Find θ .

An identical template of a different uniform material is joined to the first as in Figure 4. The weight of $CPQRSTA$ is W and that of $ADGFEB$ is $4W$. A horizontal force of magnitude X is applied at T so that AC is vertical.

(c) Find X in terms of W .

Model solution

(a) Treat the removed rectangle as negative mass and take moments about AC , because d is measured from this axis. Areas are proportional to masses for a uniform lamina.

$$(27a^2 - 6a^2)d = (27a^2)(3a) - (6a^2)(2a).$$

The triangular lamina has area $\frac{1}{2}(9a)(6a) = 27a^2$ and centroid distance $3a$ from AC . The

rectangle has area $6a^2$ and centroid distance $2a$ from AC.

$$21a^2d = 81a^3 - 12a^3 = 69a^3 \implies d = \frac{23a}{7}.$$

$$\frac{d}{a} = \frac{23}{7}.$$

((a) 3 marks)

(b) A freely suspended lamina has its centre of mass vertically below the suspension point. Find the other component of its centre of mass by taking moments about AB.

$$(27a^2 - 6a^2)y = (27a^2)(2a) - (6a^2)(1.5a).$$

$$21a^2y = 54a^3 - 9a^3 = 45a^3 \implies y = \frac{15a}{7}.$$

The centre of mass is $23a/7$ from AC and $15a/7$ from AB, so its line from A gives the required angle.

$$\tan \theta = \frac{15a/7}{23a/7} = \frac{15}{23}, \quad \theta = \tan^{-1} \left(\frac{15}{23} \right) = 33.111\dots^\circ.$$

$$\theta = 33.1^\circ \text{ (or } 33^\circ \text{ to 2 s.f.)}$$

((b) 5 marks)

(c) With AC vertical, take moments about A. The centres of mass are each $23a/7$ horizontally from AC, on opposite sides, and their weights are W and $4W$. The horizontal force at T has perpendicular distance $6a$. Equate clockwise and anticlockwise moments.

$$6aX + \left(\frac{23a}{7} \right) W = \left(\frac{23a}{7} \right) (4W).$$

$$6aX = \frac{69a}{7} W \implies X = \frac{23}{14} W.$$

$$X = \frac{23}{14} W.$$

((c) 4 marks)