



Edexcel AS Further Mathematics

8FM0/26 - Further Mechanics 2

Friday 16 May 2025

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2025 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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Question 1

Question 1

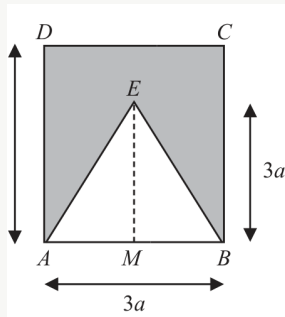


Figure 1

A uniform lamina is made by removing the isosceles triangle ABE from rectangle ABCD, where $AB = 3a$, $AD = 4a$, M is the midpoint of AB, and $EM = 3a$.

- (a) Show that the centre of mass is $\frac{13a}{5}$ from AB.
 (b) The lamina is suspended from D. Find, to the nearest degree, the angle between AD and the vertical.

Model solution

(a) Treat the removed triangle as negative mass. Since the lamina is uniform, its mass is proportional to its area, so moments of areas about AB locate the centre of mass.

part	area	distance of its centre from AB
ABCD	$12a^2$	$2a$
ABE	$\frac{1}{2}(3a)(3a) = \frac{9}{2}a^2$	$\frac{1}{3}(3a) = a$

The triangle's centroid is one third of its height from its base. The area of the remaining lamina is

$$12a^2 - \frac{9}{2}a^2 = \frac{15}{2}a^2.$$

Let x be the required distance from AB. Equating moments about AB, with the removed triangle subtracted, gives

$$\frac{15}{2}a^2x = (12a^2)(2a) - \left(\frac{9}{2}a^2\right)(a).$$

$$x = \frac{24a^3 - \frac{9}{2}a^3}{\frac{15}{2}a^2} = \frac{13a}{5}.$$

The centre of mass is $\frac{13a}{5}$ from AB.

(5)

(b) When suspended freely, the centre of mass lies vertically below D. The symmetry of the original shape about EM means that the centre of mass is horizontally $\frac{3a}{2}$ from AD. Its vertical separation below D is

$$4a - \frac{13a}{5} = \frac{7a}{5}.$$

Therefore the required angle α between AD and the vertical is found from the right triangle joining D to the centre of mass:

$$\tan \alpha = \frac{\frac{3}{2}a}{\frac{7}{5}a} = \frac{15}{14}.$$

$$\alpha = 46.97 \dots^\circ.$$

$$\alpha = 47^\circ \text{ to the nearest degree.}$$

(3)

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Question 2

Question 2

A particle moves on the x-axis with velocity

$$v = 3 - \sqrt{2t + 1}, \quad t \geq 0.$$

- (a) Find when it is instantaneously at rest.
- (b) Show that $a = \frac{1}{v-3}$.
- (c) Find its speed when it is decelerating at $\frac{4}{3} \text{ m s}^{-2}$.
- (d) Find the total distance travelled between $t = 0$ and $t = \frac{15}{2}$.

Model solution

(a) *Instantaneous rest means zero velocity, so set the given expression for v equal to zero.*

$$0 = 3 - \sqrt{2t + 1} \implies \sqrt{2t + 1} = 3 \implies 2t + 1 = 9.$$

$$t = 4 \text{ s.}$$

(2)

(b) *Acceleration is the rate of change of velocity. Differentiate the square-root expression, then use $\sqrt{2t + 1} = 3 - v$ to express the answer in terms of v .*

$$a = \frac{dv}{dt} = -\frac{1}{\sqrt{2t + 1}} = -\frac{1}{3 - v} = \frac{1}{v - 3}.$$

$$a = \frac{1}{v - 3} \text{ m s}^{-2}.$$

(3)

(c) *The particle is decelerating while initially moving in the positive direction, so its acceleration is $-\frac{4}{3} \text{ m s}^{-2}$. Substitute this in the result from part (b).*

$$-\frac{4}{3} = \frac{1}{v - 3} \implies v - 3 = -\frac{3}{4} \implies v = \frac{9}{4}.$$

$$\text{The speed is } \frac{9}{4} = 2.25 \text{ m s}^{-1}.$$

(2)

(d) *Distance requires the magnitude of displacement. The particle changes direction at $t = 4$, found in part (a), so integrate on each side of this time and add the two positive distances.*

$$s = \int (3 - \sqrt{2t + 1}) dt = 3t - \frac{1}{3}(2t + 1)^{3/2} + C.$$

Using $S(t) = 3t - \frac{1}{3}(2t + 1)^{3/2}$,

$$S(0) = -\frac{1}{3}, \quad S(4) = 3, \quad S\left(\frac{15}{2}\right) = \frac{7}{6}.$$

Velocity is positive before $t = 4$ and negative afterwards. Hence

$$\text{total distance} = \left(3 - \left(-\frac{1}{3}\right)\right) + \left(3 - \frac{7}{6}\right) = \frac{10}{3} + \frac{11}{6} = \frac{31}{6}.$$

The total distance travelled is $\frac{31}{6} \text{ m} = 5\frac{1}{6} \text{ m}.$

(4)

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Question 3

Question 3

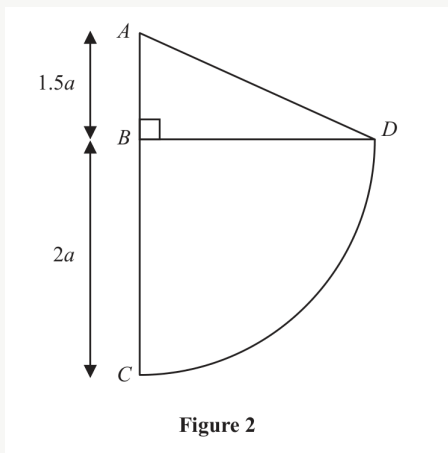


Figure 2

Uniform wire forms the framework AB, BC, BD, AD and arc CD. Here $AB = 1.5a$, $BC = BD = 2a$, CD is a circular arc of radius $2a$ and centre B, ABC is straight and ABD is a right angle.

- (a) Show that the centre of mass of arc CD is $\frac{4a}{\pi}$ from AC.
- (b) Show that the centre of mass of the framework is $\frac{17a}{2(8+\pi)}$ from AC.
- (c) The framework is pivoted at A and is held with BD horizontal by an upward vertical force at D. Its mass is M , and the magnitude of the pivot force is kMg . Find k .

Model solution

(a) Arc CD subtends $90^\circ = \frac{\pi}{2}$ at its centre B. The centre of mass of a circular arc lies on its angle bisector. The quoted arc formula gives its distance from B; resolve this distance perpendicular to AC.

$$\text{distance from B} = \frac{(2a) \sin(\pi/4)}{\pi/4} = \frac{4\sqrt{2}a}{\pi}.$$

$$\text{distance from AC} = \frac{4\sqrt{2}a}{\pi} \sin \frac{\pi}{4} = \frac{4a}{\pi}.$$

The centre of mass of arc CD is $\frac{4a}{\pi}$ from AC.

(2)

(b) Use moments of wire lengths about AC. Pieces AB and BC lie on AC, so their moments are zero. Also, $AD = \sqrt{(1.5a)^2 + (2a)^2} = 2.5a$.

wire piece	length	distance of its centre from AC
AD	$\frac{5}{2}a$	a
BD	$2a$	a
arc CD	πa	$\frac{4a}{\pi}$

The total wire length is $(8 + \pi)a$. If its centre of mass is distance x from AC, then

$$(8 + \pi)ax = \left(\frac{5}{2}a\right)a + (2a)a + (\pi a)\left(\frac{4a}{\pi}\right) = \frac{17}{2}a^2.$$

$$x = \frac{17a}{2(8 + \pi)}.$$

The centre of mass is $\frac{17a}{2(8 + \pi)}$ from AC.

(4)

(c) With BD horizontal, the force at D and the weight are vertical. Taking moments about A removes the unknown pivot force. The perpendicular distance of the force at D from A is $2a$, and that of the weight is the distance from AC found in part (b).

$$F(2a) = Mg \left(\frac{17a}{2(8 + \pi)} \right),$$

$$F = \frac{17Mg}{4(8 + \pi)}.$$

All the other forces are vertical, so vertical equilibrium gives the upward pivot force as $Mg - F$. Thus

$$kMg = Mg - \frac{17Mg}{4(8 + \pi)}$$

$$k = 1 - \frac{17}{4(8 + \pi)} = \frac{15 + 4\pi}{4(8 + \pi)}.$$

$$k = \frac{15 + 4\pi}{4(8 + \pi)}.$$

(3)

Question 4

Question 4

A car travels at constant speed in a horizontal circle of radius r .

(a) On a horizontal rough track, the maximum speed without sideways slipping is $\frac{1}{2}\sqrt{gr}$. Show that $\mu = \frac{1}{4}$.

(b) The track is now banked at angle θ to the horizontal and has coefficient of friction $\frac{1}{4}$. The minimum speed without sideways slipping is $\sqrt{\frac{4rg}{35}}$. Find $\tan \theta$.

Model solution

(a) On a horizontal track, the normal reaction balances the weight. At the maximum speed, limiting friction provides the required centripetal force towards the centre.

$$R = mg, \quad F = \frac{mv^2}{r} = \frac{m\left(\frac{1}{2}\sqrt{gr}\right)^2}{r} = \frac{1}{4}mg.$$

At limiting equilibrium $F = \mu R$, so

$$\mu = \frac{F}{R} = \frac{\frac{1}{4}mg}{mg} = \frac{1}{4}.$$

$$\mu = \frac{1}{4}.$$

(5)

(b) At the minimum speed, the car tends to slide down the bank (towards the centre), so limiting friction F acts up the slope. Take horizontal towards the centre as positive. Resolving vertically and horizontally gives

$$R \cos \theta + F \sin \theta = mg,$$

$$R \sin \theta - F \cos \theta = \frac{mv^2}{r}.$$

The given minimum speed makes $\frac{v^2}{r} = \frac{4g}{35}$, and limiting friction gives $F = \frac{1}{4}R$. Substitute both facts:

$$R \left(\cos \theta + \frac{1}{4} \sin \theta \right) = mg,$$

$$R \left(\sin \theta - \frac{1}{4} \cos \theta \right) = \frac{4}{35}mg.$$

Dividing the second equation by the first eliminates R , m and g , leaving an equation only in $\tan \theta$:

$$\frac{\tan \theta - \frac{1}{4}}{1 + \frac{1}{4} \tan \theta} = \frac{4}{35}.$$

$$35 \left(\tan \theta - \frac{1}{4} \right) = 4 \left(1 + \frac{1}{4} \tan \theta \right)$$

$$34 \tan \theta = \frac{51}{4} \implies \tan \theta = \frac{3}{8}.$$

$$\tan \theta = \frac{3}{8}.$$

(7)

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