



Edexcel AS Further Mathematics

8FM0/26 - Further Mechanics 2

November 2021

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2021 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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Question 1

Question 1

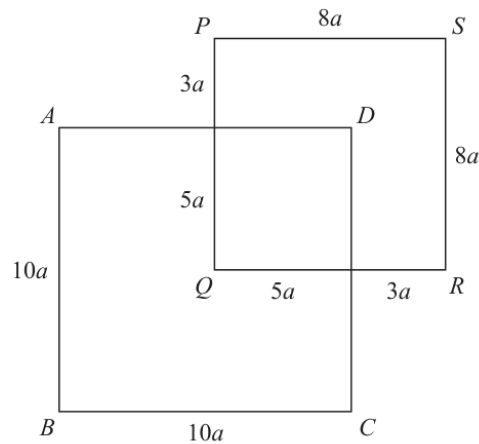


Figure 1

A uniform rod of length $72a$ is cut into pieces. The pieces are used to make two rigid

A uniform rod of length $72a$ is made into the two joined rigid squares shown. Square ABCD has side $10a$, square PQRS has side $8a$, and $AB \parallel PQ$. The intersections divide PQ in the ratio $3 : 5$ and QR in the ratio $5 : 3$.

(a) Explain why the centre of mass of square ABCD is at Q. (1)

(b) Find the distance of the centre of mass of the framework from B. (5)

Model solution

(a)

A uniform square has equal mass distribution in both of its diagonals, so its centre of mass is at their intersection. In the diagram, Q is the centre of square ABCD.

The centre of mass of square ABCD is Q.

(b)

Because the rod is uniform, mass is proportional to length. We can therefore use the perimeters of the two squares as their relative masses. Take moments about AB, so that the perpendicular distances of the two square centres are easy to read from the diagram.

$$m_{ABCD} : m_{PQRS} : m_{\text{framework}} = 40 : 32 : 72.$$

The centre of the large square is $5a$ from AB. The centre of the smaller square is $5a + 4a = 9a$ from AB. If the framework's centre is x from AB, moments give

$$40(5a) + 32(9a) = 72x, \quad x = \frac{488}{72}a = \frac{61}{9}a.$$

The arrangement is also symmetric about the diagonal through B making equal angles with AB and BC. Hence the centre is also $\frac{61}{9}a$ from BC. Its distance from B follows from Pythagoras.

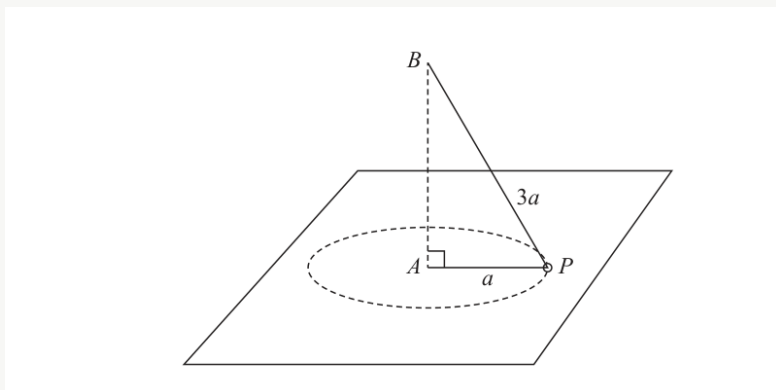
$$\text{distance from B} = \sqrt{\left(\frac{61}{9}a\right)^2 + \left(\frac{61}{9}a\right)^2} = \frac{61\sqrt{2}}{9}a.$$

$$\frac{61\sqrt{2}}{9}a$$

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Question 2

Question 2



A smooth ring P of mass m is threaded on a light inextensible string of length $4a$. One end is fixed at A on a smooth horizontal table and the other at B, vertically above A. The ring moves in a horizontal circle of radius a about AB, initially with constant angular speed $\sqrt{\frac{2g}{3a}}$. The string is taut.

- Find the normal reaction between P and the table. (6)
- As the angular speed is increased, find the angular speed when P loses contact with the table. (3)
- Explain how the fact that P is smooth has been used. (1)

Model solution

(a)

The length AP is a , so the remaining part PB has length $3a$. Since B is vertically above A, triangle APB is right-angled at A. Let θ be the angle which PB makes with the horizontal. This gives the components needed for resolving the forces.

$$\cos \theta = \frac{a}{3a} = \frac{1}{3}, \quad \sin \theta = \frac{\sqrt{(3a)^2 - a^2}}{3a} = \frac{\sqrt{8}}{3}.$$

Let the common tension in the string be T and let the upward normal reaction be N . There is no vertical acceleration, while horizontally the resultant force towards A supplies the centripetal acceleration.

$$T \sin \theta + N = mg, \quad T + T \cos \theta = ma\omega^2.$$

Here $\omega^2 = \frac{2g}{3a}$, so the horizontal equation becomes

$$T + \frac{1}{3}T = ma \left(\frac{2g}{3a} \right), \quad \frac{4}{3}T = \frac{2}{3}mg, \quad T = \frac{1}{2}mg.$$

Substitute this tension into the vertical equilibrium equation.

$$\frac{1}{2}mg \left(\frac{\sqrt{8}}{3} \right) + N = mg, \quad N = mg \left(1 - \frac{\sqrt{2}}{3} \right).$$

$$N = mg \left(1 - \frac{\sqrt{2}}{3} \right)$$

(b)

At the instant of losing contact, the table can no longer exert a normal force, so $N = 0$. The geometry of the string has not changed, so the same values of $\sin \theta$ and $\cos \theta$ apply.

$$T \left(\frac{\sqrt{8}}{3} \right) = mg \Rightarrow T = \frac{3mg}{\sqrt{8}}.$$

The two horizontal tension components still provide the centripetal force.

$$T + \frac{1}{3}T = ma\omega^2, \quad \frac{4}{3} \left(\frac{3mg}{\sqrt{8}} \right) = ma\omega^2, \quad \omega^2 = \frac{\sqrt{2}g}{a}.$$

$$\omega = \sqrt{\frac{\sqrt{2}g}{a}}$$

(c)

The ring is smooth, so there is no friction between it and the string. Therefore the tension has the same magnitude in both portions of the string on either side of P.

The tensions on the two sides of P are equal.

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Question 3

Question 3

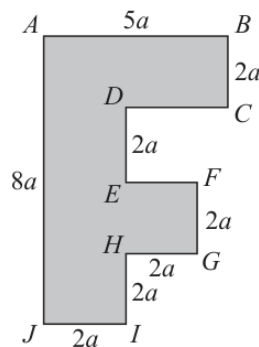


Figure 3

The uniform lamina ABCDEFGHIJ has $AJ = 8a$, $AB = 5a$, and $BC = DE = EF = FG = GH = HI = IJ = 2a$, with all corners right angles.

(a) Show that the distance of its centre of mass from AJ is $\frac{49}{26}a$. (5)

The lamina hangs in equilibrium from vertical ropes attached at A and B, with AB horizontal. Its weight is W.

(b) Find the tension in the rope at B. (3)

The rope at B breaks. The lamina then hangs freely in equilibrium from A.

(c) Find the angle between AJ and the downward vertical. (5)

Model solution

(a)

Split the stepped lamina into three non-overlapping rectangles. Since the lamina is uniform, their masses are proportional to their areas. Taking moments about AJ finds the horizontal coordinate of the combined centre of mass.

rectangle	area ratio	distance of its centre from AJ
$2a \times 8a$	16	a
$3a \times 2a$ (top right)	6	$\frac{7}{2}a$
$2a \times 2a$ (middle right)	4	$3a$

If x is the required distance from AJ, the moment of the whole lamina equals the sum of the moments of these rectangles.

$$26x = 16(a) + 6\left(\frac{7}{2}a\right) + 4(3a) = 16a + 21a + 12a = 49a, \quad x = \frac{49}{26}a.$$

$$\text{distance from AJ} = \frac{49}{26}a$$

(b)

With both ropes vertical, take moments about A. The tension at A has no moment, and the perpendicular distances of the tension at B and the weight are respectively $5a$ and

$$\frac{49}{26}a.$$

$$T_B(5a) = W \left(\frac{49}{26}a \right), \quad T_B = \frac{49}{130}W.$$

$$T_B = \frac{49}{130}W$$

(c)

When suspended from one point, a rigid body settles with its centre of mass vertically below that point. We already know its horizontal distance from A; now find its vertical distance below AB by taking moments about AB.

rectangle	area ratio	distance of its centre below AB
$2a \times 8a$	16	$4a$
$3a \times 2a$ (top right)	6	a
$2a \times 2a$ (middle right)	4	$5a$

$$26y = 16(4a) + 6(a) + 4(5a) = 90a, \quad y = \frac{90}{26}a.$$

The line from A to the centre of mass becomes the downward vertical. Therefore, using its horizontal and vertical components relative to AJ,

$$\tan \theta = \frac{\frac{49}{26}a}{\frac{90}{26}a} = \frac{49}{90}, \quad \theta = 28.5658 \dots^\circ.$$

$$\theta = 28.6^\circ \text{ (approximately } 29^\circ \text{)}$$

Question 4

Question 4

A particle P moves on the x-axis. At time $t \geq 0$, its velocity in the direction of increasing x is

$$v = 5 \sin 2t \text{ m s}^{-1}.$$

When $t = 0$, $x = 1$ and P is at rest.

- (a) Find the magnitude and direction of the acceleration when P is next at rest. (4)
- (b) Show that $1 \leq x \leq 6$. (4)
- (c) Find the total time in the first 4π seconds for which P is more than 3 m from O. (3)

Model solution

(a)

An instant of rest means $v = 0$. We need the first such time after $t = 0$, then differentiate the velocity to find acceleration.

$$5 \sin 2t = 0 \Rightarrow 2t = \pi \Rightarrow t = \frac{\pi}{2}.$$

$$\frac{dv}{dt} = 10 \cos 2t, \quad \left. \frac{dv}{dt} \right|_{t=\pi/2} = 10 \cos \pi = -10.$$

The negative sign means the acceleration is in the direction of decreasing x , which at this instant is towards the origin.

$$10 \text{ m s}^{-2} \text{ towards O}$$

(b)

Position is obtained by integrating velocity. The given initial position determines the constant of integration.

$$x = \int 5 \sin 2t \, dt = -\frac{5}{2} \cos 2t + C.$$

$$1 = -\frac{5}{2} \cos 0 + C = -\frac{5}{2} + C \Rightarrow C = \frac{7}{2}, \quad x = \frac{7}{2} - \frac{5}{2} \cos 2t.$$

Since $-1 \leq \cos 2t \leq 1$, substituting its extreme values gives the full range of x .

$$\frac{7}{2} - \frac{5}{2}(1) \leq x \leq \frac{7}{2} - \frac{5}{2}(-1), \quad 1 \leq x \leq 6.$$

$$1 \leq x \leq 6$$

(c)

The result in part (b) shows that x is always positive, so "more than 3 m from O" is simply $x > 3$. Solve the boundary equation first; the motion then repeats every π seconds because the position contains $\cos 2t$.

$$\frac{7}{2} - \frac{5}{2} \cos 2t = 3 \Rightarrow \cos 2t = \frac{1}{5}.$$

$$\alpha = \frac{1}{2} \cos^{-1} \left(\frac{1}{5} \right) = 0.684719 \dots \text{ s.}$$

In one period, $x > 3$ from $t = \alpha$ to $t = \pi - \alpha$, a duration $\pi - 2\alpha$. There are four complete periods in 4π seconds.

$$\text{total time} = 4(\pi - 2\alpha) = 4\pi - 8 \left[\frac{1}{2} \cos^{-1} \left(\frac{1}{5} \right) \right] = 7.08861 \dots \text{ s.}$$

7.09 s (approximately)

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