



Edexcel AS Further Mathematics

8FM0/27 - Decision Mathematics 1

Friday 18 May 2018

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2018 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

Contents

Question 1	2
Question 2	4
Question 3	6
Question 4	8

www.mathswithmatt.co.uk

Question 1

Question 1

Figure 1 is a network of roads. The number on each arc is the time, in minutes, to travel along that road.

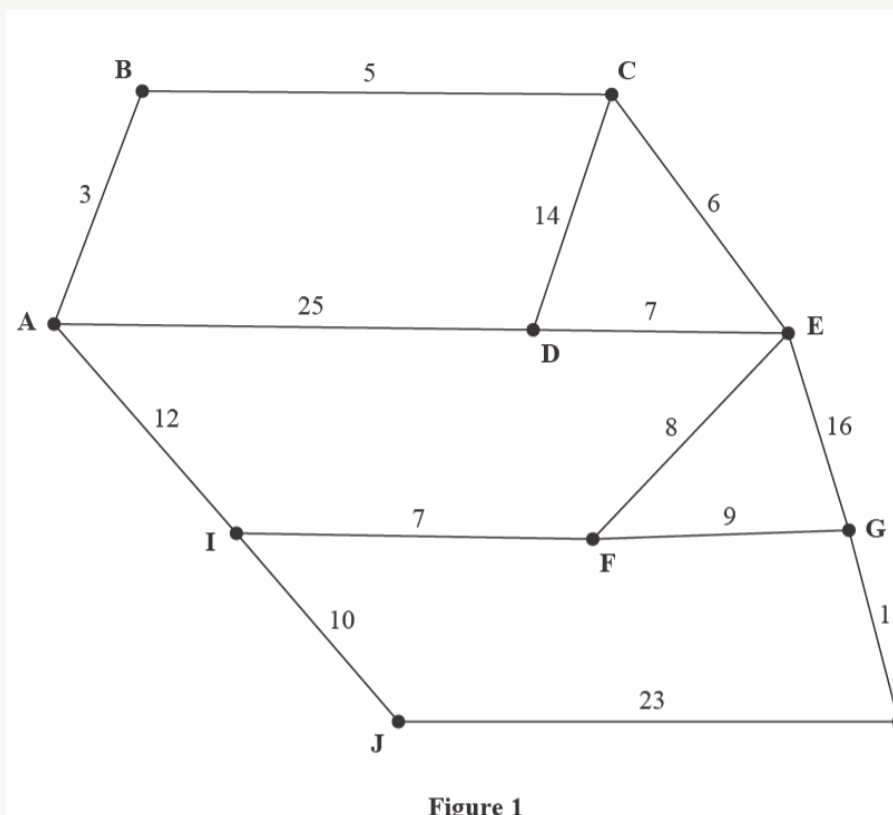


Figure 1

- (a) (i) Use Dijkstra's algorithm to find the shortest time needed to travel from A to H.
(ii) State the quickest route.
For a network with n vertices, Dijkstra's algorithm has order n^2 .
(b) If the algorithm takes 1.5 seconds when $n = 250$, calculate approximately how long, in seconds, it will take when $n = 9500$. Make the method clear.
(c) Explain why the answer to part (b) is only an approximation.

Model solution

(a)(i) Starting at A, the permanent labels and the useful tentative-label updates are as follows. A label in parentheses gives the preceding vertex.

<i>Permanent vertex and label</i>	<i>New or improved tentative labels</i>
A, 0	B = 3 (A), I = 12 (A), D = 25 (A)
B, 3	C = 8 (B)
C, 8	E = 14 (C), D = 22 (C)
I, 12	F = 19 (I), J = 22 (I)
E, 14	D = 21 (E), G = 30 (E)
F, 19	G = 28 (F)
D, 21	no improvement
J, 22	H = 45 (J)
G, 28	H = 39 (G)

The next smallest label is therefore $H = 39$, so it is permanent and is the shortest time from A to H.

The shortest time is 39 minutes.

(5)

(a)(ii) Trace back the preceding vertices from H: $H \leftarrow G \leftarrow F \leftarrow I \leftarrow A$.

The quickest route is A--I--F--G--H.

(1)

(b) Treating the running time as proportional to n^2 , the scale factor is

$$\left(\frac{9500}{250}\right)^2 = 38^2 = 1444.$$

$$1.5 \times 1444 = 2166.$$

Approximately 2166 seconds.

(2)

(c) Order n^2 specifies only that n^2 is the dominant term in the running time. It does not say that the running time is exactly proportional to n^2 for these particular values of n ; lower-order terms and fixed overheads are being ignored.

(1)

Question 2

Question 2

A simply connected graph is a connected graph in which any two vertices are directly connected by at most one arc and no vertex is directly connected to itself.

- (a) Given that a simply connected graph has exactly four vertices, write down (i) the minimum number of arcs it can have and (ii) the maximum number of arcs it can have.
- (b) (i) Draw a simply connected graph with exactly four vertices and exactly five arcs.
(ii) State, with justification, whether the graph is Eulerian, semi-Eulerian or neither.
- (c) By considering the orders of the vertices, explain why there is only one simply connected graph with exactly four vertices and exactly five arcs.

Model solution

(a)(i) A connected graph on four vertices can be a tree, which has one fewer arc than vertices.

The minimum number of arcs is 3.

(1)

(a)(ii) In a simple graph each pair of distinct vertices can be joined at most once. There are $\binom{4}{2} = 6$ such pairs.

The maximum number of arcs is 6.

(1)

(b)(i) Label the vertices A, B, C, D and use the arcs AB, BC, CD, DA, AC. This is a connected simple graph with four vertices and five arcs.

(i) e.g.



(ii) The graph has exactly two Eulerian

Edges: AB, BC, CD, DA, AC

Orders: $\text{ord}(A) = 3, \text{ord}(B) = 2, \text{ord}(C) = 3, \text{ord}(D) = 2$

(1)

(b)(ii) Exactly two vertices, A and C, have odd order. A connected graph with exactly two odd vertices is semi-Eulerian.

The graph is semi-Eulerian.

(2)

(c) By the handshaking lemma, the sum of the vertex orders is twice the number of arcs:

$$\sum \text{ord}(v) = 2 \times 5 = 10.$$

The possible positive orders, in non-decreasing order and with each at most 3, are 1, 3, 3, 3 and 2, 2, 3, 3. The first is impossible: every vertex of order 3 is joined to all the other vertices, so three vertices cannot all have order 3 while the fourth has order 1.

Thus the orders must be 2,2,3,3. Each order-3 vertex is joined to every other vertex. Consequently the two order-2 vertices cannot be joined to each other; all five remaining possible arcs must be present. Hence the graph is uniquely K_4 with the arc between the two order-2 vertices removed. (5)

www.mathswithmatt.co.uk

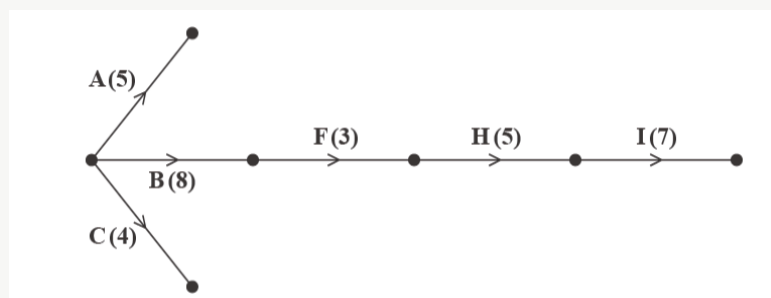
Question 3

Question 3

The following table gives the activities for a building project. An activity is shown by an arc; its time is in days; and the final column gives its immediately preceding activities.

Activity	Time taken (days)	Immediately preceding activities
A	5	–
B	8	–
C	4	–
D	14	A
E	10	A
F	3	B, C, E
G	7	C
H	5	D, F, G
I	7	H
J	9	H

The partially completed activity network is shown below.



- Add the missing activities and necessary dummies to the activity network.
- Complete the network to show the early event times and late event times.
- State the critical activities.

At the beginning of the project, activity G is no longer required.

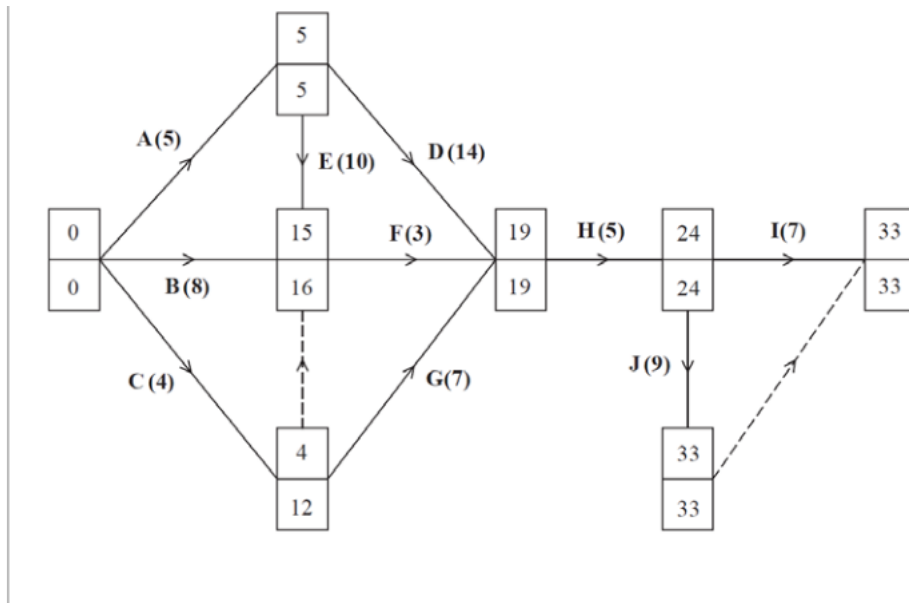
- Explain the effect, if any, on (i) the shortest completion time and (ii) the timing of the remaining activities.

Model solution

(a) Complete the network as follows. From the end of A draw D(14) to the event at which H starts, and draw E(10) to the end of B. Draw a dummy from the end of C to the end of B, so that F depends on B, C and E. Draw G(7) from the end of C to the event at which H starts. From the end of H draw J(9) to a new event, and draw a dummy from that new event to the final event; this makes the project depend on both I and J. (3)

(b) Forward pass: the early event times are 0, 5, 15, 4, 19, 24, 33 at the successive relevant events. In particular, the event before F is $\max(8, 4, 5 + 10) = 15$, the event before H is $\max(5 + 14, 15 + 3, 4 + 7) = 19$, and the final event is $\max(24 + 7, 24 + 9) = 33$.

Backward pass from final time 33: the corresponding late event times are 0, 5, 16, 12, 19, 24, 33. Thus the completed event labels (early above late) are: start 0/0; end of A 5/5; end of B/E 15/16; end of C 4/12; start of H 19/19; end of H 24/24; final event 33/33.



(3)

(c) Critical activities have zero float, so their start and finish events have equal early and late times.

The critical activities are A, D, H and J.

(1)

(d)(i) G is not critical, so removing it does not alter the critical path A--D--H--J.

There is no effect: the shortest completion time remains 33 days.

(1)

(d)(ii) The only affected activity is C. Without the requirement to precede G, its latest finish is governed by F: it may finish at time 16, so it may start at time 12. Previously its latest finish was time 12 and it had to start by time 8.

C may now start 4 days later, at time 12 (or finish as late as time 16); the timings of the other remaining activities are unchanged.

(2)

Question 4

Question 4

A factory manager plans production of cabinets over three weeks. Week 3 production cannot be fewer than the total in weeks 1 and 2; week 3 production is at most twice week 2 production; and the combined production in weeks 2 and 3 is at most 125. The production costs per cabinet in weeks 1, 2 and 3 are £250, £275 and £200 respectively. The objective is to minimise

$$250x + 275y + 200z.$$

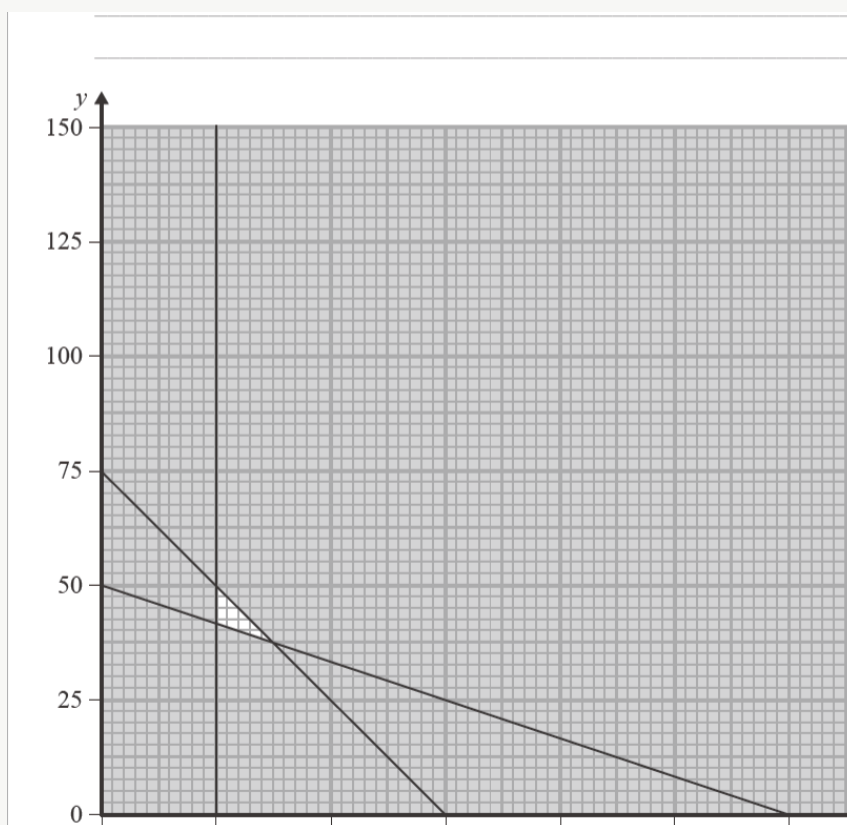
(a) Explain what x, y, z represent.

(b) Write down the constraints in terms of x, y, z .

Exactly 150 cabinets must be produced. This gives the reduced constraints

$$x + y \leq 75, \quad x + 3y \geq 150, \quad x \geq 25, \quad y \geq 0.$$

The diagram supplied for the graphical method is shown below.



(c) (i) Use a graphical approach to determine the integer production schedule that minimises the cost.

(ii) Find the minimum total cost.

Model solution

(a) x is the number of cabinets produced in week 1, y is the number produced in week 2, and z is the number produced in week 3. (1)

(b) Translating the three conditions, together with non-negativity, gives

$$x + y \leq z, \quad z \leq 2y, \quad y + z \leq 125, \quad x, y, z \geq 0.$$

(2)

(c)(i) Since exactly 150 cabinets are required, $z = 150 - x - y$. Substitute in the objective:

$$P = 250x + 275y + 200(150 - x - y) = 50x + 75y + 30000.$$

The feasible region is bounded by $x = 25$, $x + y = 75$ and $x + 3y = 150$. Its vertices are

$$\left(25, \frac{125}{3}\right), \quad (25, 50), \quad \left(\frac{75}{2}, \frac{75}{2}\right).$$

For the objective line $50x + 75y = \text{constant}$, the objective values (omitting the constant 30000) at these vertices are

$$50(25) + 75\left(\frac{125}{3}\right) = 4375, \quad 50(25) + 75(50) = 5000, \quad 50\left(\frac{75}{2}\right) + 75\left(\frac{75}{2}\right) = 4687.5.$$

Thus, as the objective line is moved towards smaller values, it first meets the feasible region at $(25, 125/3)$. As cabinet numbers must be integers, test the nearby feasible integer points. At $(25, 42)$,

$$25 + 3(42) = 151 \geq 150, \quad 25 + 42 = 67 \leq 75,$$

so the point is feasible. With $y = 41$, the constraint $x + 3y \geq 150$ requires $x \geq 27$: $(26, 41)$ is infeasible because $26 + 3(41) = 149 < 150$, while the nearest feasible point is $(27, 41)$. Its objective value is

$$50(27) + 75(41) = 4425 > 4400 = 50(25) + 75(42).$$

For $y \geq 42$, $x \geq 25$ gives $50x + 75y \geq 4400$; for $y \leq 41$, $x \geq 150 - 3y$ gives $50x + 75y \geq 7500 - 75y \geq 4425$. Hence $(25, 42)$ is the optimal integer point.

$$z = 150 - 25 - 42 = 83.$$

Produce 25 cabinets in week 1, 42 in week 2 and 83 in week 3.

(7)

(c)(ii) The total cost for this integer schedule is

$$250(25) + 275(42) + 200(83) = 6250 + 11550 + 16600 = 34400.$$

finalansTheminimumtotalcostis\$34 400.

(1)