



Edexcel AS Further Mathematics

8FM0/27 - Decision Mathematics 1

Thursday 16 May 2019

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2019 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

Contents

Question 1	2
Question 2	4
Question 3	5
Question 4	7
Question 5	9

www.mathswithmatt.co.uk

Question 1

Question 1

- (a) Draw the complete graph K_5 .
 (b) (i) Explain, in graph-theory terms, what “semi-Eulerian” means.
 (ii) Draw two semi-Eulerian subgraphs of K_5 , each with five vertices but with different numbers of edges.
 (c) Explain why a graph with five vertices of orders 1, 2, 2, 3, 4 cannot be a tree.

Model solution

(a) K_5 is the complete graph on five vertices, so every pair of distinct vertices must be joined. Label the vertices A, B, C, D, E; the required ten edges are

AB, AC, AD, AE, BC, BD, BE, CD, CE, DE.

A complete graph on five vertices, with all $\binom{5}{2} = 10$ edges drawn.

(1)

(b)(i) For an Euler trail to start and finish at different vertices, precisely the two end vertices must have odd order. This is exactly the defining feature of a semi-Eulerian graph.

A semi-Eulerian graph has exactly two vertices of odd order; all its other vertices have even order.

(1)

(b)(ii) We need two edge sets taken from K_5 , each using all five vertices and giving exactly two odd orders. The following are two valid drawings specified by their edges; joining the listed pairs produces the required subgraphs.

subgraph	edges	vertex orders (A, B, C, D, E)
1	AB, BC, CD, DE	1, 2, 2, 2, 1
2	AB, BC, CD, DE, EA, AC	3, 2, 3, 2, 2

The first has four edges and the second has six, and each has exactly two odd-order vertices.

For example, the two subgraphs above.

(2)

(c) Use the handshaking lemma to convert the listed vertex orders into a number of edges. A tree on n vertices must have exactly $n - 1$ edges.

$$\text{sum of orders} = 1 + 2 + 2 + 3 + 4 = 12 = 2|E|,$$

$$|E| = 6.$$

But a tree with five vertices has $5 - 1 = 4$ edges. Since this graph has six edges, it cannot be a tree.

It has 6 edges, whereas a tree with five vertices has only 4 edges.

(2)

www.mathswithmatt.co.uk

Question 2

Question 2

The algorithm below approximates $I = \int_A^B x^4 dx$ by the trapezium rule.

$$H = \frac{B - A}{N}, \quad C = \frac{H}{2}, \quad D = A^4 + B^4,$$

then, successively for $E = A + H, A + 2H, \dots, B - H$, replace D by $D + 2E^4$, and output $F = CD$.
For $A = 1, B = 3$ and $N = 4$:

- (a) (i) complete the algorithm table; (ii) state its final output.
(b) Calculate the percentage error, to 3 significant figures, between the exact value of I and this approximation.

Model solution

(a)(i) The clue $H = (B - A)/N$ and the repeated addition of $2E^4$ shows that the algorithm is accumulating the interior trapezium-rule terms. First calculate the fixed values.

$$H = \frac{3 - 1}{4} = 0.5, \quad C = \frac{H}{2} = 0.25, \quad D = 1^4 + 3^4 = 82.$$

Each pass increases E by 0.5 and then adds twice the fourth power of that interior value to D .

value of E	new value of D	comment
1.5	$82 + 2(1.5)^4 = 92.125$	continue
2	$92.125 + 2(2)^4 = 124.125$	continue
2.5	$124.125 + 2(2.5)^4 = 202.25$	continue
3	202.25	now $E = B$

At $E = B$, the algorithm moves to the output step, so

$$F = CD = 0.25(202.25) = 50.5625.$$

(i) The table entries are as above; (ii) $F = 50.5625$.

(4)

(b) Find the exact integral before comparing it with the numerical approximation. The percentage error is measured relative to the exact value.

$$I = \int_1^3 x^4 dx = \left[\frac{x^5}{5} \right]_1^3 = \frac{243 - 1}{5} = 48.4.$$

$$\text{percentage error} = \frac{|50.5625 - 48.4|}{48.4} \times 100 = 4.4679 \dots \%$$

4.47% (to 3 significant figures).

(3)

Question 3

Question 3

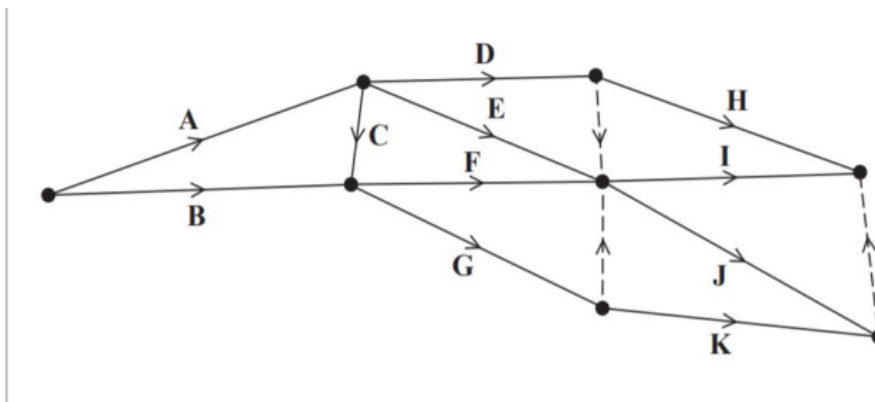
The immediate predecessors of the activities are

activity	A	B	C	D	E	F	G	H	I	J	K
predecessors	—	—	A	A	A	B, C	B, C	D	D, E, F, G	D, E, F, G	G

- (a) Draw the activity-on-arc network using the minimum number of dummies.
Every activity has the same duration.
(b) Explain why B cannot be critical.
(c) State the other activities that are not critical.

Model solution

(a) In an activity-on-arc network, merge events only when they have exactly the same required preceding activities. Dummies are needed to retain a precedence condition without creating an extra activity. The following network has the minimum three dummies (shown dashed).



In particular, C follows A; F and G start only after both B and C; I waits for D, E, F, G; and J waits for D, E, F, G.

The activity-on-arc network shown, with three dummy arcs.

(5)

(b) Compare the time needed to reach the start of F (or G). Both B and the two-activity chain A, C must be complete first. Since all activities have equal duration, A, C take twice as long as B. Thus B can be delayed while A and C are being completed, so it has float and cannot be critical.

B is not critical because it finishes before the required chain A, C can be complete.

(1)

(c) Applying the same equal-duration comparison to the network identifies the remaining activities with float.

D, E and H.

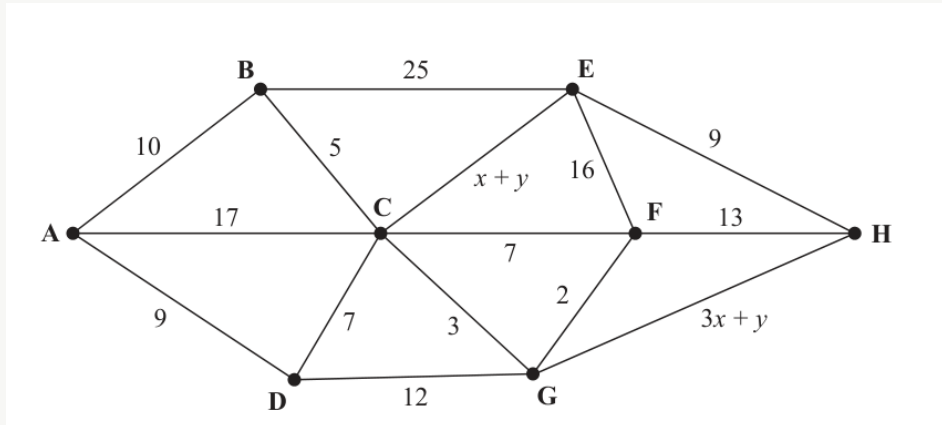
(1)

www.mathswithmatt.co.uk

Question 4

Question 4

In the weighted network below, the arc weights $CE = x + y$ and $GH = 3x + y$, where $x, y > 0$ and $7 < x + y < 20$. There are three shortest paths from A to H.



(a) Use Dijkstra's algorithm to find x and y .

An inspection route starts at A, ends at H, traverses every arc at least once, and has minimum length.

(b) State the arcs traversed twice. (c) State how many times C appears. (d) Find the route length.

Model solution

(a) *Dijkstra's algorithm is appropriate because all the weights are positive and we need shortest-path information from A. Record each permanent label and the alternative temporary labels that matter.*

permanent order	vertex and label	relevant updates
1	A : 0	B = 10, C = 17, D = 9
2	D : 9	C = $\min(17, 9 + 7) = 16$, G = 21
3	B : 10	C = $\min(16, 10 + 5) = 15$, E = 35
4	C : 15	E = $\min(35, 15 + x + y)$, F = 22, G = 18
5	G : 18	F = $\min(22, 18 + 2) = 20$, H = $18 + 3x + y$
6	F : 20	H = $\min(18 + 3x + y, 20 + 13) = \min(18 + 3x + y, 33)$
7	E	H = $\min(33, 24 + x + y)$
8	H	three routes give 33, $18 + 3x + y$, $24 + x + y$

The question says that all three routes to H have the same minimum length, so equate the three displayed route lengths.

$$18 + 3x + y = 33 \implies 3x + y = 15,$$

$$24 + x + y = 33 \implies x + y = 9.$$

Subtracting the second equation from the first isolates x .

$$2x = 6 \implies x = 3, \quad y = 9 - 3 = 6.$$

$$x = 3, \quad y = 6.$$

(7)

(b) An open route from A to H may leave only A and H odd. The odd vertices in the original network that must be paired are B and D, so duplicate their shortest connecting path.

$$B \rightarrow C \rightarrow D \quad \text{has length } 5 + 7 = 12.$$

Arcs BC and CD are traversed twice.

(1)

(c) Vertex C is used on four visits in the resulting Euler trail: the duplicated arcs add two incidences at C to its original six incidences.

C appears 4 times.

(1)

(d) Add the duplicated shortest path to the total network weight, then use the values of x and y .

$$135 + 4x + 2y + 12 = 135 + 4(3) + 2(6) + 12 = 171.$$

171 units.

(1)

www

Question 5

Question 5

Let x, y, z be respectively the numbers of roses, hydrangeas and peonies. At least three-fifths of all flowers are roses; for every two hydrangeas there are at most three peonies; exactly 1000 flowers are ordered. Costs are £1, £5 and £4 respectively.

- (a) Formulate the linear programme in x and y only, with simplified integer-coefficient inequalities. Ben then orders the minimum permitted number of roses.
 (b) Find (i) the number of each flower which minimises cost, and (ii) the total cost.

Model solution

(a) *Begin with the cost before eliminating z . This makes the meaning of each coefficient clear.*

$$\text{Minimise } P = x + 5y + 4z.$$

Translate each verbal condition. The "at least three-fifths" condition is a fraction of the total, and "for every 2 hydrangeas, at most 3 peonies" gives $2z \leq 3y$.

$$x \geq \frac{3}{5}(x + y + z), \quad 3y \geq 2z, \quad x + y + z = 1000.$$

The total is fixed, so eliminate z using $z = 1000 - x - y$. Substitution into the objective gives

$$P = x + 5y + 4(1000 - x - y) = 4000 - 3x + y.$$

The roses condition becomes $x \geq \frac{3}{5}(1000) = 600$. For the hydrangea--peony condition, substitute for z and rearrange without fractions.

$$3y \geq 2(1000 - x - y) \implies 2x + 5y \geq 2000.$$

Minimise $P = 4000 - 3x + y$, subject to $x \geq 600$, $2x + 5y \geq 2000$, $x, y \geq 0$ and $x + y \leq 1000$, with x, y integral.

(7)

(b)(i) *The instruction fixes x at its least allowed value, $x = 600$. With this value, use the second constraint to find the least y ; this minimises $P = 4000 - 3x + y$.*

$$2(600) + 5y \geq 2000 \implies 5y \geq 800 \implies y \geq 160.$$

Take $y = 160$, then use the total of 1000 to find the peonies.

$$z = 1000 - 600 - 160 = 240.$$

(i) 600 roses, 160 hydrangeas and 240 peonies.

(b)(ii) *Apply the stated unit costs to this order.*

$$600(1) + 160(5) + 240(4) = 600 + 800 + 960 = 2360.$$

(ii) £2360.

(3)

www.mathswithmatt.co.uk