



Edexcel AS Further Mathematics

8FM0/27 - Decision Mathematics 1

Friday 16 May 2025

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2025 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

Contents

Question 1	2
Question 2	4
Question 3	6
Question 4	8

www.mathswithmatt.co.uk

Question 1

Question 1

(a) The list 16, 10, 25, 30, 13, 12, 28, 22, 23, 20 is to be sorted into ascending order using a bubble sort, starting at the left.

The weighted arcs in the network are

$AB = 13$, $AC = 10$, $AD = 30$, $BC = 12$, $BE = 22$, $BF = 23$, $CD = 28$, $CF = 20$, $DF = 25$, $EF = 16$.

(b) Use Prim's algorithm, starting at A, to find a minimum spanning tree and list the arcs in selection order.

(c) Draw the tree and state its weight.

Model solution

(a) A left-to-right bubble-sort pass compares neighbouring entries and swaps them when they are in the wrong order. Thus the largest unsorted entry bubbles to the right-hand end on each pass.

pass	state of the list at the end of the pass									
1	10	16	25	13	12	28	22	23	20	30
2	10	16	13	12	25	22	23	20	28	30
3	10	13	12	16	22	23	20	25	28	30
4	10	12	13	16	22	20	23	25	28	30
5	10	12	13	16	20	22	23	25	28	30

The list is now in ascending order, so the sort is complete.

10, 12, 13, 16, 20, 22, 23, 25, 28, 30.

(3)

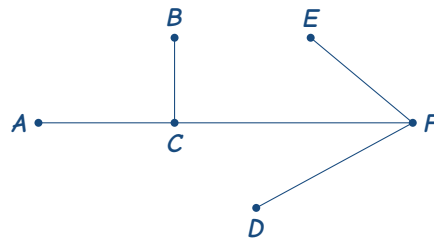
(b) Prim's algorithm grows one connected tree. At each stage choose the least-weight arc joining a vertex already in the tree to a vertex outside it. Starting at A:

tree vertices before selection	least eligible arc	new vertex
A	AC (10)	C
A, C	BC (12)	B
A, B, C	CF (20)	F
A, B, C, F	EF (16)	E
A, B, C, E, F	DF (25)	D

AC, BC, CF, EF, DF, in that order.

(2)

(c)(i) The minimum spanning tree is obtained by retaining precisely the five arcs selected by Prim's algorithm.



The minimum spanning tree is shown.

(1)

(c)(ii) *Add the weights of the five tree arcs; no other arcs are included.*

$$10 + 12 + 20 + 16 + 25 = 83.$$

The weight of the minimum spanning tree is 83.

(1)

Question 2

Question 2

The activity precedences are

Activity	immediately preceding activities
A, B, C	—
D	B
E	A, B
F	B
G	B
H	C, D
I	G, H
J	E
K	E, F, I
L	G, H
M	G, H

(a) Explain why at least one dummy is needed. (b) Draw an activity-on-arc network using the minimum number of dummies. Each activity takes 3 hours. (c) State the minimum completion time. (d) One activity is to be extended by one hour without changing that time; list the possible activities.

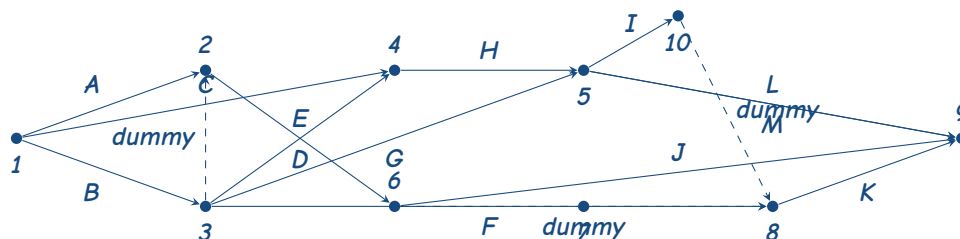
Model solution

(a) A dummy is needed when two activities have different precedence requirements but their event structure would otherwise make them have the same requirements. Here D is preceded by B only, whereas E is preceded by both A and B. A dummy is needed to pass the completion of B to the start event for E, without making D depend on A.

For example, D needs B only but E needs both A and B, so a dummy is required.

(1)

(b) Use activity-on-arc: an activity is an arrow, and an event is reached only when every arrow entering it has been completed. The three dashed arrows below are dummies of zero duration. They preserve the different predecessor sets while using the minimum required structure.



In particular, the dummy from the end of B to the end of A makes E wait for A and B. The two later dummies make the start of K wait for E, F, and I, while the common end events for (C, D) and for (G, H) express the remaining joint precedences.

The activity-on-arc network is shown; the dashed arrows are dummies.

(5)

(c) All activities have duration 3 hours, so find the largest number of activities on a route from the start event to the finish event. One critical route is $B \rightarrow D \rightarrow H \rightarrow I \rightarrow K$, containing five activities.

$$5 \times 3 = 15 \text{ hours.}$$

The minimum completion time is 15 hours.

(1)

(d) An activity can be extended by one hour only if it has at least one hour of float. The activities on the critical route cannot be chosen; calculating the event times shows that the remaining activities with sufficient float are listed below.

A, C, E, F, G, J, L or M.

(1)

Question 3

Question 3

Figure 2 is a weighted road network with total weight 153, with roads

$$\begin{aligned} AB = 15, AC = 10, AE = 14, AG = 30, BC = 4, BD = 3, BH = 12, \\ CD = 9, CE = 2, DE = 6, DF = 8, DH = 6, EF = 4, FG = 10, FH = 9, GJ = 3, HJ = 8. \end{aligned}$$

(a) Classify the graph as Eulerian, semi-Eulerian, or neither. (b) Use Dijkstra's algorithm to find the quickest path from A to J. Road AE is then blocked. Ruby must cover each available road at least once, starting and finishing at B: (c) find the roads to repeat and the journey time. Finally, she may start anywhere and finish at A: (d) give the best start junction and the time saved.

Model solution

(a) Euler classification is determined by vertex degrees. A connected graph with exactly two odd-degree vertices is semi-Eulerian. Counting the roads at each junction gives odd degree only at D and G.

The graph is semi-Eulerian, because exactly two vertices, D and G, have odd degree.

(1)

(b)(i) Dijkstra's algorithm is appropriate because every road time is positive. Starting from A, permanently label the unlabelled junction with the smallest tentative time, then update the times through it.

permanent label	final time	new or improved tentative times
A	0	B = 15, C = 10, E = 14, G = 30
C	10	B = 14, D = 19, E = 12
E	12	D = 18, F = 16
B	14	D = 17, H = 26
F	16	G = 26, H = 25
D	17	H = 23
H	23	J = 31
G	26	J = 29
J	29	

Trace back from J: its final label came via G, then F, E, C, and A.

The quickest path is $A \rightarrow C \rightarrow E \rightarrow F \rightarrow G \rightarrow J$.

(5)

(b)(ii)

The shortest time is 29 minutes.

(1)

(c)(i) With AE blocked, remove its weight 14. The odd vertices are then A, D, E, G. Route inspection pairs the odd vertices and repeats the shortest paths for the pairing with the

least total length.

pairing	shortest paths used	extra time (minutes)
(A, D), (E, G)	$A \rightarrow C \rightarrow B \rightarrow D$ and $E \rightarrow F \rightarrow G$	$17 + 14 = 31$
(A, E), (D, G)	$A \rightarrow C \rightarrow E$ and $D \rightarrow H \rightarrow J \rightarrow G$	$12 + 17 = 29$
(A, G), (D, E)	$A \rightarrow C \rightarrow E \rightarrow F \rightarrow G$ and $D \rightarrow E$	$26 + 6 = 32$

The least total is 29, so repeating its two shortest paths makes every vertex even and permits a closed route from B.

Traverse AC, CE, DH, HJ and JG twice.

(4)

(c)(ii) Every available road is walked once, then the selected duplicate roads add the route-inspection total.

$$(153 - 14) + 29 = 168.$$

Ruby's journey time is 168 minutes.

(1)

(d)(i) For an open route ending at A, the start and finish vertices should be the two vertices left odd. Leaving G and A odd means only D and E need pairing.

Ruby should start at G.

(1)

(d)(ii) The new route repeats only DE, of length 6, rather than duplicate roads totalling 29.

$$29 - 6 = 23.$$

She saves 23 minutes.

(1)

Question 4

Question 4

George bakes x sponge cakes and y fruit cakes. The displayed line has intercepts $(0, 20)$ and $(16, 0)$, with the region above it shaded. He must bake at least four fruit cakes; for every two sponge cakes, at most three fruit cakes; and he has 2.8 kg of butter, requiring 200 g per sponge cake and 100 g per fruit cake. He makes £8 profit per sponge cake and £5 per fruit cake. Write the inequalities, graph the feasible region, and use vertices to choose the whole-number production plan that maximises profit.

Model solution

(a) The line through $(0, 20)$ and $(16, 0)$ has equation $5x + 4y = 80$. The shaded side excludes the origin, so the permitted side containing the origin is the $\leq$ side.

$$5x + 4y \leq 80.$$

(1)

(b)(i) Translate each sentence directly. "At least" gives $\geq$; the ratio statement compares $3x$ with $2y$; and convert 2.8 kg to 2800 g before simplifying the butter inequality.

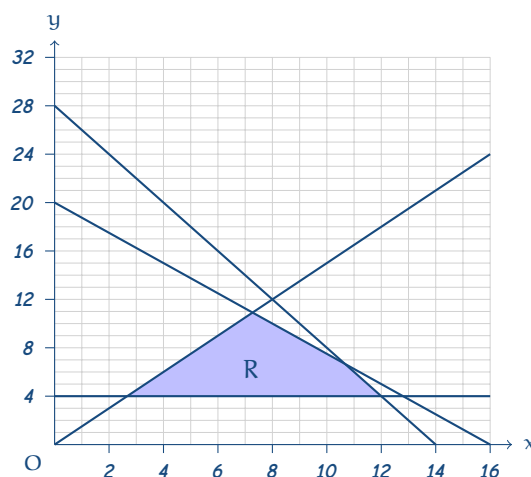
$$\begin{aligned} y &\geq 4, \\ 3x &\geq 2y, \\ 200x + 100y &\leq 2800 \implies 2x + y \leq 28. \end{aligned}$$

$$y \geq 4, \quad 3x \geq 2y, \quad 2x + y \leq 28.$$

(3)

(b)(ii) Draw the boundary lines $y = 4$, $y = \frac{3}{2}x$, and $2x + y = 28$, then retain the common side satisfying all four inequalities, including $5x + 4y \leq 80$. The feasible region R is the quadrilateral with the following vertices, in order:

$$\left(\frac{8}{3}, 4\right), \quad (12, 4), \quad \left(\frac{32}{3}, \frac{20}{3}\right), \quad \left(\frac{80}{11}, \frac{120}{11}\right).$$



The shaded feasible region is R.

(2)

(c) For a linear objective function, the maximum over a feasible region occurs at a vertex. Use $P = 8x + 5y$ and test each vertex of R.

(x, y)	$P = 8x + 5y$
$(\frac{8}{3}, 4)$	$\frac{124}{3} = 41\frac{1}{3}$
$(12, 4)$	116
$(\frac{32}{3}, \frac{20}{3})$	$\frac{356}{3} = 118\frac{2}{3}$
$(\frac{80}{11}, \frac{120}{11})$	$\frac{1240}{11} = 112\frac{8}{11}$

The continuous optimum is $(\frac{32}{3}, \frac{20}{3})$. Cakes must be whole numbers. For each feasible integer x , take the greatest permitted integer y :

x	3	4	5	6	7	8	9	10	11	12
<i>greatest feasible integer y</i>	4	6	7	9	10	10	8	7	6	4
P	44	62	75	93	106	114	112	115	118	116

The largest feasible integer profit is therefore obtained at (11, 6).

George should bake 11 sponge cakes and 6 fruit cakes each day, giving a profit of £118.

(5)