



Edexcel AS Further Mathematics

8FM0/27 - Decision Mathematics 1

November 2021

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2021 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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Question 1

Question 1

The list 3.5, 6.3, 2.9, 5.4, 3.1, 2.8, 3.7, 1.7, 4.1, 3.3, 2.2 is to be sorted into descending order. Use bubble sort, quick sort and first-fit decreasing bin packing as specified. Bins have capacity 11.5.

Model solution

(a)(i) In a descending bubble-sort pass, compare adjacent entries from left to right and swap whenever the left entry is smaller. Thus the largest value is moved to the far left during this pass.

3.5, 6.3, 2.9, 5.4, 3.1, 2.8, 3.7, 1.7, 4.1, 3.3, 2.2 $\rightarrow$ 6.3, 3.5, 5.4, 3.1, 2.9, 3.7, 2.8, 4.1, 3.3, 2.2, 1.7.

After one pass: 6.3, 3.5, 5.4, 3.1, 2.9, 3.7, 2.8, 4.1, 3.3, 2.2, 1.7.

(1)

(a)(ii) There are 11 entries, so one left-to-right pass compares each adjacent pair once: $11 - 1 = 10$ comparisons. Counting the exchanges made in the pass above gives seven swaps.

10 comparisons and 7 swaps.

(2)

(b) The question supplies the list after its second bubble-sort pass. Quick sort is appropriate here because each pivot separates values greater than it from values smaller than it. To keep the required descending order, write the greater group first. Middle-right pivots are underlined.

start : 6.3, 5.4, 3.5, 3.1, 3.7, 2.9, 4.1, 3.3, 2.8, 2.2, 1.7
 pass 1 : 6.3, 5.4, 3.5, 3.1, 3.7, 4.1, 3.3 | 2.9 | 2.8, 2.2, 1.7
 pass 2 : 6.3, 5.4, 3.5, 3.1, 3.7, 4.1, 3.3 | 2.9 | 2.8, 2.2, 1.7
 6.3, 5.4, 3.5, 3.7, 4.1, 3.3 | 3.1 | 2.9 | 2.8 | 2.2 | 1.7
 pass 3 : 6.3, 5.4, 4.1 | 3.7 | 3.5, 3.3 | 3.1 | 2.9 | 2.8 | 2.2 | 1.7
 pass 4 : 6.3 | 5.4 | 4.1 | 3.7 | 3.5 | 3.3 | 3.1 | 2.9 | 2.8 | 2.2 | 1.7

Every remaining group now has one entry, so the partitioning is complete.

6.3, 5.4, 4.1, 3.7, 3.5, 3.3, 3.1, 2.9, 2.8, 2.2, 1.7.

(3)

(c) First-fit decreasing uses the descending list just obtained. For each number, test bins in numerical order and put it into the first bin with sufficient remaining capacity.

bin	contents	total
1	6.3, 4.1	10.4
2	5.4, 3.7, 2.2	11.3
3	3.5, 3.3, 3.1	9.9
4	2.9, 2.8, 1.7	7.4

Four bins, packed as shown.

(3)

(d) To decide whether fewer bins could work, use the total size to obtain a lower bound. No packing can use fewer bins than the total size divided by one bin capacity, rounded up.

$$\frac{6.3 + 5.4 + 4.1 + 3.7 + 3.5 + 3.3 + 3.1 + 2.9 + 2.8 + 2.2 + 1.7}{11.5} = \frac{39}{11.5} = 3.391 \dots$$

At least 4 bins are therefore necessary, and part (c) achieves this bound.

Yes; the four-bin packing is minimum.

(2)

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Question 2

Question 2

For the activity network in Figure 1, activity durations are in hours and $5 < x < 10$. Complete the precedence information, find event times and critical activities, determine x , and give a cascade chart.

Model solution

(a) In an activity-on-arc network, an activity can start only after every activity entering its start event has finished. Reading these dependencies from the network gives:

activity	immediately preceding activities	activity	immediately preceding activities	activity	immediately preceding activities
A	—	G	B, C, E	M	—
B	—	H	B, C, E	N	—
C	—	I	B, C, E	P	—
D	A	J	D, G	Q	—
E	A	K	D, G		
F	C	L	F, I		

The completed precedence table is shown.

(2)

(b) Use a forward pass to find early event times: at each event take the largest incoming early time plus duration. Then use a backward pass from the completion time 38: at each event take the smallest outgoing late time minus duration. Dummy arcs have duration zero. The event positions identify exactly where each pair is placed on Diagram 1.

event position	start	upper left	centre left	lower left	upper middle	lower middle	centre right	upper right
early time	0	7	11	6	17	19	29	29
late time	0	7	11	11	21	19	29	29

For example, event 7 has early time $\max(11 + 4, 19 + 10, 17 + 0) = 29$, while event 5 has late time $\min(32 - 11, 29 - 0, 38 - 16) = 21$.

Early and late event times are as tabulated.

(4)

(c) A critical activity has zero float, so its early start equals its latest start. Following the zero-float route through the network identifies the critical activities.

A, E, I, L, Q.

(1)

(d) The float of J is the latest time its end event can occur, minus its earliest start and duration. Here its start event has early time 17, its duration is 11, and its end event has late time $38 - x$. The stated float is four hours.

$$(38 - x) - 17 - 11 = 4 \quad \Rightarrow \quad x = 6.$$

$$x = 6 \text{ hours.}$$

(1)

(e) A cascade chart records each activity at its earliest possible start, with its float shown by the possible extension to its latest start. The table gives all bars needed to draw it; intervals are measured in hours from time zero. A zero float identifies a critical activity.

activity	earliest start	finish	latest start	float
A	0	7	0	0
B	0	8	3	3
C	0	6	5	5
D	7	12	16	9
E	7	11	7	0
F	6	13	12	6
G	11	17	15	4
H	11	15	25	14
I	11	19	11	0
J	17	28	21	4
K	17	33	22	5
L	19	29	19	0
M	19	25	32	13
N	28	34	32	4
P	29	34	33	4
Q	29	38	29	0

The cascade-chart schedule is given by the start, finish and float data above; its critical activities are A, E, I, L, Q.

(4)

Question 3

Question 3

Donald bakes brownies x , flapjacks y and muffins z . Formulate the given situation as a linear programming problem in x and y only.

Model solution

Income is the quantity of each cake multiplied by its selling price, so initially

$$\text{maximise } P = 1.5x + y + 1.25z.$$

The fixed total of brownies and muffins allows z to be eliminated:

$$x + z = 48 \quad \Rightarrow \quad z = 48 - x.$$

Translate each remaining condition before substituting. "At least 5 brownies for every 3 flapjacks" gives $3x \geq 5y$. Muffins are at most 40% of all cakes, so $z \leq 0.4(x + y + z)$, equivalently $2x + 2y \geq 3z$. The ingredient statement gives

$$\frac{x}{60} + \frac{y}{45} + \frac{z}{35} \leq 1 \quad \Rightarrow \quad 21x + 28y + 36z \leq 1260.$$

Now substitute $z = 48 - x$ and simplify, so the problem is expressed using only x and y :

$$P = 1.5x + y + 1.25(48 - x) = 0.25x + y + 60,$$

$$3x \geq 5y, \quad 5x + 2y \geq 144, \quad 15x - 28y \geq 468.$$

The numbers of cakes cannot be negative, so $x \geq 0$ and $y \geq 0$.

$$\text{Maximise } P = 0.25x + y + 60, \text{ subject to } 3x \geq 5y, \quad 5x + 2y \geq 144, \\ 15x - 28y \geq 468, \quad x \geq 0, \quad y \geq 0.$$

(9)

Question 4

Question 4

Dijkstra's algorithm has been applied to the network in Figure 2. Determine the shortest-path information and then use the given Prim's-algorithm information to find the two missing arc weights.

Model solution

(a) In Dijkstra's table, the permanent label at the destination is the length of the shortest path from the start. The final label at G is 66.

66.

(1)

(b) A node's working value is the path length from A through its predecessor. Subtract the predecessor's permanent value from the new working value to recover each arc weight. For example, $BC = 35 - 25 = 10$ and $FG = 66 - 63 = 3$.

arc	AB	AC	AD	BC	BE	CD	CF	CG	DG	EG	FG
weight	25	37	45	10	31	7	28	39	28	12	3

The arc weights are as tabulated.

(3)

(c) Trace backwards from G through the predecessors that produced its successive smallest labels: $G \leftarrow F \leftarrow C \leftarrow B \leftarrow A$.

$A \rightarrow B \rightarrow C \rightarrow F \rightarrow G$, or ABCFG.

(1)

(d) Prim's order tells us exactly which six arcs form the minimum spanning tree. Adding their weights and using its total weight gives one equation in CE and EF:

$$25 + 10 + 7 + CE + EF + 3 = 80.$$

The specified shortest path from A to F via E is $A \rightarrow B \rightarrow E \rightarrow F$. Its stated length therefore gives

$$25 + 31 + EF = 67 \quad \Rightarrow \quad EF = 11.$$

Substitution into the spanning-tree equation now gives

$$45 + CE + 11 = 80 \quad \Rightarrow \quad CE = 24.$$

$CE = 24$ and $EF = 11$.

(3)