



Edexcel AS Further Mathematics

8FM0/28 - Decision Mathematics 2

Friday 17 May 2024

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2024 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

Contents

Question 1	2
Question 2	3
Question 3	5
Question 4	7

www.mathswithmatt.co.uk

Question 1

Question 1

Figure 1 is a capacitated directed network from S to T ; arc labels are capacities and circled labels are a feasible flow. (a) State its value. (b) Explain why CD and CG cannot both be saturated. (c) Find the capacities of C_1, C_2 . (d) Give an augmenting route of weight 6 saturating BF . It is then applied. (e) Prove the increased flow is maximal.

Model solution

(a) The flow value is the total leaving the source.

$$39 + 16 + 35 = 90.$$

The flow value is 90.

(1) (b) Both arcs would require $21 + 19 = 40$ units leaving C , but its only incoming arc has capacity 35. Thus $35 < 40$, so both cannot be saturated.

CD and CG cannot both be saturated.

(1) (c) Add capacities of arcs crossing each cut from its source side to its sink side.

$$\text{cap}(C_1) = 26 + 18 + 24 + 21 + 19 = 108, \quad \text{cap}(C_2) = 24 + 19 + 17 + 14 + 40 = 114.$$

$$\text{cap}(C_1) = 108, \quad \text{cap}(C_2) = 114.$$

(2) (d) Follow residual capacity; its bottleneck is 6 and BF has residual capacity 6.

$$S \rightarrow B \rightarrow F \rightarrow J \rightarrow T \quad (+6).$$

$SBFJT$, of weight 6.

(1) (e) The augmentation raises the flow to 96. In the augmented network, draw the cut through the saturated arcs EH, EF, BF, DJ, DG and CG . Adding the capacities of the arcs crossing this cut in its forward direction gives

$$\text{cap}(\text{cut}) = 96.$$

The flow value equals this cut capacity; by the max-flow min-cut theorem, maximum flow = minimum cut = 96.

The increased flow is maximal.

(3)

Question 2

Question 2

Players A, B, C, D, E must be assigned one each to rounds P, Q, R, S, T, to maximise score.

	P	Q	R	S	T
A	32	40	35	41	37
B	38	—	40	27	33
C	41	28	37	36	35
D	35	33	38	36	—
E	40	38	—	39	34

Cells BQ, DT, ER are forbidden. (a) Explain the Hungarian-

algorithm modification. (b) Reducing rows first, find an optimal allocation and score.

Model solution

(a) The Hungarian algorithm minimises. Subtract each score from 50, then enter a large value, e.g. 100, in forbidden cells so they cannot be selected.

Use $50 - \text{score}$, then put 100 in BQ, DT, ER.

(2) (b)(i) Form the cost matrix, reduce rows and then columns.

	P	Q	R	S	T
A	18	10	15	9	13
B	12	100	10	23	17
C	9	22	13	14	15
D	15	17	12	14	100
E	10	12	100	11	16

→

	P	Q	R	S	T
A	9	0	6	0	0
B	2	89	0	13	3
C	0	12	4	5	2
D	3	4	0	2	84
E	0	1	90	1	2

Three lines (row A, columns P and R) cover all zeros. The least uncovered entry is 1; subtract it from uncovered entries and add it to double-covered entries.

	P	Q	R	S	T
A	10	0	7	0	0
B	2	88	0	12	2
C	0	11	4	4	1
D	3	3	0	1	83
E	0	0	90	0	1

Four lines now cover zeros, for example rows A, E and columns P, R. Repeat the adjustment by 1.

	P	Q	R	S	T
A	11	0	8	0	0
B	2	87	0	11	1
C	0	10	4	3	0
D	3	2	0	0	82
E	1	0	91	0	1

Five independent zeros give one assignment in every row and column.

$A \rightarrow T, B \rightarrow R, C \rightarrow P, D \rightarrow S, E \rightarrow Q.$

(5) (b)(ii) Use original scores:

$$37 + 40 + 41 + 36 + 38 = 192.$$

Maximum expected score = 192.

(1)

www.mathswithmatt.co.uk

Question 3

Question 3

		X	Y	Z	
	A	4	-2	-5	
Haruki's zero-sum-game pay-offs are	B	1	4	-3	(a) Test stability. Y is removed. (b)
	C	-1	6	1	
	D	-4	5	3	

Give Meera's reduced matrix. (c) Given Meera plays X with probability p , find her strategy, Haruki's game value and options never played. (d) If (C, X) changes to k and values are now equal, find k .

Model solution

(a) Compare Haruki's maximin with Meera's minimax.

$$\text{row minima} = -5, -3, -1, -4 \Rightarrow -1; \quad \text{column maxima} = 4, 6, 3 \Rightarrow 3.$$

Since $-1 \neq 3$, the game is not stable.

(2) (b) Transpose and negate, because Meera's gain is Haruki's loss.

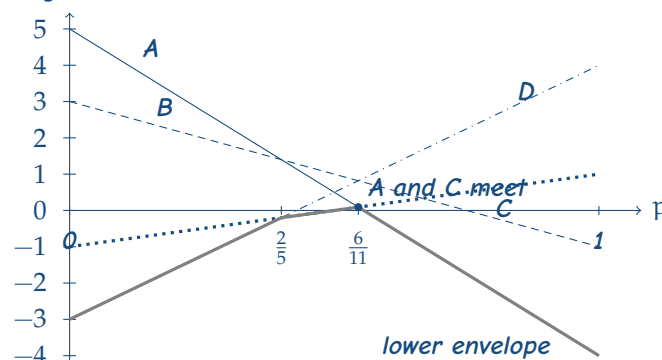
	A	B	C	D
X	-4	-1	1	4
Z	5	3	-1	-3

The reduced matrix is as displayed.

(1) (c)(i) Calculate Meera's expected gain for each Haruki choice. On a graph against p , she maximises the lower envelope.

A	$5 - 9p$
B	$3 - 4p$
C	$-1 + 2p$
D	$-3 + 7p$

expected gain to Meera



The optimum is where active lines A,C meet.

$$5 - 9p = -1 + 2p \Rightarrow p = \frac{6}{11}.$$

Meera plays X with probability $\frac{6}{11}$, Z with $\frac{5}{11}$.

(5) (c)(ii) Meera's value is the intersection height; Haruki's is its negative.

$$-1 + 2\left(\frac{6}{11}\right) = \frac{1}{11}.$$

Haruki's value is $-\frac{1}{11}$.

(1) (c)(iii) Only A and C form the lower envelope.

Haruki never plays B or D.

(1) (d) Equal values in a zero-sum game mean value 0. From A, $5 - 9p = 0$, so $p = \frac{5}{9}$. For C, Meera's gains are $-k$ under X and -1 under Z.

$$-k\left(\frac{5}{9}\right) - \frac{4}{9} = 0 \Rightarrow k = -\frac{4}{5}.$$

$$k = -\frac{4}{5}.$$

(4)

Question 4

Question 4

Peter deposits £D initially and £M monthly. $u_{n+1} = 1.025u_n + 1800$. (a) Find M. Given $u_1 = 6925$, (b) solve the recurrence, (c) find D, and (d) find algebraically when it exceeds £20000.

Model solution

(a) The annual addition is twelve monthly payments.

$$12M = 1800 \Rightarrow M = 150.$$

$$M = \$150.$$

(1) (b) The homogeneous solution is $A(1.025)^n$. A constant trial is appropriate for the constant addition.

$$\lambda = 1.025\lambda + 1800 \Rightarrow \lambda = -72000, \quad u_n = A(1.025)^n - 72000.$$

$$6925 = 1.025A - 72000 \Rightarrow A = 77000.$$

$$u_n = 77000(1.025)^n - 72000.$$

(5) (c) The initial deposit is u_0 .

$$D = 77000 - 72000 = 5000.$$

$$D = \$5000.$$

(1) (d) Use the explicit solution and logarithms; $1.025 > 1$.

$$77000(1.025)^n - 72000 > 20000 \Rightarrow (1.025)^n > \frac{92}{77} \Rightarrow n > \frac{\log(92/77)}{\log(1.025)} = 7.20795 \dots$$

The value first exceeds £20000 after 8 years.

(3)