



Edexcel AS Further Mathematics

8FM0/28 - Decision Mathematics 2

Friday 16 May 2025

Questions and Fully Worked Model Solutions

This booklet places the **question and the solution together**. The mathematics is set out using proper fractions, powers, roots and exact notation, with extra written explanation so that a student can see *why* each step is being taken.

Solutions have been checked against the Pearson Edexcel Summer 2025 mark scheme. The aim is a clear **full-mark model response**, not merely a list of final answers.

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Question 1

Question 1

Five workers A, B, C, D, E are to be assigned one each to five tasks J, K, L, M, N. The profits (in pounds) are

	J	K	L	M	N
A	38	33	40	35	32
B	26	24	27	25	23
C	33	29	—	30	27
D	36	—	41	37	33
E	32	27	31	29	25

Worker C cannot do L, and worker D cannot do K. (a) Explain how to modify the table so that the Hungarian algorithm can be used. (b) Reducing rows first, use the Hungarian algorithm to find the maximum total profit.

Model solution

(a) The Hungarian algorithm is a minimisation algorithm. Convert each permitted profit p into a cost $50 - p$, where 50 is larger than every profit. This reverses the preference order: a larger profit gives a smaller cost. The forbidden cells must never be selected, so replace each dash by a large cost, for example 100.

	J	K	L	M	N
A	12	17	10	15	18
B	24	26	23	25	27
C	17	21	100	20	23
D	14	100	9	13	17
E	18	23	19	21	25

(2)

(b) First reduce each row by its smallest entry. This creates at least one zero in every row without changing which complete assignment has minimum cost. The row minima are 10, 23, 17, 9, 18, respectively. Then reduce each column of the new table by its smallest entry; the column minima are 0, 3, 0, 2, 4.

	J	K	L	M	N
A	2	4	0	3	4
B	1	0	0	0	0
C	0	1	83	1	2
D	5	88	0	2	4
E	0	2	1	1	3

The zeros can be covered by only three lines, for example rows B and D, and column J. Since fewer than five lines are required, an assignment of five independent zeros is not yet possible, so the table is not optimal. The smallest uncovered entry is 1. Subtract 1 from each uncovered entry and add 1 at intersections of two covering lines; leave singly covered

entries unchanged.

	J	K	L	M	N
A	2	3	0	2	3
B	2	0	1	0	0
C	0	0	83	0	1
D	5	87	0	1	3
E	0	1	1	0	2

Now four lines suffice to cover all zeros, for example rows A, B, D and column J. This is still fewer than five, so repeat the improvement. The least uncovered entry is again 1.

	J	K	L	M	N
A	1	2	0	1	2
B	2	0	2	0	0
C	0	0	84	0	1
D	4	86	0	0	2
E	0	1	2	0	2

This time five lines are needed to cover all the zeros, so the Hungarian stopping condition is met and the table is optimal. Choose independent zeros, one in each row and column:

$A \rightarrow L, \quad B \rightarrow N, \quad C \rightarrow K, \quad D \rightarrow M, \quad E \rightarrow J.$

Maximum profit = $40 + 23 + 29 + 37 + 32 = 161.$

Assign A to L, B to N, C to K, D to M, and E to J; the maximum total profit is \$161.

(7)

Question 2

Question 2

Figure 1 is a capacitated directed network from S to T, with a feasible flow shown. (a) State the two conditions for a feasible flow. (b) List the saturated arcs. (c) Find the capacities of cuts C_1 and C_2 . (d) Give a flow-augmenting route increasing the flow by four units. (e) Give the resulting flow pattern. (f) Prove that this is a maximum flow. The network data in Figure 1 are flow/capacity:

arc	SA	SB	AC	AD	BD	BE	CD	CG	DG	DF	EF	FG	FT
flow/capacity	63/70	44/48	31/31	32/45	20/20	24/24	0/8	31/39	20/20	32/36	6/6	21/21	14/14

Model solution

(a) A feasible flow must respect both the pipe capacities and conservation of flow at intermediate vertices.

On every arc, $0 \leq \text{flow} \leq \text{capacity}$. At every vertex other than S and T, total inflow equals total outflow.

(2)

(b) A saturated arc is one whose flow equals its capacity. Reading these directly from the diagram gives

AC, BD, BE, DG, EF, FG, FT.

(1)

(c)(i) The capacity of a cut is the sum of capacities of its forward arcs. For C_1 , these are AC, AD, BD, DF, EH.

$$\text{cap}(C_1) = 31 + 45 + 20 + 6 + 25 = 127.$$

127

(1)

(c)(ii) For C_2 , add the capacities on SB, CD, DF, FT.

$$\text{cap}(C_2) = 48 + 8 + 45 + 80 = 181.$$

181

(1)

(d) A flow-augmenting route may use a reverse arc to cancel part of an existing flow. The route below has residual capacity 4: on $F \rightarrow E$ it uses the reverse of $E \rightarrow F$, whose current flow is 6.

$S \rightarrow A \rightarrow D \rightarrow F \rightarrow E \rightarrow H \rightarrow T.$

(1)

(e) Send four extra units along this route. Hence increase the flows on the forward arcs SA, AD, DF, EH, HT by 4, but decrease EF by 4, because that arc was traversed backwards. All other flows are unchanged. The resulting flow pattern is recorded below as *flow/capacity*.

arc	SA	SB	AC	AD	BD	BE	CD	CG	DG	DF	EF	FG
<i>flow/capacity</i>	67/70	44/48	31/31	36/45	20/20	24/24	0/8	31/39	20/20	36/36	2/6	21/21

For example, flow is conserved at F: $36 + 2 = 21 + 14 + 3 = 38$. The total flow out of S is now $67 + 44 = 111$.

The augmented flow has value 111, with arc flows as in the table.

(2)

(f) To prove maximality, use the max-flow min-cut theorem. In the residual network after augmentation, the vertices reachable from S are S, A, B, D. Thus the forward arcs crossing from this source side to the other side are AC, BE, DG, DF. Each is saturated in the final flow and

$$\text{cap}(\text{cut}) = 31 + 24 + 20 + 36 = 111.$$

The value of the flow is also 111. A flow equal in value to a cut capacity is maximum by the max-flow min-cut theorem.

The flow of value 111 is a maximum flow.

(3)

Question 3

Question 3

The pay-off matrix for Layla is

	X	Y	Z
P	1	-2	2
Q	-4	3	-5
R	-1	1	-3

- (a) Find each player's play-safe strategy and decide whether the game is stable. (b) Remove R. (i) Find Layla's expected pay-off for each of Mohsin's choices if she plays P and Q equally often. (ii) Use a graphical method to find Layla's optimal mixed strategy.

Model solution

(a)(i) Layla protects herself by choosing the row with the greatest row minimum (the maximin). Mohsin protects himself by choosing the column with the smallest column maximum (the minimax).

	X	Y	Z	row minimum
P	1	-2	2	-2
Q	-4	3	-5	-5
R	-1	1	-3	-3
column maximum	1	3	2	

The greatest row minimum is -2, in row P, and the smallest column maximum is 1, in column X.

Layla's play-safe strategy is P; Mohsin's play-safe strategy is X.

(2)

(a)(ii) A stable solution requires the maximin and minimax values to be equal.

There is no stable solution, since $-2 \neq 1$.

(1)

(b)(i) With P and Q each played with probability 0.5, take the mean of the two pay-offs in each column.

$$X : 0.5(1) + 0.5(-4) = -1.5,$$

$$Y : 0.5(-2) + 0.5(3) = 0.5,$$

$$Z : 0.5(2) + 0.5(-5) = -1.5.$$

Expected pay-offs: X : -1.5, Y : 0.5, Z : -1.5.

(1)

(b)(ii) Let p be the probability that Layla plays P, so $1 - p$ is the probability that she plays Q, where $0 \leq p \leq 1$. For each fixed choice by Mohsin, Layla's expected pay-off is a straight

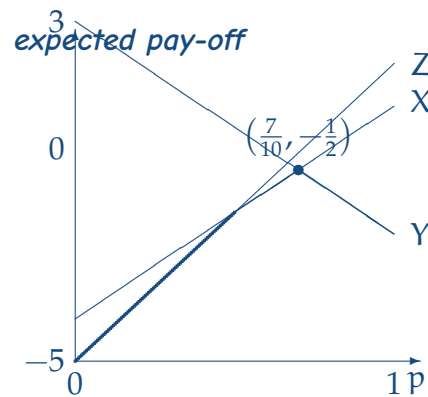
line in p :

$$X : p(1) + (1-p)(-4) = 5p - 4,$$

$$Y : p(-2) + (1-p)(3) = -5p + 3,$$

$$Z : p(2) + (1-p)(-5) = 7p - 5.$$

On a graph of expected pay-off against p , Mohsin will choose the lowest of these lines, so Layla must maximise the lower envelope. The graph below shows all three lines for $0 \leq p \leq 1$; the heavier sections form the lower envelope.



Thus the required graphical maximum of the lower envelope is the intersection of the X and Y lines at $(\frac{7}{10}, -\frac{1}{2})$. The plotting values are:

p	$X : 5p - 4$	$Y : -5p + 3$	$Z : 7p - 5$
0	-4	3	-5
$\frac{7}{10}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{10}$
1	1	-2	2

These points give the three straight lines for the graphical construction; the lower envelope is formed by Z first, then X, then Y.

$$7p - 5 = 5p - 4 \Rightarrow p = \frac{1}{2},$$

so the lower envelope changes from Z to X at $p = \frac{1}{2}$. Its highest point occurs where X and Y meet:

$$5p - 4 = -5p + 3 \Rightarrow 10p = 7 \Rightarrow p = \frac{7}{10}.$$

$$\text{At } p = \frac{7}{10} : \quad 5p - 4 = -\frac{1}{2}, \quad -5p + 3 = -\frac{1}{2}, \quad 7p - 5 = -\frac{1}{10}.$$

So the lower envelope has value $-\frac{1}{2}$ there, and Mohsin is indifferent between X and Y.

Layla should play P with probability $\frac{7}{10}$ and Q with probability $\frac{3}{10}$.

(7)

Question 4

Question 4

A sequence $\{u_n\}$, for $n \geq 1$, satisfies

$$4u_{n+1} + 2u_n = 3n - 5, \quad u_1 = -2.$$

(a) Solve the recurrence relation. (b) Hence determine, with justification, how many terms are negative.

Model solution

(a) First divide by 4, then solve the associated homogeneous recurrence and add a particular solution. The non-homogeneous term is linear in n , so try a linear particular solution $u_n = an + b$.

$$u_{n+1} = -\frac{1}{2}u_n + \frac{3}{4}n - \frac{5}{4} \Rightarrow r = -\frac{1}{2},$$

$$u_n^{(c)} = A \left(-\frac{1}{2}\right)^n.$$

Substitute $u_n^{(p)} = an + b$ into the full recurrence:

$$4(a(n+1) + b) + 2(an + b) = 3n - 5.$$

Comparing coefficients of n , and then constants, gives

$$6a = 3 \Rightarrow a = \frac{1}{2},$$

$$4a + 6b = -5 \Rightarrow 2 + 6b = -5 \Rightarrow b = -\frac{7}{6}.$$

Therefore

$$u_n = A \left(-\frac{1}{2}\right)^n + \frac{1}{2}n - \frac{7}{6}.$$

Use $u_1 = -2$ to find A :

$$-2 = -\frac{1}{2}A + \frac{1}{2} - \frac{7}{6} \Rightarrow -\frac{1}{2}A = -\frac{4}{3} \Rightarrow A = \frac{8}{3}.$$

$$u_n = \frac{8}{3} \left(-\frac{1}{2}\right)^n + \frac{1}{2}n - \frac{7}{6}.$$

(6)

(b) Separate even and odd values of n , because $\left(-\frac{1}{2}\right)^n$ changes sign. For even n , the exponential term is positive. At $n = 2$,

$$u_2 = \frac{8}{3} \left(\frac{1}{2}\right)^2 + 1 - \frac{7}{6} = \frac{1}{2} > 0.$$

For all later even n , both $\frac{8}{3}(\frac{1}{2})^n$ and $\frac{1}{2}n - \frac{7}{6}$ are positive, so every even term is positive. For odd n ,

$$u_1 = -2 < 0, \quad u_3 = \frac{8}{3} \left(-\frac{1}{2}\right)^3 + \frac{3}{2} - \frac{7}{6} = 0.$$

For subsequent odd n , the negative exponential term becomes closer to zero while $\frac{1}{2}n - \frac{7}{6}$ increases, so all are positive.

There is exactly one negative term, namely u_1 .

(3)

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